CM 12

2018년 2월 19일 월요일 오후 7:54

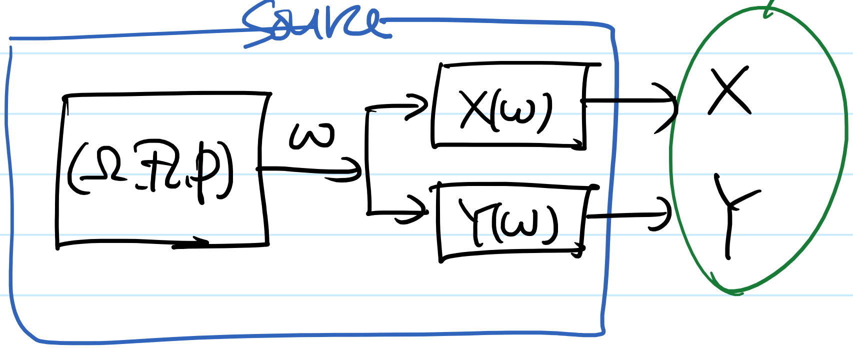
☐ Ann.

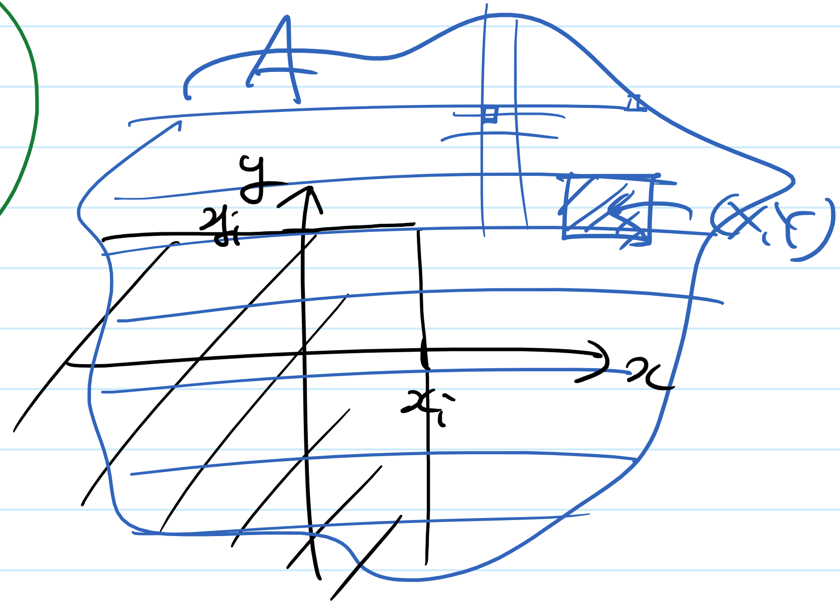
- HW. , state...

☐ Review

Ch. 6

bivariate random variable

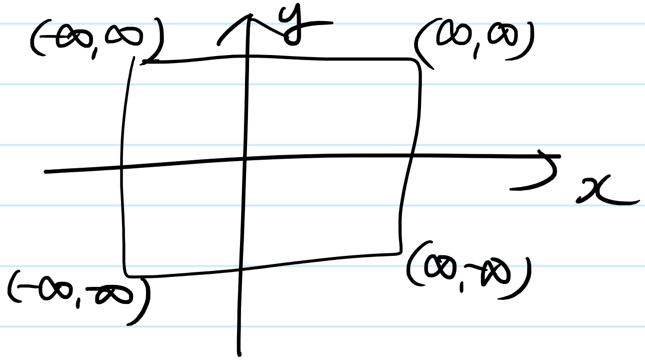




1) $(\Omega, \mathcal{F}, P)$, $X(\omega), Y(\omega)$
2) $(\mathbb{R}^2, \mathcal{B}^2, P_{X,Y})$
3) $\because \forall A \in \mathcal{B}^2$ $(\Rightarrow P_{X,Y}(A))$

A is obtained by uncountably infinite set operations on $\{(x,y) : x \le x_i, y \le y_i\}$

$i \in I$ ← countable, uncountable



Joint CDF

$$F_{X,Y}(x,y) \triangleq \Pr(X \le x, Y \le y)$$

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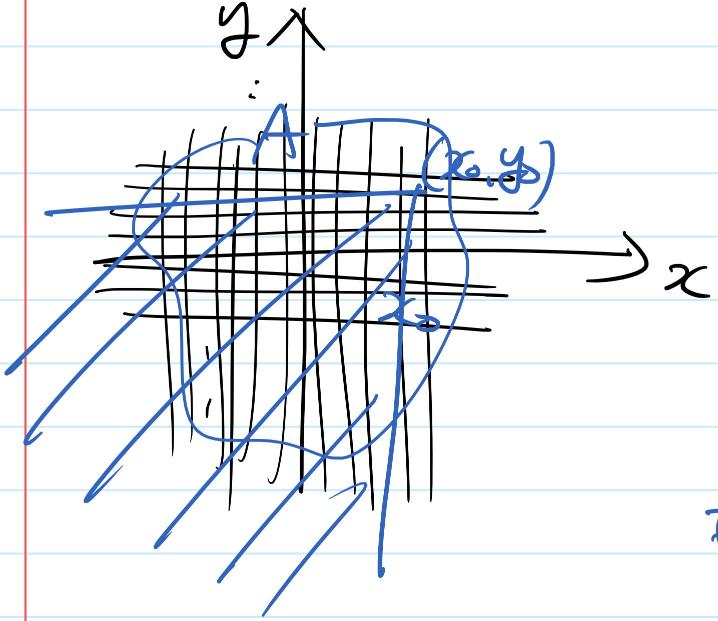
Ⅱ Joint PDF



Scatter plot (x_1, y_1) ⋮ (x_{106}, y_{106})

2-D histogram

$f_{X,Y}(x,y)$ ··· ← intuitively defined.

$$F_{X,Y}(x,y) = \int_{-\infty}^{y}\int_{-\infty}^{x} f_{X,Y}(\alpha,\beta)\,d\alpha\,d\beta$$

- Leibnitz Rule

$$x \to \boxed{b(x)} \xrightarrow{y} \boxed{F(y)} \to z$$

$$\int f(x)\,dx = F(x) + C \quad \left(\Rightarrow \frac{dF(x)}{dx} = f(x)\right)$$

HW generalized version

$$\frac{d}{dx}\int_{a(x)}^{b(x)} f(x)\,dx = \frac{d}{dx}\Big(F(b(x)) - F(a(x))\Big) = f(b(x))\,b'(x) - f(a(x))\,a'(x)$$

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- $$\begin{cases} \dfrac{\partial F_{X,Y}}{\partial x} = \displaystyle\int_{-\infty}^{y} f_{X,Y}(x,\beta)\,d\beta \\ \dfrac{\partial F_{X,Y}}{\partial y} = \displaystyle\int_{-\infty}^{x} f_{X,Y}(\alpha,y)\,d\alpha \end{cases}$$

$$\frac{\partial}{\partial x}\boxed{F_{X,Y}(x,\infty)} = \frac{\partial}{\partial x}\Pr(X\le x, Y\le\infty) = \frac{\partial}{\partial x}\Pr(X\le x) = \frac{d}{dx}F_X(x) = f_X(x)$$

$$\begin{cases} f_X(x) = \displaystyle\int_{-\infty}^{\infty} f_{X,Y}(x,\beta)\,d\beta \\ f_Y(y) = \displaystyle\int_{-\infty}^{\infty} f_{X,Y}(\alpha,y)\,d\alpha \end{cases}$$

✓ - $\dfrac{\partial^2}{\partial x\,\partial y}F_{X,Y} = f_{X,Y}(x,y)$

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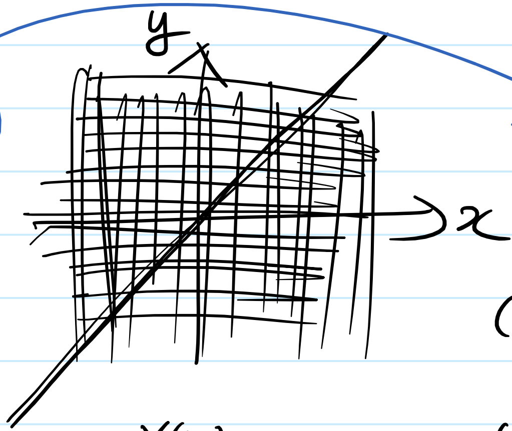
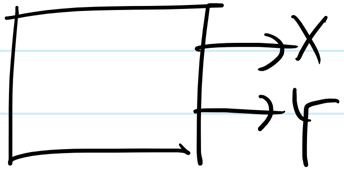
I. $X=Y$ 와 $\{X=Y\}$의 차이.

- $X=Y$ ($\Leftrightarrow$ $X(\omega)=Y(\omega)$, $\forall \omega \in \Omega$) "$X(\omega)$ equals $Y(\omega)$ everywhere." "X equals Y surely."



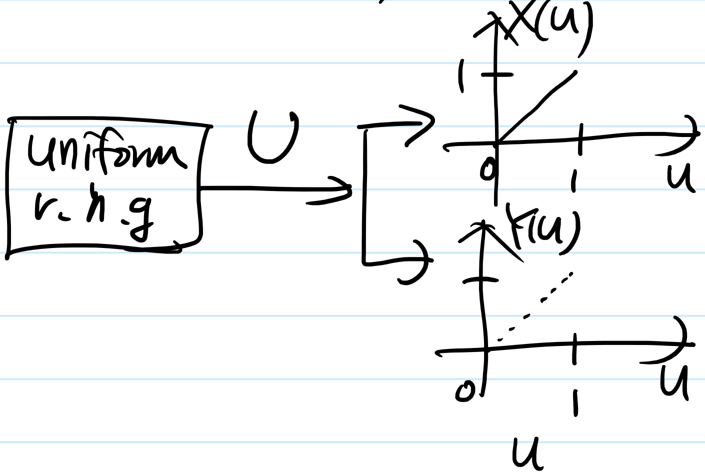
$Pr(\{X=Y\})=1$ "$X=Y$ with probability One w.p. 1" "$X=Y$ almost surely"



(x_1, y_1)

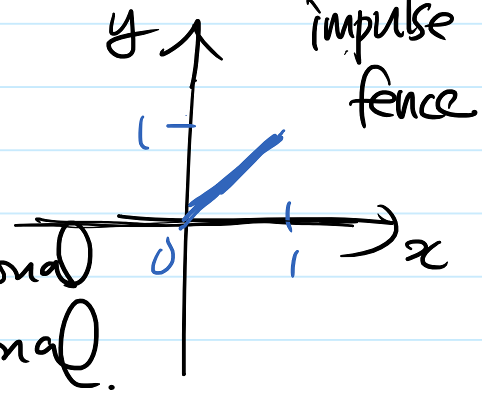
$\vdots$

(x_{10^6}, y_{10^6})

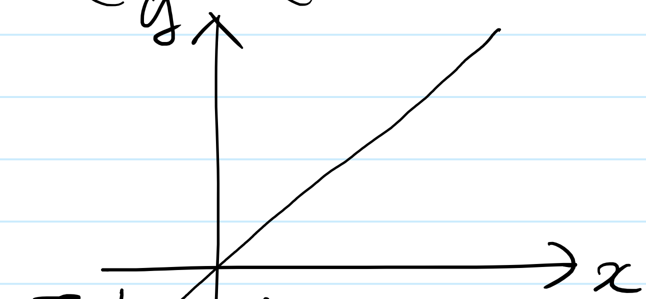


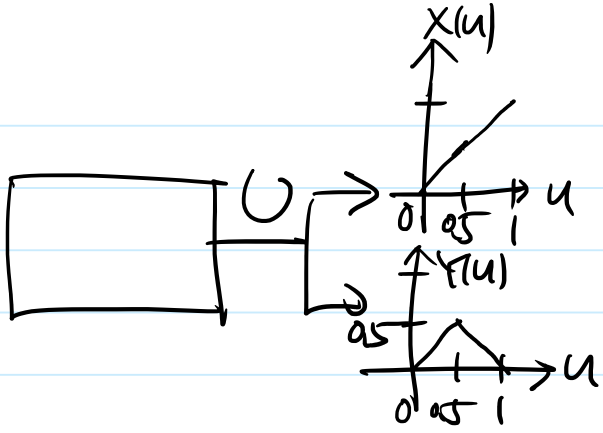
$$X(u)=u$$

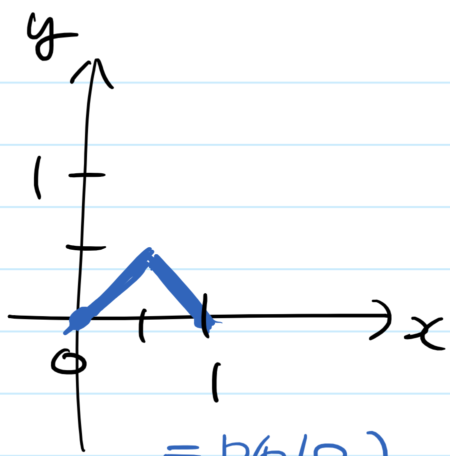
$$Y(u)=\begin{cases} u, & \text{for } u \text{ irrational} \\ 0, & \text{for } u \text{ rational.} \end{cases}$$



$\{X=Y\}$ ← event







□ Independence

Independence of two events A & B

$$P(A\cap B)=P(A)P(B|A)=P(B)P(A|B)$$

$$P(B|A)=P(B|\Omega)=P(B)$$

$$P(A|B)=P(A|\Omega)=P(A)$$

" " " random variables X & Y.

Def. " any event in terms of X &
" " " " " Y are independent events."

Lemma. Independent r. variables exist!

" . iff $F_{X,Y}(x,y)=\Pr(\{X\le x\}\cap\{Y\le y\})=\Pr(X\le x)\Pr(Y\le y)$
$=F_X(x)F_Y(y),\quad \forall (x,y)\in\mathbb{R}^2$

"X & Y are independent" iff $f_{X,Y}(x,y)=f_X(x)f_Y(y)$, " "

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II Theorem 6-1