DS-CDMA and its PSD

Spread spectrum systems

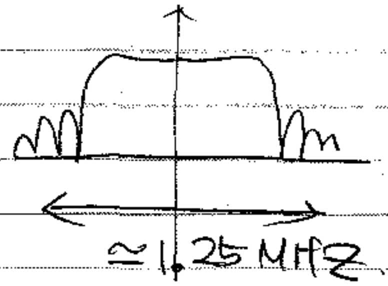
- Spread spectrum (**SS**)
  - Direct-sequence spread-spectrum (**DS/SS**)
  - Frequency-hopping spread-spectrum (**FH/SS**)
  - various hybrid SS systems

- A spread-spectrum system is a kind of wideband system where the data symbol rate is much smaller than the total system bandwidth, i.e., the spectrum of the system is much larger than the symbol transmission rate.

  In terms of the excess bandwidth, usually a SS system has $\beta \gtrsim 10, 100, 200 \ldots$

  Some people use the term SS even when $\beta \simeq 2, 3 \ldots$

- IS-95



The cellular standard IS-95 has the bandwidth around 1.25MHz in passband, but the BPSK data symbol is transmitted at the rate around 10 KHz.

$$\therefore \beta = 1.25 \times 10^{6} \times 10^{-4} - 1 = 124.$$

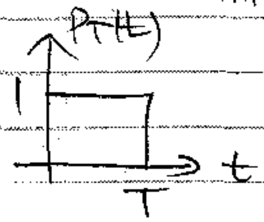
○ Direct-sequence Spread-spectrum system

- Signal model (Not practical but simple)

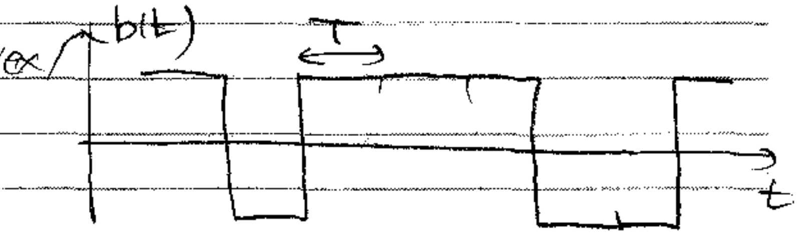
Consider a data signal

$$b(t) = \sum_{m=-\infty}^{\infty} b[m]\, p_T(t-mT)$$

where



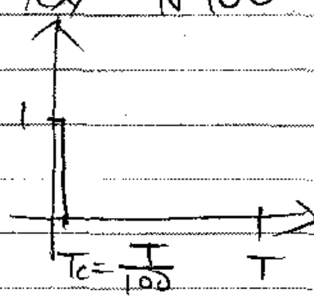
/ex/



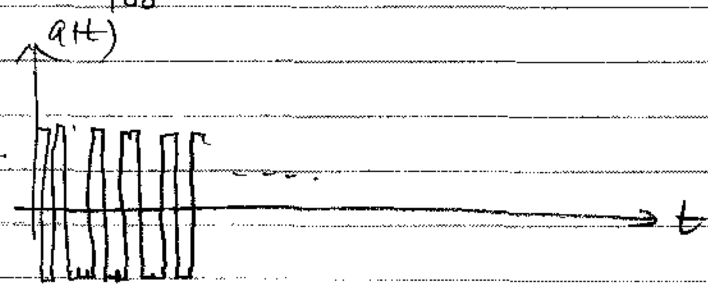
Also consider a spreading signal

$$a(t) = \sum_{n=-\infty}^{\infty} a[n]\, p_{T_c}(t-nT_c)$$

where $\underbrace{NT_c = T}_{\text{a large number}}$ /ex/ $N=100$ and $\{a[n]\}_n$

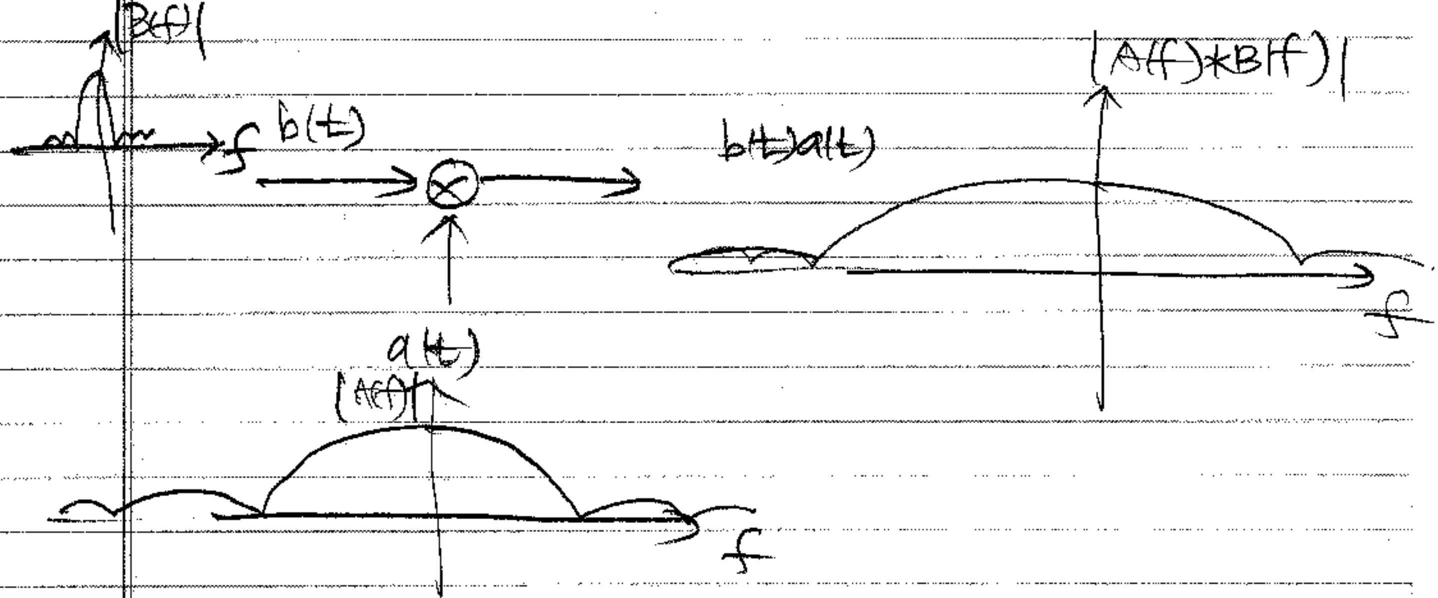


Is a pseudo-noise (**PN) sequence**, e.g., generated by an m-sequence generator ..



Q1. What happens in the frequency domain if b(t) is multiplied by a(t)?

A. $b(t)$ is a narrowband signal, while
$a(t)$ " " wideband "
Thus $b(t)a(t)$ becomes a wideband signal



Note that the spectrum of the signal is spread over a wide frequency band.

- Jargons.

  $P_{Tc}(t)$ is called the **chip pulse** or the chip waveform

  $1/T_c$ is called the chip rate or the **chipping rate**

  $\{a[n]\}_{n=-\infty}^{\infty}$ is called the spreading sequence

  N is called the **spreading factor**

- Q2. What is the optimum receiver?
  Q3. Why do we use a DS/SS system?

○ Optimal receiver

- The signal $b(t)a(t)$ can be rewritten as

$$\sum_{m=-\infty}^{\infty} b[m]\, p_m(t-mT)$$

where $p_m(t) \triangleq \sum_{n=0}^{N-1} a[n+mN]\, p_{T_c}(t-nT_c)$.

Then, this is a linearly modulated signal with a symbol waveform $p_m(t)$ for the mth symbol $d[m]$

- If a **PN** sequence/**code** generator has the period of N, then

$$p_m(t) = p(t) \quad \forall m.$$

This DS/SS system is called **a short-code system**

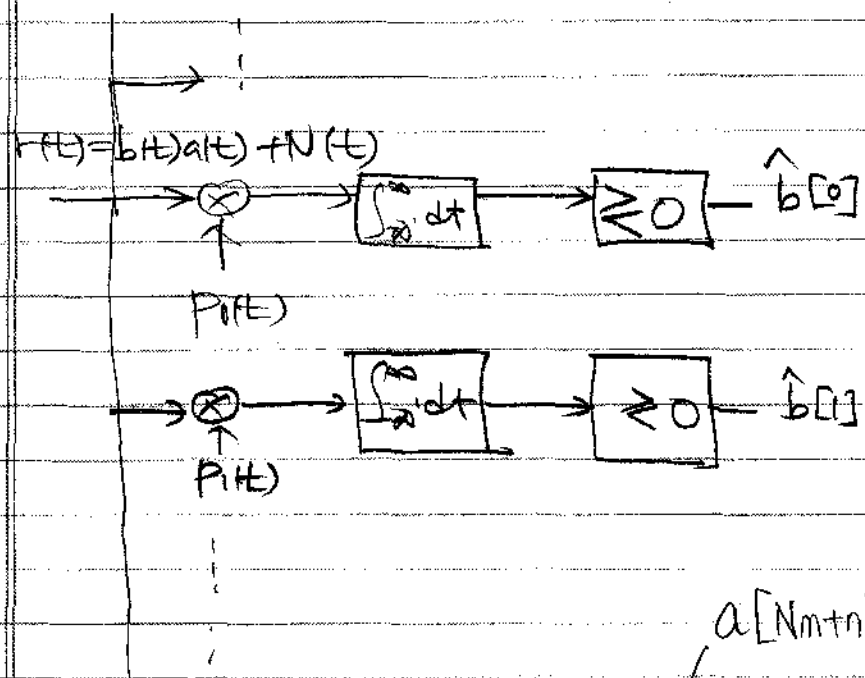
- If a PN code has the period much greater than N, then $p_m(t)$ is a time-varying symbol waveform. (Linearly time-varying modulation)

This DS/SS system is called **a long-code system.** Usually, a long-code system has a periodically time-varying symbol waveform with a period greater than 1.

(ICI)

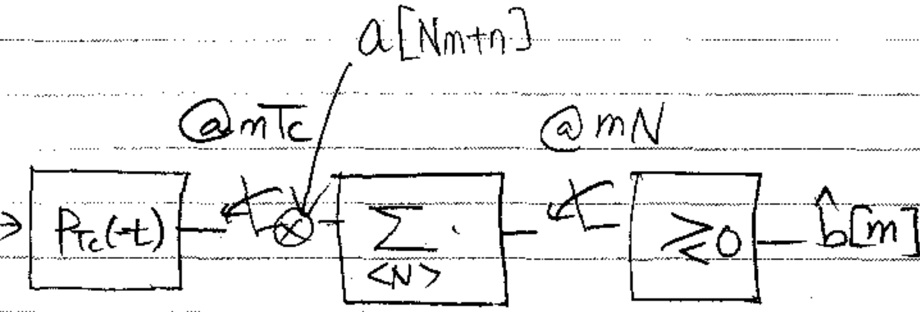
- Since $p_{T_c}(t)$ induces no **interchip interference**, the decision can be made symbol-by-symbol (in this special case, bit-by-bit) by symbol-waveform

the matched filters as



or equivalently

$\equiv$ r(t) $\longrightarrow$



using the **chip-matched filter**. This structure is applicable for any PS/SS w/ zero ICI.

○ Signal model & PSD

- $$s(t) = \sum_{m=-\infty}^{\infty} d[m]\, p_m(t - mNT_c)$$

where

$$p_m(t) \triangleq \sum_{n=0}^{N-1} a[n+mN]\, p(t - nT_c)$$

where $p(t)$ is **the chip pulse** usually satisfying the zero ICI condition

$$\int_{-\infty}^{\infty} p(t-mT_c)\, p(t-m'T_c)^* dt = T_c\, \delta_{m,m'}$$

- Received signal in AWGN

$$r(t) = \sqrt{2P} \sum_{m=-\infty}^{\infty} d[m] \sum_{n=0}^{N-1} a[n+mN]\, p(t-nT_c-mNT_c)\, e^{j\theta} + N(t)$$

Where P is the received signal power,
θ is the phase angle, and
$N(t)$ is the proper-complex AWGN w/
$S_{NN}(f) = 2N_0$

- For a long-code system with a very large period, $\{a[n]\}_{n=-\infty}^{\infty}$ is often modeled as

(i) **BPSK spreading**
a sequence of i.i.d. random variables w/
$Pr(a[n]=1) = Pr(a[n]=-1) = 1/2$

(ii) **QPSK spreading**
a " " " " " "
$Pr(a[n]=e^{j\frac{\pi}{4}}) = Pr(a[n]=e^{j\frac{3}{4}\pi}) = Pr(a[n]=e^{j\frac{5}{4}\pi})$
$= Pr(a[n]=e^{j\frac{7}{4}\pi}) = 1/4$.

- The data sequence $\{d[m]\}_{m=-\infty}^{\infty}$ is a coded sequence of (BPSK / QPSK / M-PSK / ⋮) symbols.

- Q. What is the PSD of the DS/SS signal w/ QPSK symbols & QPSK spreading?

A. Since $\{a[n]\}_n$ is an i.i.d. sequence,

$\{d[m]\,a[n+mN]\}_{(m,n)}$ has i.i.d. elements.

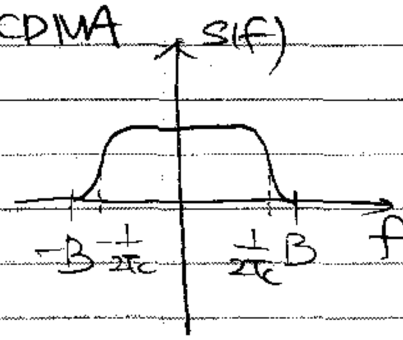
Therefore, for the computation of the PSD, we can view $s(t)$ as

$$s(t) = \sum_{n=-\infty}^{\infty} \hat{a}[n]\, p(t-nT_c)$$

∴ The PSD is given by

$$S(f) = \frac{P}{T_c} \underbrace{|P(f)|^2}_{\text{energy spectral density of the chip pulse}}$$

• WCDMA



/ex/ SRRC chip pulse w/ roll-off factor 22% $\beta_c = 0.22$

$$\Rightarrow \quad BT_c = \frac{1+\beta_c}{2}$$

where β_c is the chip excess bandwidth

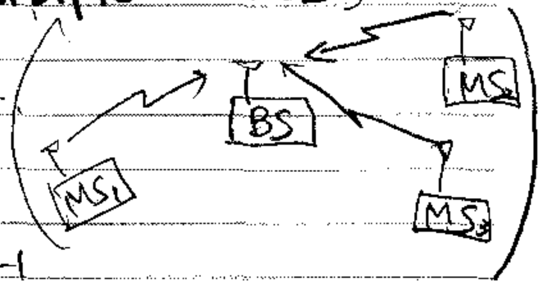
Since $NT_c = T_s$, the symbol excess bandwidth is computed from $BT_s = \frac{1+\beta_s}{2} = BNT_c$

$$= \frac{N(1+\beta_c)}{2}$$

$$\Rightarrow \quad \beta_s = N(1+\beta_c) - 1 \qquad \beta_s \gg 1$$

DS-CDMA

(Direct-sequence code-division **multiple-access**)



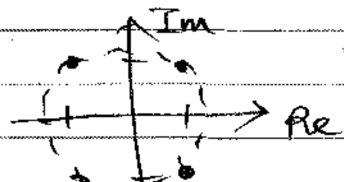
- The original name is DS/SSMA
- The signal model

$$\underbrace{r(t)}_{\text{the complex envelope of the received signal}} = \sum_{k=1}^{K} \underbrace{h_k(t)}_{\text{channel from the kth user to the base station}} * \left\{ \sum_{m=-\infty}^{\infty} d_k[m] \sum_{n=0}^{N-1} a_k[n+mN]\, p(t-nT_c-mNT_c) \right\} + N(t)$$

where $\{d_k[m]\}_{m=-\infty}^{\infty}$ is the data sequence of the kth user w/ $E\{d_k[m]\}=0$, $E\{d_k[m]^2\}=0$, $E\{|d_k[m]|^2\}=1$.

$\{a_k[n]\}_{n=-\infty}^{\infty}$ is the spreading sequence of the kth user.

- In cdmaOne & cdma2000, two PN sequence generators with a very long period are used with different initial **code phases** for different users & I&Q for QPSK spreading



- The PN sequences w/ different code phases look like independent sequences of i.i.d.

elements.

- The PSD of the signal in AWGN

Since $r(t)$ is the summation of K uncorrelated signals & noise

$$S(f) = \sum_{k=1}^{K} S_k(f) + 2N_0$$

$$= \sum_{k=1}^{K} \frac{P_k}{T_c} |P(f)|^2 + 2N_0$$

where
P_k is the signal power of the kth user

- The PSD in frequency selective linear channels corrupted by AWGN

$$S(f) = \sum_{k=1}^{K} \frac{|H_k(f)P(f)|^2}{T_c} + 2N_0$$

$$= \frac{|P(f)|^2}{T_c} \sum_{k=1}^{K} |H_k(f)|^2 + 2N_0.$$