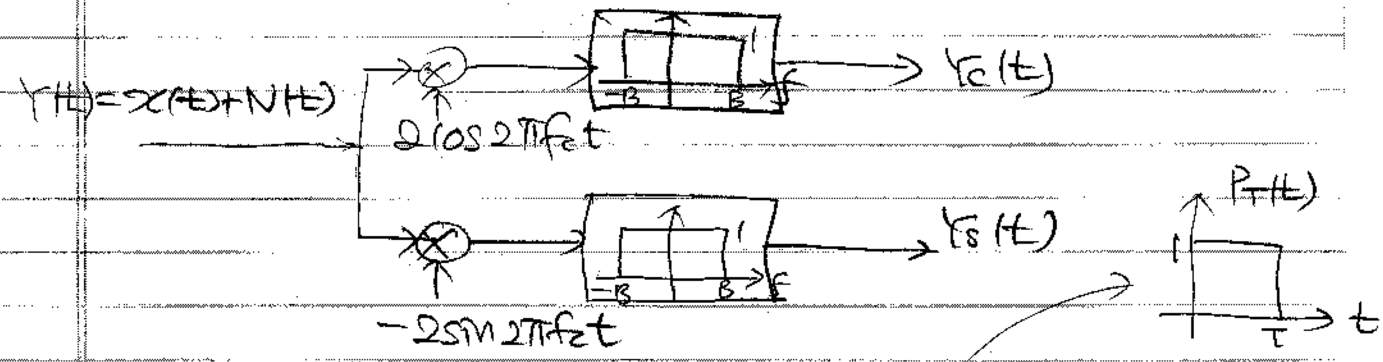
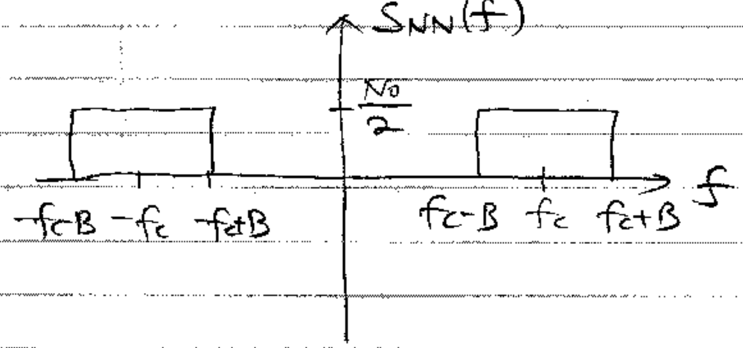
○ Example 1

- Consider the following quadrature demodulation.



where
$$x(t) = \mathrm{Re}\left\{ \sqrt{2P} \sum_{m=-\infty}^{\infty} b[m]\, p_T(t-mT)\, e^{j2\pi f_c t} \right\}$$

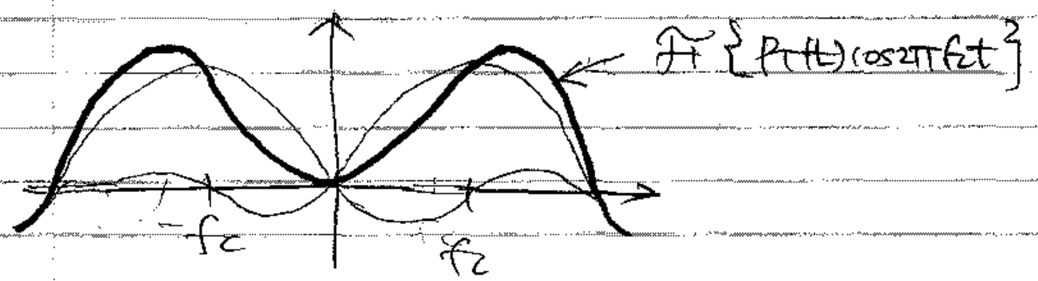
and $N(t)$ is a **bandlimited AWGN w/ two-sided PSD** $N_0/2$, center frequency f_c, and the BW $2B$



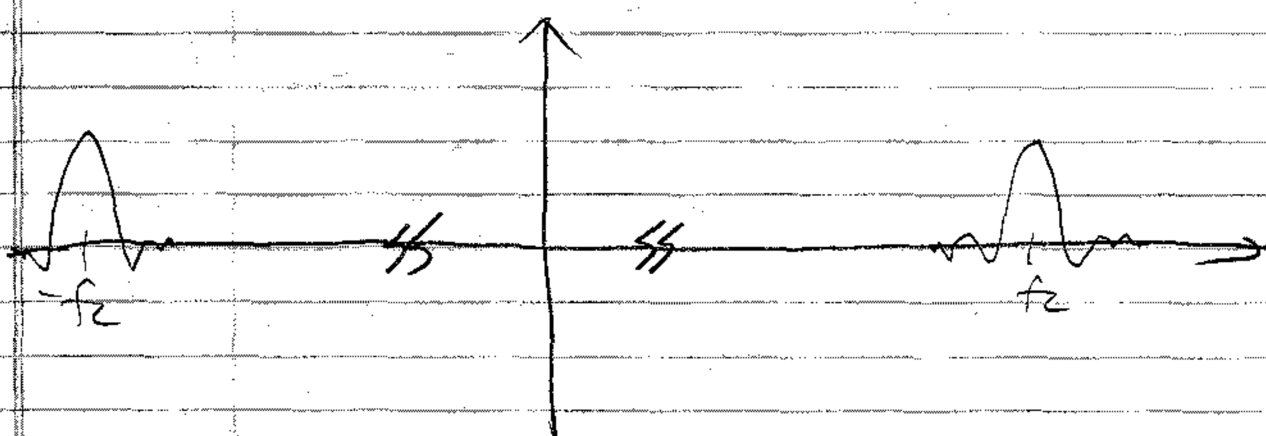
- We assume that the BW of the bandpass signal $x(t)$ is much smaller than the carrier frequency f_c. This assumption is called **the narrowband assumption**.

- Q1 What is the complex envelope of $x(t)$?

A) Since $P_T(t)$ has infinite support in frequency domain, $\sqrt{2P} \sum_{m=-\infty}^{\infty} b[m]\, p_T(t-mT)$ is not the complex envelope in the strict sense. In extreme, if $\frac{1}{T} \simeq f_c$, we have the spectrum of $p_T(t)\cos 2\pi f_c t$ given by



Fortunately, thanks to the narrowband assumption, $\frac{1}{T} \ll f_c$, which gives us the spectrum of $p_T(t)\cos 2\pi f_c t$ given by



We no longer have aliasing! Therefore, we can claim that

$$x_c(t) \approx \sqrt{2P}\sum_{m=-\infty}^{\infty} b[m]\, p_T(t-mT)$$

- Q2. Let $x_c(t)$ be the complex envelope of $x(t)$. Then, is it always true that

$$r_c(t) + j\, r_s(t) = x_c(t) \quad ?$$

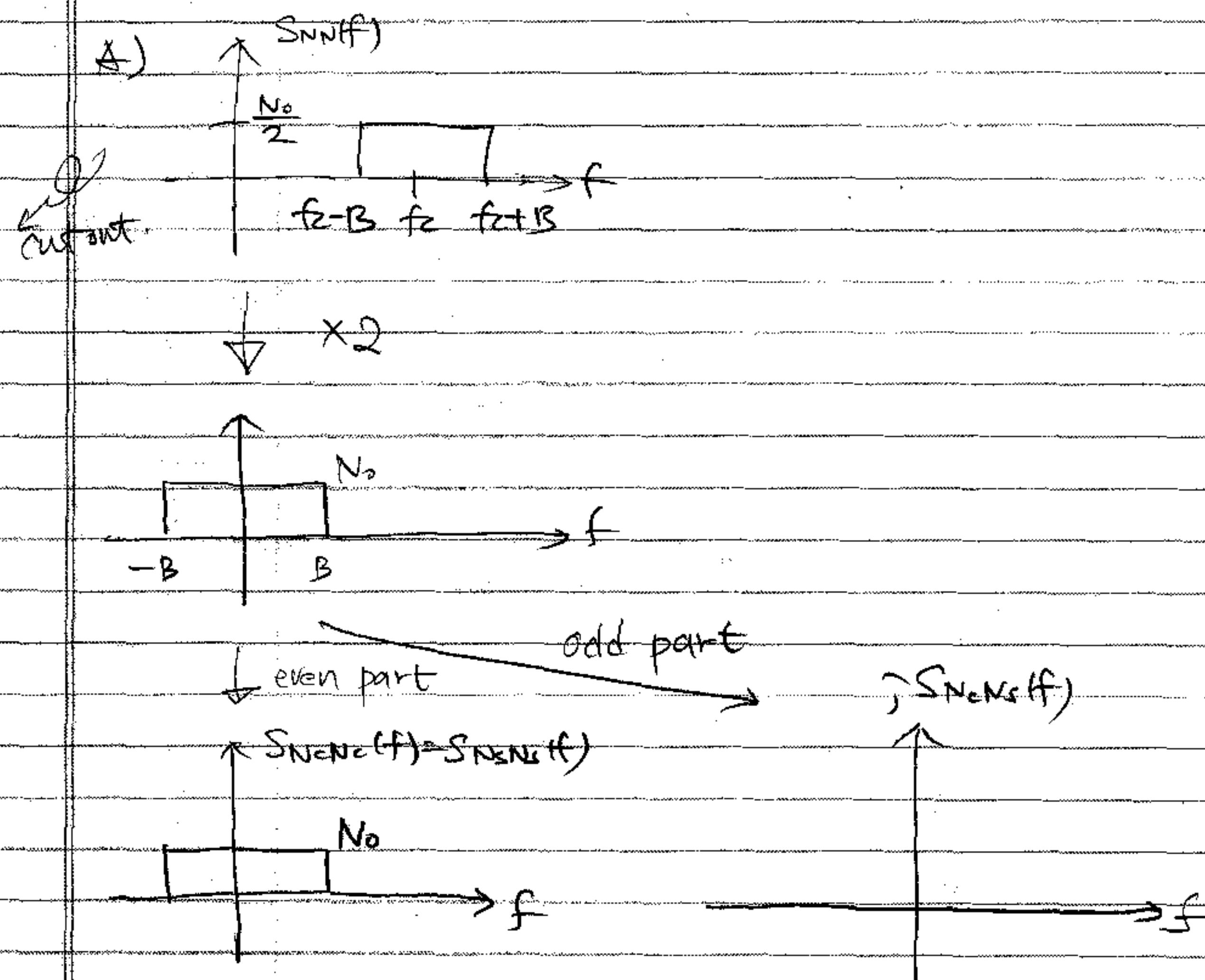
A). No. We need in this case $B \gg \frac{1}{T}$. Otherwise, some sidelobes of $P_T(t)$ are filtered out by the ideal LPF w/ bandwidth B

· Q3 Let $r_c(t) + j r_s(t) = x_\ell(t) + N_\ell(t)$

$= x_\ell(t) + (N_c(t) + jN_s(t))$

Find $S_{N_cN_c}(f)$, $S_{N_sN_s}(f)$, and $jS_{N_cN_s}(f)$.

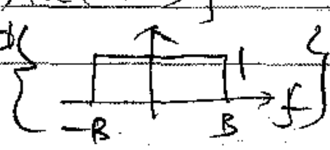
A)



· Q4. What is the autocorrelation function of $N_\ell(t)$?

A) $E\{N_\ell^*(t)\,N_\ell(t+\tau)\} = E\{(N_c(t)+jN_s(t))^*\,(N_c(t+\tau)+jN_s(t+\tau))\}$

$= 2E\{N_c(t)\,N_c(t+\tau)\}$

$= 2N_0\,\mathcal{F}^{-1}\{$  $\}$

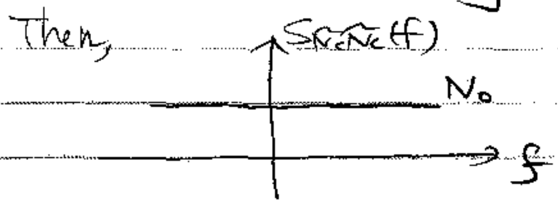
- When the narrowband assumption holds and the ideal LPF has large enough bandwidth, the **complex-baseband equivalent of the received signal** is

$$x_R(t) + \tilde{N}(t)$$

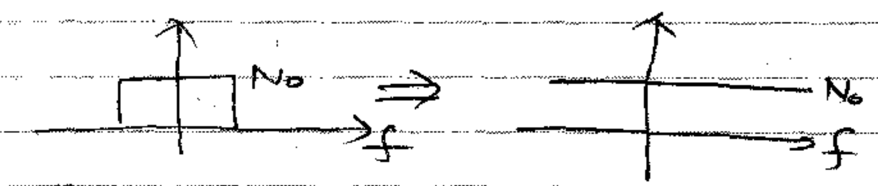
Where $\tilde{N}(t)$ is a proper-complex AWGN with two-sided PSD $2N_0$.

Let $\tilde{N}(t) = \tilde{N}_c(t) + j\tilde{N}_s(t)$

Then,



The reason why



is because $\delta(\tau)$ is much easier to handle than $\mathrm{sinc}(\cdots)$.

○ Example 2

- Consider a **specular multipath channel**

diffusive

dispersive channel = frequency selective channel

$$h(t) = a\delta(t) + b\delta(t-\Delta).$$

with the channel input given by $x(t) = \mathrm{Re}\{\sqrt{2P}\sum_{m=-\infty}^{\infty} b[m]\, p_T(t-mT)\, e^{j(2\pi f_c t+\theta)}\}$.

- Q1. Find the channel output.

A) The channel output is

$$y(t) = h(t) * x(t) = a\,x(t) + b\,x(t-\Delta)$$

$$= \mathrm{Re}\left\{\sqrt{2P}\sum_{m=-\infty}^{\infty} b[m]\left(a\,p_T(t-mT) + b\,p_T(t-\Delta-mT)\right.\right.$$
$$\left.\left. e^{-j2\pi f_c \Delta}\right) e^{j(2\pi f_c t + \theta)}\right\}$$

- Q2. What is the complex envelope of $y(t)$?

A) From Q1,

$$y_\ell(t) = \sqrt{2P}\sum_{m=-\infty}^{\infty} b[m]\left(a\,p_T(t-mT) + b\,p_T(t-\Delta-mT)e^{-j2\pi f_c \Delta}\right)$$
$$\times\left(e^{j\theta}\right).$$

- It turns out that, effectively, $h(t)$ has the complex envelope

$$h_\ell(t) = 2\left(a\delta(t) + b\delta(t-\Delta)\,e^{-j2\pi f_c \Delta}\right)$$

because

$$y_\ell(t) = \frac{1}{2}x_\ell(t) * h_\ell(t)$$

The reason why we call this an **effective complex envelope** is that actually $h(t)$ does not have the complex envelope because it is not band-limited.

Of course, we can redefine $h(t)$ by bandpass filtering it with a BPF w/ center f_c and bandwidth B. However, this approach violates "simplicity" we pursue by using complex signals.

- Note that the specular multipath channel

$$h(t) = a\delta(t) + b\delta(t-\Delta)$$

introduces not only the delayed multipath in $y_e(t)$ but also the phase shift $e^{-j2\pi f_c \Delta}$, which is **dependent on** the center frequency f_c