# Summary of Handout #9



## ○ PAM (Pulse Amplitude Modulation)

· PAM in a wide sense

$$s(t) = \mathrm{Re}\left\{\sqrt{2P}\sum_{m=-\infty}^{\infty} d[m]\, p(t-mT)\, e^{j2\pi f_c t}\right\}$$

where $P$ is the average signal power
$d[m]$ is the $m$th data symbol satisfying
$E\{d[m]\}=0,\ E\{|d[m]|^2\}=1$
$p(t)$ is the symbol waveform with

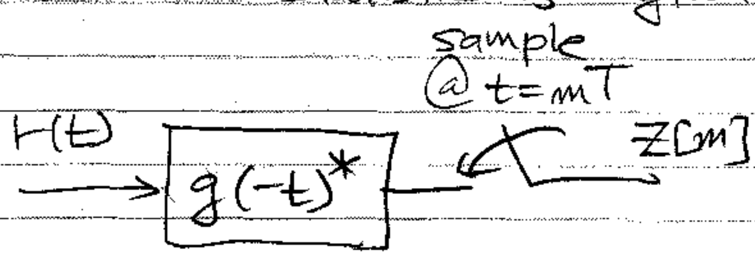
$$\int_{-\infty}^{\infty}|p(t)|^2\,dt = T$$

· Optimal receiver front-end

If the received signal is modeled in the complex baseband as

$$r(t) = \sqrt{2P}\sum_{m=-\infty}^{\infty} d[m]\, g(t-mT) + N(t)$$

where $N(t)$ is a proper-complex AWGN with $S_{NN}(f) = 2N_0$,
the optimal receiver front-end that converts the continuous-time observation to a sufficient discrete-time statistic is given by



- Properties of $Z[m]$

$$Z[m] = \sqrt{2p} \sum_{n=-\infty}^{\infty} d[n]\, \tilde{g}((m-n)T) + N[m]$$

where

$\tilde{g}(t)$ is the time-autocorrelation $g(-t)^* * g(t)$ of $g(t)$ and

$$N[m] \triangleq \int_{-\infty}^{\infty} N(t) g(t-mT)^* dt$$

$$Z[m] = \sqrt{2p} \left[ \underbrace{\sum_{n \neq m} d[n]\, \tilde{g}((m-n)T)}_{\text{the ISI}} + d[m]\tilde{g}(0) \right]$$

$$+ N[m]$$

$$N[m] \sim CN(0, 2N_0 \tilde{g}(0))$$

$$E\{N[m]^* N[n]\} = 2N_0\, \tilde{g}((m-n)T)$$

○ Nyquist's theorem

• Nyquist's zero ISI criterion.

Let $x(t)$ is a finite-energy signal. Then,

$$x(nT)=\begin{cases} C\ (>0), & \text{for } n=0 \\ 0, & \text{for } n\in\mathbb{Z}\setminus\{0\} \end{cases} \quad \cdots (*)$$

is called the Nyquist zero ISI criterion for sampling rate $1/T$.

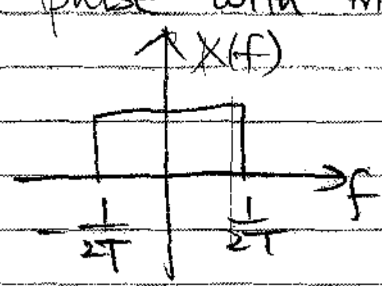
• Nyquist's theorem.

$$\sum_{n=-\infty}^{\infty} X\left(f+\frac{n}{T}\right) = C' \text{ (a constant)} \quad \forall f$$

• Nyquist's minimum bandwidth condition

If $x(t)$ satisfies $(*)$, then the bandwidth of $x(t)$ is at least $\frac{1}{2T}$.

• Nyquist pulse with minimum bandwidth




$$\overset{FT}{\Longleftrightarrow} \quad x(t) \propto \mathrm{sinc}\left(\frac{t}{T}\right)$$

• In optimal receiver front-end,

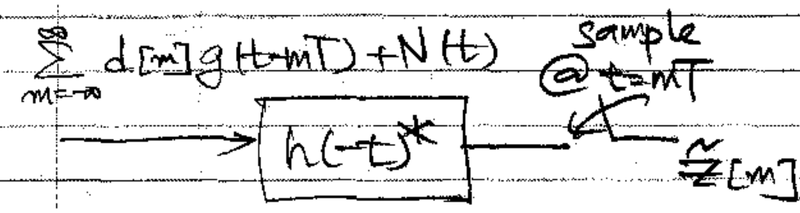
iff $\tilde{g}(t)$ satisfies the Nyquist condition w/ rate $1/T$ $\Leftrightarrow \sum_{n=-\infty}^{\infty} |G(f+\frac{n}{T})|^2 = C$

then

$\{Z[m]\}_m$ contains no ISI

$\{N[m]\}_m$ becomes an i.i.d. sequence.

• Consider a suboptimal receiver front-end:




Then,

$\{\tilde{Z}[m]\}_m$ has no ISI iff

$$\sum_{m=-\infty}^{\infty} G(f+\frac{m}{T}) H^*(f+\frac{m}{T}) = C', \ \forall f$$

$\{\tilde{N}[m]\}_m$ becomes an i.i.d. sequence iff

$$\sum_{n=-\infty}^{\infty} \left| H(f+\frac{n}{T}) \right|^2 = C'', \ \forall f.$$

## Excess bandwidth

- From the fact that the minimum bandwidth for zero ISI is $\frac{1}{2T}$ [Hz] in baseband, we defined the excess bandwidth $\beta$ via the relation

$$BT = \frac{1+\beta}{2}$$

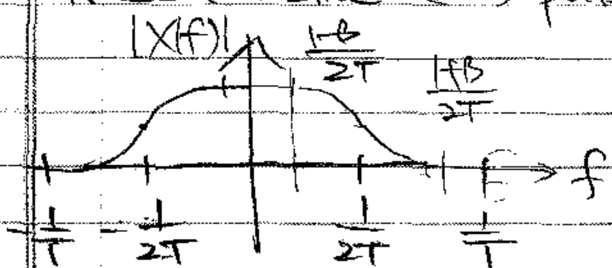
where $B$ is the bandwidth of the pulse in baseband, $1/T$ is the data symbol Tx rate.

- The Nyquist pulse with minimum bandwidth has $\beta = 0$.
  $\Leftrightarrow$ If $\beta < 0$, the ISI is inevitable.

- Usually, $\beta > 0$ is chosen. $\beta = 0.5 \Rightarrow B = 1.5 \times \frac{1}{2T}$
  $\beta = 1 \Rightarrow B = 2 \times \frac{1}{2T}$
  ⋮

- If $0 < \beta < 1$, the system is called a narrowband system.
  If $\beta \gg 1$, " " " " " wideband system.

- Raised cosine (RC) pulses : a representative class of Nyquist pulses.



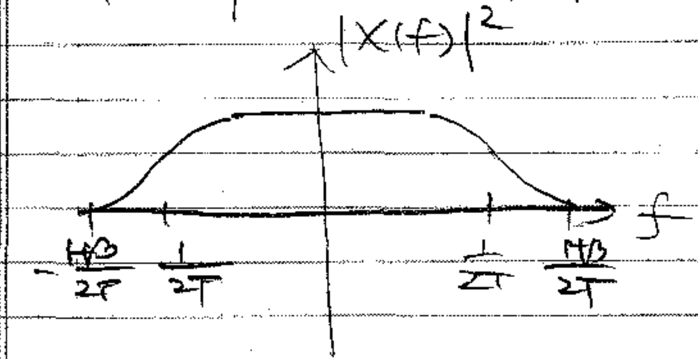

$x(t)$

$0 \le \beta \le 1$

$\beta = 0 \Leftrightarrow x(t) \propto \mathrm{sinc}(\frac{t}{T})$

representative

- Square-root raised-cosine (SRRC) pulses: a class of square-root Nyquist pulses $x(t)$




- $0 < \beta \le 1$

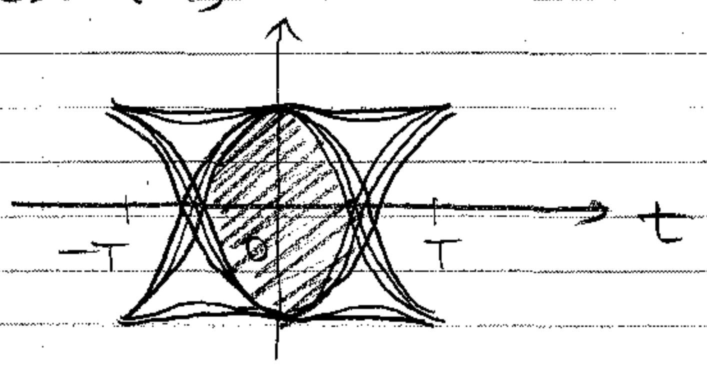
## ○ Eye diagram.

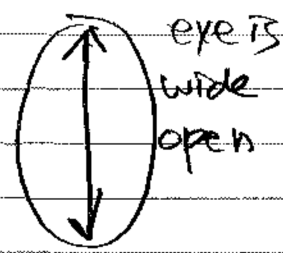
- To check the ISI property of $x(t)$ qualitatively we often use the eye diagram

- Observed signal

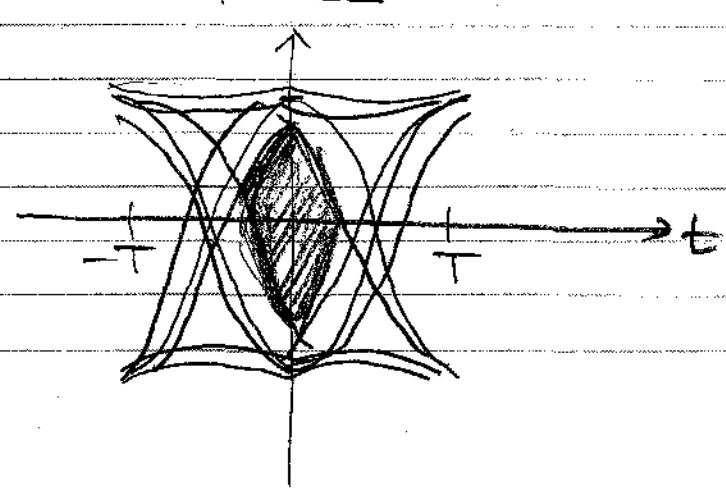
$$\sum_m d[m]\, x(t-mT)$$

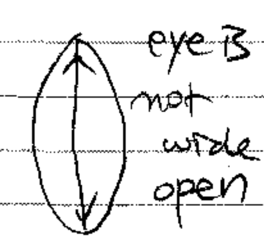
ex/ $d[m] \in \{1, -1\}$




No ISI



- timing jitter resistance

The sampler may have timing jitters in generating $\{z[m]\}_m$.

By spending more excess bandwidth, we can make the pulse more jitter resistant.