EECE 695F Year 2013 Fall

Midterm Exam #1. Sep. 28

(25 pts)

1. An I-Q modulator is used to generate a real-valued bandpass signal $x(t)$. However, direct upconversion from real baseband to real passband suffers from a serious problem in implementation especially when the center frequency is very high. What is this problem? To find the answer, solve the following problems.

(5 pts)
(a) In implementation, can you use a complex-valued signal input complex-valued signal output system? If not, what kind of system you must use only?

(5 pts)
(b) Write $x(t)$ by using only real-valued signals.

(5 pts)
(c) Draw the block diagram of an I-Q modulator by using the result in (b).

(5 pts)
(d) Find the part in the diagram in (c) that is affected by the center frequency.

(5 pts)
(e) Now, discuss what is the implementation issue related to a high center frequency.

2 (20 pts) Suppose that an LTI system w/ real-valued input & output signals has the impulse response

$$h(t) = \sum_{i=1}^{L} \alpha_i \delta(t-t_i).$$

We consider driving this system by using either one of the following two signals;

$$x_1(t) = Re\{x_\ell(t) e^{j2\pi f_1 t}\}$$

$$x_2(t) = Re\{x_\ell(t) e^{j2\pi f_2 t}\}$$

where $f_1 \neq f_2$.

Answer the following questions.

(a) (10 pts) Find $(\beta_i)_{i=1}^{L}$ and $(\gamma_i)_{i=1}^{L}$ that satisfy

$$x_1(t) * h(t) = Re\left\{\left(\sum_{i=1}^{L} \beta_i x_\ell(t-t_i)\right) e^{j2\pi f_1 t}\right\}$$

and

$$x_2(t) * h(t) = Re\left\{\left(\sum_{i=1}^{L} \gamma_i x_\ell(t-t_i)\right) e^{j2\pi f_2 t}\right\}$$

(b) (10 pts) Let $h_{e,1}(t)$ & $h_{e,2}(t)$ be defined by

$$h_{e,1}(t) \triangleq \sum_{i=1}^{L} \beta_i \delta(t-t_i)$$

and $$h_{e,2}(t) \triangleq \sum_{i=1}^{L} \gamma_i \delta(t-t_i).$$

Find the Fourier transforms $H_{e,1}(f)$ & $H_{e,2}(f)$ of $h_{e,1}(t)$ & $h_{e,2}(t)$, respectively and discuss whether $|H_{e,1}(f)|$ & $|H_{e,2}(f)|$ are identical or not.

3. (15 pts) Let X and Y be the real and the imaginary parts of a complex Gaussian random variable Z, i.e.,

$$Z \triangleq X + jY$$

where $j \triangleq \sqrt{-1}$. When $X \sim N(0, \sigma^2)$, $Y \sim N(0, 4\sigma^2)$, and $E[XY] = 0$ w/ $\sigma^2 > 0$, answer the following questions.

(a) (5 pts) Prove or disprove that Z is proper.

(b) (5 pts) Sketch the set defined by

$$\left\{(x,y) : \frac{x^2}{\cancel{4}} + \frac{y^2}{4} \le A^2\right\}$$

when $A > 0$.

(c) (5 pts) Find

$$\Pr\left(\frac{X^2}{\cancel{4}} + \frac{Y^2}{4} \le A^2\right)$$

by using only elementary functions.

4. (20 pts) Suppose that Z_1 and Z_2 be two correlated zero-mean proper-complex Gaussian random variables satisfying

$$E\{|Z_1|^2\} = \sigma_1^2 \quad w/ \ \sigma_1 > 0$$

$$E\{|Z_2|^2\} = \sigma_2^2 \quad w/ \ \sigma_2 > 0$$

and

$$E\{Z_1^* Z_2\} = \rho \sigma_1 \sigma_2.$$

In this problem, we are going to find the condition on the value of ρ. Answer the following questions.

(a) (5 pts) To start w/ a simplified version of this problem, describe a similar problem you already learned in your undergraduate course.

(b) (5 pts) In your problem in (a), how can you show that

$$-1 \underline{\underline{<}} \frac{E\{X_1 X_2\}}{\sqrt{E\{X_1^2\} E\{X_2^2\}}} \underline{\underline{<}} 1 \quad ?$$

(c) (10 pts) By using a similar technique to (b), find all possible values of ρ.

5. (20 pts) In this problem, we are going to derive

$$\Pr(\|\mathbf{Z}-\underline{\mu}\| \geq \|\mathbf{Z}-\underline{c}\|)$$

when $\mathbf{Z} \sim \mathcal{CN}(\underline{\mu}; 2\sigma^2 I_N, \underline{O}_N)$ by using the Q-function. Answer the following questions.

(a) (5 pts) Simplify the problem to a problem involving only a real random variable & $N=1$. Then, solve it.

(b) (5 pts) Repeat (a) w/ $N=2$

(c) (5 pts) Repeat (a) w/ $N=2M$ with $M \in \mathbb{N}$.

(d) Solve the original problem.

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