# *Complex Baseband Representation of Real Band-pass Signals: Main Result*

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- Title dissection
  - Representation
    - $x(t)$: a time function
    - $X(f) \triangleq \mathcal{F}\{x(t)\}$: a frequency domain representation of $x(t)$
  - Complex baseband signal $\stackrel{?}{\equiv}$ real bandpass signal
  - In what sense?
  - Why **real bandpass** signals and systems do we deal with?
  - What is the advantage of handling **complex baseband** representation of real bandpass signals and systems?

- Why real bandpass signals?
  - Electrical signals are real-valued.

$$\begin{cases} \text{current} \\ \text{voltage} \end{cases}$$

  - Fig.

- In many systems, spectrum is a valuable resource. For example, in a cable for CATV, we want to put as many channel signals as possible.
- Fig.

- What is the advantage of handling complex baseband signals instead of real bandpass signals?
  - Two real numbers are equivalent to one complex number
    - There are many cases where handling a complex number is much easier than handling two real numbers.
  - In real bandpass signaling, it can be shown that two real baseband signals contain "all" the information of the real bandpass signal

$$\underset{\text{one real bandpass}}{x(t)} \equiv \underset{\text{two real baseband}}{(x_c(t), x_s(t))} \equiv \underset{\text{one complex baseband}}{z(t) \triangleq x_c(t) + \mathrm{j}x_s(t)}$$

- The main result
  - Given
    - $x(t)$, a real-valued "bandpass" signal with finite energy and
    - $X(f) \triangleq \mathcal{F}\{x(t)\}$ that has a finite support $f \in (0, B]$,
  - we choose $f_c \in (B/2, B)$ and $\theta \in [0, 2\pi)$. ($\leftarrow$ NOT unique!)
  - Then, there **exists** a unique pair $x_c(t)$ and $x_s(t)$ of real-valued band-limited **baseband** signals such that

$$\begin{aligned} x(t) &= x_c(t)\cos(2\pi f_c t + \theta) - x_s(t)\sin(2\pi f_c t + \theta) \\ &= \mathrm{Re}\{(x_c(t) + \mathrm{j}x_s(t))e^{\mathrm{j}(2\pi f_c t + \theta)}\} \end{aligned}$$

    where $\mathrm{j} \triangleq \sqrt{-1}$.

  - $e^{\mathrm{j}\phi} = \cos\phi + \mathrm{j}\sin\phi$ (Euler's identity)

- $x_c(t)\cos(2\pi f_c t + \theta)$: in-phase component of $x(t)$
  $-x_s(t)\sin(2\pi f_c t + \theta)$: quadrature component of $x(t)$. ($2\pi/4 = 90^o$)

- The complex baseband signal $x_c(t) + \mathrm{j}x_s(t)$, the center frequency $f_c$, and the phase $\theta$ contain all the information carried by $x(t)$.

- The complex baseband signal $x_c(t) + \mathrm{j}x_s(t)$ is called **the complex envelope** of the real bandpass signal $x(t)$ and often denoted by $x_l(t)$.

- $x_c(t)$: real part of $x_l(t)$. Often called the in-phase component of $x_l(t)$
  $x_s(t)$: imaginary part of $x_l(t)$. Often called the quadrature component of $x_l(t)$.

– It is obvious that,
if $x_c(t)$ and $x_s(t)$ are real-valued baseband signals having bandwidth $B'$,
then $x(t) \triangleq x_c(t)\cos(2\pi f_c t + \theta) - x_s(t)\sin(2\pi f_c t + \theta)$ is a real-valued bandpass signal for $f_c > B'$.

– The main result says that a kind of the converse of this statement is also true.

– How can we prove?

  * In the time domain, we have no idea.
    We tackle the problem in the frequency domain.