EECE 695F: Advanced Digital Communications

Midterm Exam # 2 (Spring 2015)

Time allowed: 4.5 hours

Name: ______________________________________

Problem 1: _____ / 20

Problem 2: _____ / 20

Problem 3: _____ / 20

Problem 4: _____ / 10

Problem 5: _____ / 20

Problem 6: _____ / 5

Problem 7: _____ / 5

Total: _____ / 100

Problem 1. (50 points) Answer the following questions. You can answer either in English or in Korean. Use more than 3 sentences to each answer.

(a) (5 points) Sketch the block diagram of a transmitter that employs a digital baseband modem, two DACs, a two-stage upconverter to the RF, and an antenna. Properly place filters where necessary.

(b) (5 points) Explain the difference between upper-side tuning and lower-side tuning in two-stage upconverter in (a).

(c) (5 points) Sketch the block diagram of a heterodyne receiver. Properly place filters where necessary.

(d) (5 points) Explain what is the image band problem and how we solve it.

Problem 2. (20 points) Answer the following questions. You can answer either in English or in Korean.

(a) (5 points) Describe Nyquist's sampling theorem. What is the Nyquist rate in this theorem?

(b) (5 points) Describe Nyquist's criterion and theorem for zero ISI. What is the Nyquist condition in this theorem?

(c) (5 points) Describe the sampling and interpolation in the time and the frequency domains.

(d) (5 points) Describe the upsampling and downsampling in the time and the frequency domains.

Problem 3. (20 points) Let $(x[n])_{n=-\infty}^{\infty}$ be a deterministic DT signal and $p(t)$ be a complex-valued baseband CT signal of finite energy $E > 0$ and bandwidth $B > 0$. When a complex-valued CT signal $y(t)$ is defined as

$$y(t) \triangleq \sum_{n=-\infty}^{\infty} x[n]p(t - nT),$$

answer the following questions.

(a) (1 point) State the definition of the DTFT $X(e^{j2\pi f})$ of $(x[n])_{n=-\infty}^{\infty}$.

(b) (1 point) State the definition of the CTFT $P(f)$ of $p(t)$.

(c) (5 points) When $p(t)$ is a sinc pulse with bandwidth $1/2T$, find and sketch $P(f)$. Recall that the energy of $p(t)$ is E.

(d) (1 points) In (c), find and sketch $p(t)$.

(e) (5 points) When $x[0] = 1, x[1] = -1$, and $x[n] = 0$ elsewhere; and $p(t)$ is a sinc pulse with bandwidth $1/2T$ as found in (c) and (d); sketch $y(t)$. What are the values of $y(t)$ at $t = -T, 0, T/2, T$, and $2T$?

(f) (1 point) When $x[0] = 1, x[1] = 1$, and $x[n] = 0$ elsewhere; and $p(t)$ is a sinc pulse with bandwidth $1/2T$ as found in (c) and (d); sketch $y(t)$. What are the values of $y(t)$ at $t = -T, 0, T$, and $2T$?

(g) (5 points) Derive the CTFT $Y(f)$ of $y(t)$ in terms of $X(e^{j2\pi f})$, $P(f)$, and T.

(h) (1 points) Sketch the CTFT $Y(f)$ of $y(t)$, when $X(e^{j2\pi f})$ is a triangular pulse with height 1 and support $-1/2 \leq f < 1/2$, and $p(t)$ is a sinc pulse with bandwidth $1/2T$ as found in (c) and (d).

Problem 4. (10 points) In Problem 3, when a real-valued CT signal $z(t)$ is defined as

$$z(t) \triangleq \mathrm{Re}\{y(t)\exp(\mathrm{j}2\pi f_c t)\},$$

answer the following questions.

(a) (5 points) Derive the CTFT $Z(f)$ of $z(t)$ in terms of the CTFT $Y(f)$ of $y(t)$.

(b) (1 point) State the CTFT $Z(f)$ of $z(t)$ in terms of the DTFT $X(e^{\mathrm{j}2\pi f})$ of $(x[n])_{n=-\infty}^{\infty}$, the CTFT $P(f)$ of $p(t)$, and T.

(c) (1 point) Sketch $Z(f)$ when $f_c \geq B$ and $y(t)$ is given as Problem 3-(h).

(d) (1 point) Repeat (c), when $f_c = 1/(4T)$.

(e) (1 point) Find the energy of $y(t)$ in terms of $X(e^{\mathrm{j}2\pi f})$, $P(f)$, and T.

(f) (1 point) Find the energy of $z(t)$ in terms of $X(e^{\mathrm{j}2\pi f})$, $P(f)$, and T. Compare this value with that found in (e) when $f_c \geq B$.

Problem 5. (20 points) Let $(X[n])_{n=-\infty}^{\infty}$ be a proper-complex wide-sense stationary DT random process and $p(t)$ be a complex-valued baseband CT signal of finite energy E and bandwidth B. When the auto-correlation $\tilde{p}(\tau)$ of $p(t)$ is defined as

$$\tilde{p}(\tau) \triangleq \int_{-\infty}^{\infty} p(t)p^*(t-\tau)dt$$

and when $Y(t)$ is defined as

$$Y(t) \triangleq \sum_{n=-\infty}^{\infty} X[n]p(t-nT),$$

for some $T > 0$, answer the following questions.

(a) (1 point) When the auto-correlation function of $(X[n])_{n=-\infty}^{\infty}$ is defined as

$$R_{XX}[m] \triangleq \mathbb{E}\{X[n]X^*[n-m]\},$$

what is a property of $R_{XX}[m]$ as a function of m?

(b) (1 point) State the definition of the pseudo auto-correlation function $\tilde{R}_{XX}[m]$ of $(X[n])_{n=-\infty}^{\infty}$. What is a property of $\tilde{R}_{XX}[m]$ as a function of m?

(c) (3 points) State the relation between the power spectral density $S_{XX}(e^{j2\pi f})$ and the auto-correlation function $R_{XX}[m]$ of $(X[n])_{n=-\infty}^{\infty}$. What is a property of $S_{XX}(e^{j2\pi f})$ as a function of f? Is it always even symmetrical?

(d) (3 points) When the auto-correlation function of $Y(t)$ is defined as

$$R_{YY}(t, t-\tau) \triangleq \mathbb{E}\{Y(t)Y^*(t-\tau)\}$$

find $R_{YY}(t, t-\tau)$ in terms of $R_{XX}[m]$, $p(t)$, and T.

(e) (5 points) Show first that

$$\sum_{n=-\infty}^{\infty} p(t-nT)p^*(t-nT-(\tau-mT))$$

is periodic with period T, and then find its time average in terms of $\tilde{p}(\tau)$.

(f) (2 points) Using the results in (d) and (e), find the time-averaged auto-correlation function $\mathcal{R}_{YY}(\tau)$ of $Y(t)$ in terms of $R_{XX}[m]$, $\tilde{p}(t)$, and T.

(g) (1 point) Find the CTFT of $\tilde{p}(\tau)$ in terms of $P(f)$ and T.

(h) (4 points) Using the results in (f) and (g), derive the PSD $S_{YY}(f)$ in terms of $S_{XX}(e^{j2\pi f})$, $P(f)$, and T.

Problem 6. (5 points) In Problem 5, when a real-valued CT signal $Z(t)$ is defined as

$$Z(t) \triangleq \mathrm{Re}\{Y(t)\exp(\mathrm{j}2\pi f_c t)\},$$

answer the following questions.

(a) (1 point) Sketch the PSD of $Y(t)$ when $S_{XX}(e^{\mathrm{j}2\pi f})$ is a triangular pulse with height 1 and support $-1/2 \leq f < 1/2$, and $p(t)$ is a sinc pulse with bandwidth $1/2T$ as found in Problem 3-(c) and 3-(d).

(b) (1 point) State the PSD of $S_{ZZ}(f)$ of $Z(t)$ in terms of the PSD $S_{XX}(e^{\mathrm{j}2\pi f})$ of $(X[n])_{n=-\infty}^{\infty}$, the CTFT $P(f)$ of $p(t)$, and T.

(c) (1 point) Sketch $S_{ZZ}(f)$ when $f_c \geq B$ and $Y(t)$ is given as (a) and $f_c \geq 1/(2T)$.

(d) (1 point) Find the power of $Y(t)$ in (a).

(f) (1 point) Find the power of $Z(t)$ in (c). Compare this value with that found in (e) when $f_c \geq B$.

Problem 7. (5 points) Let $(X_1[n])_{n=-\infty}^{\infty}$ and $(X_2[n])_{n=-\infty}^{\infty}$ be proper-complex wide-sense stationary DT random processes and independent to each other. When $Z_k(t)$ for $k = 1$ and 2 are defined as

$$Z_k(t) \triangleq \mathrm{Re}\{\sum_{n=-\infty}^{\infty} X_k[n]p(t-nT)\exp(\mathrm{j}2\pi f_k t)\}$$

where $f_1 > f_2 \gg 1/(2T)$ and other parameters are similarly defined as before, consider a real-valued bandpass signal given by

$$W(t) = Z_1(t) + Z_2(t).$$

Answer the following questions.

(a) (1 point) When $(X_1[n])_{n=-\infty}^{\infty}$ consists of i.i.d. proper-complex random variables of mean zero and variance one, and $p(t)$ is a rectangular pulse with duration T and height 1; sketch the PSD $S_{Z_1Z_1}(f)$ of $Z_1(t)$.

(b) (1 point) Find the PSD $S_{WW}(f)$ of $W(t)$ in terms of the PSDs $S_{Z_1Z_1}(f)$ and $S_{Z_2Z_2}(f)$ of $Z_1(t)$ and $Z_2(t)$.

(c) (1 point) In (a), when $(X_2[n])_{n=-\infty}^{\infty}$ is independent and identically distributed to $(X_1[n])_{n=-\infty}^{\infty}$, and $f_1 - f_2 \gg 0$, sketch $S_{WW}(f)$ of $Z_W(t)$.

(d) (1 point) Repeat (c), when $f_1 - f_2 = 1/T$.

(e) (1 point) Explain what happens to the PSD of $W(t)$ if $(X_1[n])_{n=-\infty}^{\infty}$ and $(X_2[n])_{n=-\infty}^{\infty}$ are correlated?