EECE 695F: Advanced Digital Communications

Final Exam (Spring 2015)

Time allowed: 4.5 hours

Name: ______________________________

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Problem 1. (40 points) Answer the following questions. You can answer either in English or in Korean.

(a) (5 points) Discuss when the direct upconversion transmitter is implementable.

(b) (5 points) Explain in the time and the frequency domains about sampling, interpolation, upsampling, and downsampling.

(c) (5 points) Describe a QAM signal in the time domain and in the frequency domain. For the frequency-domain description, provide its PSD.

(d) (5 points) Describe the shape of the pdf of an improper-complex Gaussian random variable.

(e) (5 points) Explain what are the Nyquist sampling theorem and the Nyquist theorem for zero ISI.

(f) (5 points) Explain how a DFE is designed when a linear feedforward filter is given. You may provide examples with MMSE and ZF criteria.

(g) (5 points) Sketch a block diagram of a CT PLL that is used in some receivers to regenerate the carrier signal without decision direction. What does the acronym VCO represent? What is the function of the VCO in the PLL?

(h) (5 points) State different levels of synchronization in the receiver.

Problems 2–5. Suppose that a real-valued band-limited transmitted signal is given by

$$X(t) = \mathrm{Re}\left\{ \left(\sum_{m=-\infty}^{\infty} d[m]s(t - mT_s - \Delta) \right) Ae^{\mathrm{j}(2\pi f_c t + \theta_c)} \right\}, \tag{1}$$

where $A > 0$ and θ_c are, respectively, the magnitude and the phase of the carrier with frequency f_c and that the channel is given by

$$h(t) = \sum_{l=1}^{L} h_l \delta(t - \tau_l), \tag{2}$$

where h_l and τ_l are the gain and the propagation delay of the lth path, respectively. When the received signal that is observed just before the down-conversion mixing is modeled as

$$Y(t) = X(t) * h(t) + N(t) \tag{3}$$

where $N(t)$ is a real-valued bandpass AWGN noise with two-sided PSD $N_0/2$, answer the following questions.

Problem 2. (20 points) In this problem, you are asked questions on the transmitter side. Assume that the symbol sequence $(d[n])_n$ in Eq. (1) consists of i.i.d. random variables with $\Pr(d[n] = 1) = \Pr(d[n] = -1) = 1/2$ and that the transmit pulse $s(t)$ is the square-root raised cosine pulse with roll-off factor 0.5.

(a) (5 points) Sketch the block diagram of a transmitter that consists of a digital baseband modem, all real-input real-output components, and no intermediate frequency.

(b) (3 points) Sketch the block diagram of an upper-side tuned transmitter that consists of a digital IF modem with IF frequency f_{IF} and all real-input real-output components.

(c) (2 points) Sketch the block diagram of a lower-side tuned transmitter that consists of a digital IF modem with IF frequency f_{IF} and all real-input real-output components.

(d) (5 points) When the passband bandwidth of $X(t)$ is 20 [MHz], find T_s and sketch $|S(f)|^2$.

(e) (5 points) When $|S(0)|^2 = B > 0$ in (d), find the average transmit power and the average bit energy E_b in the transmitted signal in terms only of A and B.

Problem 3. (20 points) In this problem, you are asked questions on the channel. Define the complex envelope $\tilde{X}(t)$ of $X(t)$ such that

$$X(t) = \mathrm{Re}\left\{\tilde{X}(t)e^{\mathrm{j}2\pi f_c t}\right\}.$$

(a) (5 points) Find $\tilde{X}(t)$.

(b) (1 point) Find $X(t) * h(t)$ in terms of $X(t)$ and h_l.

(c) (2 points) Derive the complex envelope of $X(t) * h(t)$ in terms of $\tilde{X}(t)$ and h_l.

(d) (2 points) Using the result in (c), find $\tilde{h}_l$ that satisfies

$$X(t) * h(t) = \mathrm{Re}\left\{\left[\tilde{X}(t) * \left(\sum_{l=1}^{L} \tilde{h}_l \delta(t - \tau_l)\right)\right] e^{\mathrm{j}2\pi f_c t}\right\}.$$

Is $\tilde{h}_l$ real-valued? Is it the same as h_l?

(e) (3 points) In (c), how can you approximate the complex envelope of $X(t) * h(t)$ in terms of $\tilde{X}(t)$ and h_l, when $\tau_1 = 0$ and $\max_{i \neq j} |\tau_i - \tau_j| \ll 1/W$?

(f) (2 points) In (e), justify the Rayleigh frequency-flat channel model.

(g) (5 points) Discuss a channel model when L is very large but $\max_{i \neq j} |\tau_i - \tau_j| \ll 1/W$ is not satisfied.

Problem 4. (20 points) In this problem, you are asked questions on the receiver side.

(a) (5 points) Sketch the block diagram of a receiver that consists of a direct down-converter and a digital baseband modem. Specify which signal is $Y(t)$. Also describe the characteristic of a bandpass filter that needs to be used before the down-conversion.

(b) (5 points) When the channel is well approximated to a frequency-flat channel as in Problem 3-(e), find $Y(t)$ in terms of $d[n]$, $s(t)$, and A but without using the Euler number e.

(c) (8 points) In (a) and (b), how can you reduce the number of the branches that are connected to the baseband modem? Sketch your new block diagram.

(d) (2 points) In (c), what happens if $s(t)$ is not real-valued but complex-valued?

Problem 5. (15 points) In this problem, you are asked questions on the noise $N(t)$. Let $\tilde{N}(t)$ be the complex envelope of $N(t)$. Define $N_I(t)$ and $N_Q(t)$ as the real and the imaginary parts of $\tilde{N}(t)$, respectively.

(a) (2 points) Sketch the PSD of the real-valued bandpass AWGN $N(t)$.

(b) (2 points) Find the relation among $N(t)$, $N_I(t)$, and $N_Q(t)$ with and without using the Euler number e.

(c) (3 points) Sketch the PSDs of $N_I(t)$ and $N_Q(t)$. Also sketch the cross PSD of $N_I(t)$ and $N_Q(t)$.

(d) (3 points) Find the probability distribution of the random vector $\underline{N}_I = [N_I(0), N_I(1/W), N_I(2/W), N_I(3/W)]^{\mathcal{T}}$. Repeat it for the random vector $\underline{N}_Q = [N_Q(0), N_Q(1/W), N_Q(2/W), N_Q(3/W)]^{\mathcal{T}}$.

(e) (5 points) In (d), write the probability distribution density function of the random vector $\underline{N} = \underline{N}_I + \underline{N}_Q$.

Problem 6. (15 points) In this problem, you are asked questions on the baseband modem. Assume that the channel is frequency-flat but the receiver does not know the carrier phase of the received signal.

(a) (1 point) Find the complex baseband signal $\tilde{Y}(t)$ that is effectively processed by the baseband modem in terms of $\tilde{X}(t), h, \theta$, and $\tilde{N}(t)$.

(b) (4 points) Repeat (a) in terms of $A, d[n], s(t), \Delta, h, \theta, \theta_c$, and $\tilde{N}(t)$.

(c) (4 points) Sketch a matched filter front end that processes $\tilde{Y}(t)$ found in (b). Carefully specify your sampling instants.

(d) (1 points) Compare your sampling rate in (c) with the symbol transmission rate in Nyquist's theorem for zero ISI.

(e) (4 points) Find the pdf of the noise sample at the output of your receiver front end.

(f) (1 points) What is the auto-correlation function of the noise sample sequence at the output of your receiver front end?

Problem 7. (15 points) In this problem, you are asked questions on the baseband modem. Assume that the channel is not frequency-flat.

(a) (1 point) What is the bandwidth of the complex baseband signal $\tilde{Y}(t)$ that is effectively processed by the baseband modem?

(b) (4 points) When the filter front end matched to $s(t)$ is used to process $\tilde{Y}(t)$, what is Nyquist's minimum sampling rate?

(c) (5 points) Discuss whether the filter front end matched to $s(t)$ followed by a symbol rate sampler is in general an optimal front end or not.

(d) (3 points) Justify the claim that the filter front end matched to $p(t) = s(t) * \tilde{h}(t)$ followed by a symbol rate sampler is not an optimal front end in general.

(e) (2 point) In (d), when does the front end become optimal?

Problem 8. (15 points) In this problem, we continue the investigation started in Problem 7. Let $p(t)$ be defined again as

$$p(t) = s(t) * \tilde{h}(t),$$

and let $\hat{p}(t)$ be defined as

$$\hat{p}(t) = p(t) * p^*(-t).$$

(a) (5 points) Find $\tilde{Y}(t)$ in terms of $A, \theta_c, d[n], p(t)$, and $\tilde{N}(t)$.

(b) (2 points) Find the desired component at the output of the optimal matched filter front end followed by a sampler.

(b) (2 points) In (b), find the ISI component at the output of the optimal matched filter front end followed by a sampler.

(d) (1 point) What is the autocorrelation function of the noise component at the output of the optimal matched filter front end followed by a sampler.

(e) (5 point) By using the results in (b)-(d), find the matched filter bound on the BER performance of any equalizer.