OFDM and its PSD

○ FDM vs OFDM.

- Frequency-division multiplexing (FDM) has non-overlapping frequency bands for each data stream.

- To the contrary, orthogonal frequency-division multiplexing has overlapping but still orthogonal frequency bands for each data stream.

○ Signal model (Not practical but simple)

- The complex baseband equivalent of the OFDM signal is given by

$$Y(t) = \sum_{k=1}^{K} X_k e^{j2\pi f_k t}, \quad 0 \le t \le T_0 \quad \cdots (*)$$

where

$$f_k - f_{k'} = \frac{k-k'}{T_0}, \quad k, k' = 1, 2, \cdots, K$$

$$f_1 + f_K = 0 \quad \text{(to make the center freq} = 0\text{.)}$$

X_k is the kth **sub-carrier** data symbol.
T_0 is called the **OFDM symbol period**.

- K symbols are transmitted in the time duration T_0

- Note that

$$\int_0^{T_0} e^{j2\pi f_k t} e^{-j2\pi f_{k'} t} dt = \int_0^{T_0} e^{j2\pi \frac{k-k'}{T_0} t} dt = T_0 \delta_{k,k'}.$$

Real-valued BP signal is

$\Rightarrow \mathrm{Re}\{\tilde{r}(t)e^{j2\pi f_c t}\}$.

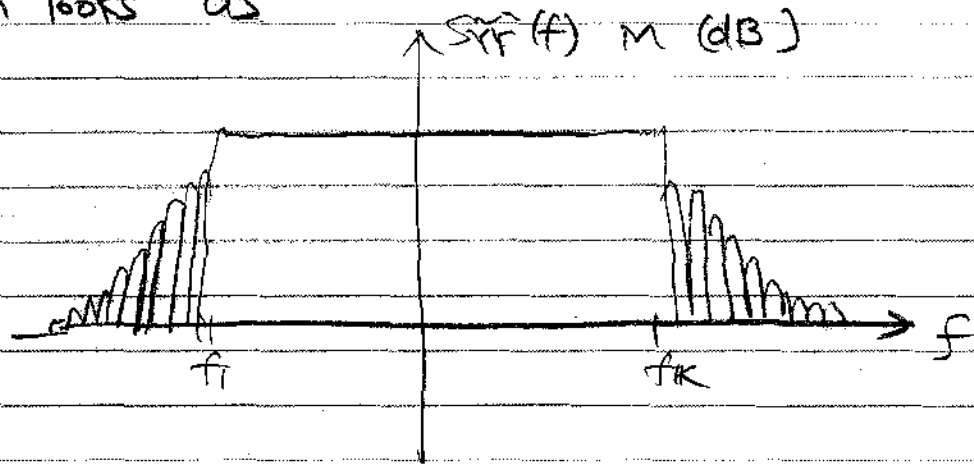
- To transmit more than one symbols per subcarrier, we modify the signal model as

$$\tilde{r}(t) = \sum_{k=1}^{K}\left(\sum_{m=-\infty}^{\infty} X_k[m]\, p_{T_0}(t-mT_0)\right) e^{j2\pi f_k t} \quad \cdots (**)$$

- The PSD of this signal can be obtained as

$$S_{\tilde{r}\tilde{r}}(f) = \sum_{k=1}^{K} \frac{|P_{T_0}(f-f_k)|^2}{T_0} \propto \sum_{k=1}^{K} \frac{1}{T_0} \cdot \mathrm{sinc}^2\big((f-f_k)T_0\big)$$

which looks as



- The PSD is almost flat in the middle because

$$\sum_{n=-\infty}^{\infty} \mathrm{sinc}^2(t-n) = 1 \quad \forall t$$

↑ why? Use Nyquist's zero ISI theorem.

~~At~~ the right edge, the effect of $\mathrm{sinc}^2((f-f_K)T_0)$ is dominating, which decays at the rate of $\frac{1}{(f-f_K)^2}$.

- Q. Is it done?

A. No!

The OFDM is especially robust in multipath channel when the **cyclic prefix (CP)** is used in signaling. The signal models (*) & (**) do not have it.

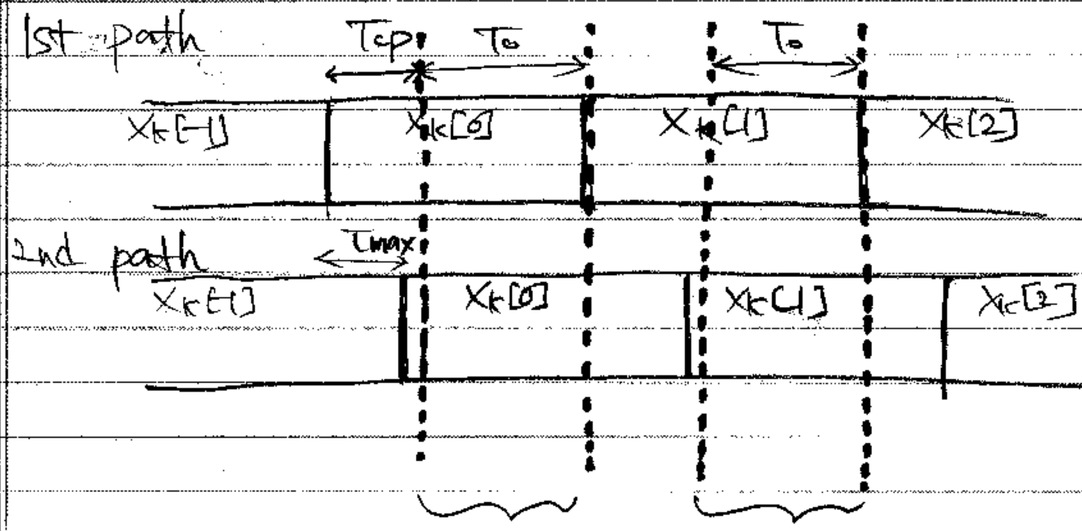
○ Cyclic prefix and its effect on PSD.

- The signal model

$$\hat{r}(t) = \sum_{k=1}^{K} \left\{ \sum_{m=-\infty}^{\infty} X_k[m] \, P_{T_0+T_{cp}}(t - m(T_0+T_{cp})) \right\} e^{j2\pi f_k t} \quad \cdots (***)$$

- A multipath channel

$$h(t) = \alpha\delta(t) + \beta\delta(t-\tau_{max})$$



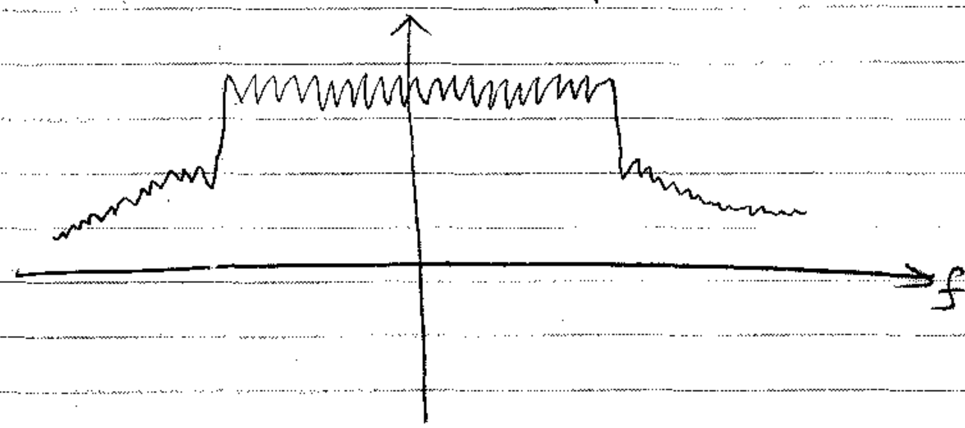
observed interval in Rx for $X_k[0]$

Of course, this scheme in Rx is suboptimal & lose some energy in demodulation.

- As far as $T_{cp} \geq \tau_{max}$, the K subcarriers are orthogonal!! (why?)

- The PSD.

$$S_{\hat{f}\hat{f}}(f) \propto \sum_{k=1}^{K} \frac{\mathrm{sinc}^2\left((f-f_k)(T_0+T_{cp})\right)}{T_0+T_{cp}}$$



- Q. Is it the full story?

A. No!

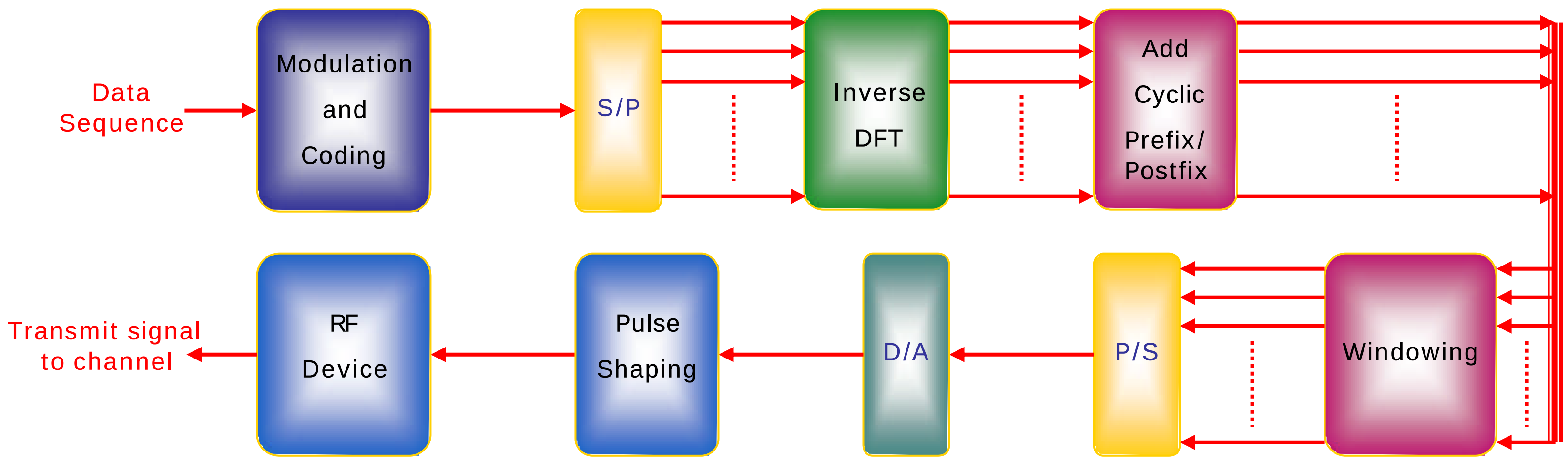
Introduction to OFDM

❖ Characteristics of OFDM (Orthogonal Frequency Division Multiplexing)

- Parallel data transmission with very long symbol duration
  - Robust under multi-path channels
- Transformation of a frequency-selective channel into N frequency-flat channels
- Cyclic prefix (CP) is used to efficiently eliminate inter-symbol interference (ISI).
- High spectral efficiency of the orthogonal subcarriers
- Efficient implementation of the transmitter-receiver pair using Inverse Fast Fourier transform (IFFT) and Fast Fourier transform (FFT)
- High peak-to-average power ratio (PAPR) and sensitivity to carrier frequency offset (CFO)

Introduction to OFDM (cont.)

❖ General system structure of OFDM transmitter



Review of Non-IDFT-Based Signal Model and its PSD

❖ **Non-IDFT-based** signal model for OFDM system

- Common continuous-time signal model in many literatures for **a single OFDM symbol in complex baseband** would be

$$Y(t)=\sum_{k=1}^{K} X_k e^{j2pf_k t},\quad 0 \le t < T_0$$

where X_k is the data symbol transmitted by the kth subcarrier whose carrier frequency is f_k .

- The length of a single OFDM symbol and the subcarrier frequencies are related as

$$f_k - f_{k'} = \frac{k-k'}{T_0}$$

which leads to the orthogonality of the signals $e^{j2pf_k t}$ for $k=1,2,\ldots,K$ in the observation interval with time duration T_0 .

- For simplicity, assume that X_k 's are uncorrelated, proper complex, zero-mean random variables with unit variance.

Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ **Non-IDFT-based** signal model for OFDM system (cont.)

- For the transmission of **more than one OFDM symbol**, the signal model is modified to

$$\hat{Y}(t)=\sum_{k=1}^{K}\left(\sum_{m=-\infty}^{\infty}X_k[m]w_C(t-mT_b)\right)e^{j2pf_k t}$$

which $X_k[m]$ is the kth subcarrier data symbol for time-index m, $1/T_b$ is the OFDM block transmission rate and $w_C(t)$ is the windowing function.

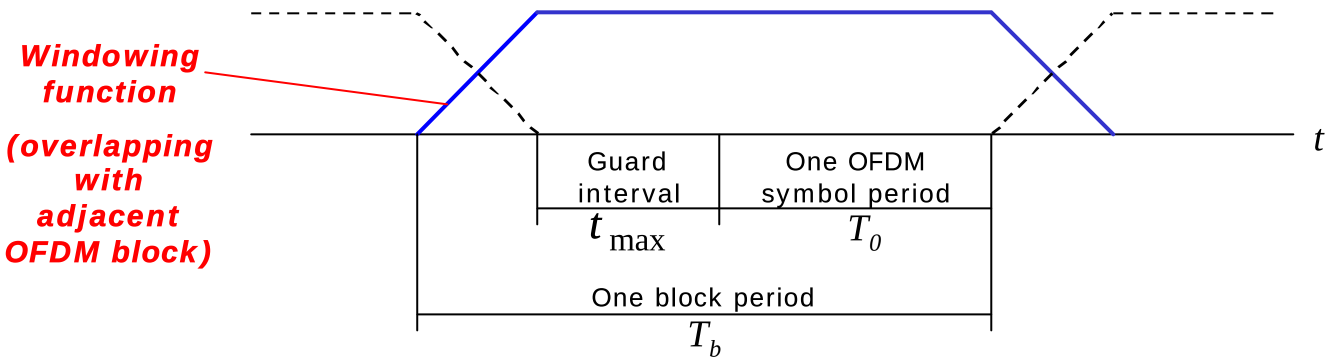
- T_b is the OFDM block transmission time and selected as larger than or equal to $T_0+t_{\max}$ where $t_{\max}$ is the maximum delay spread of the channel.

- The shape of the roll-off of the windowing function $w_C(t)$ is usually designed to reduce the inter-carrier interference (ICI) under carrier frequency offset (CFO).

  ➔ If no CFO, not only IBI but also ICI can be completely removed.

❖ Example of **non-IDFT based** continuous-time signal structure with windowing

$$\hat{Y}(t)=\sum_{k=1}^{K}\left(\sum_{m=-\infty}^{\infty} X_k[m]\,w_C(t-mT_b)\right)e^{j2pf_k t}$$



Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model

- The PSD of this signal model is given by

$$S_{\hat{Y}\hat{Y}}(f) = \frac{1}{T_b} \sum_{k=1}^{K} \left| W_C(f - f_k) \right|^2$$

where $W_C(f)$ is the continuous-time Fourier transform (CTFT) of the windowing function $w_C(t)$.

- Note that $X_k[m]$'s are uncorrelated, proper complex, zero-mean random variables with unit variance.

- If a wideband analog filter is additionally used, if some subcarriers are null-out, and if different transmit power is allocated for each subcarrier, signal model can be rewritten to a more general form shown in the next page.

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Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model (cont.)

- General form of non-IDFT-based signal model

$$Z(t) = p(t) * \left[\sum_{k=1}^{K} \left(\sum_{m=-\infty}^{\infty} a_k X_k[m] w_C(t - mT_b) \right) e^{j2pf_k t} \right]$$

  - $p(t)$ is the complex envelope of the bandpass transmit filter.

  - a_k is the complex weighting factor for the kth subcarrier.

  (a_k is zero. ➔ The kth subcarrier is called a *null subcarrier*.)

  - $X_k[m]$ is the kth subcarrier data symbol for time-index m.

  - $w_C(t)$ is the continuous-time windowing function.

  - $1/T_b$ is the OFDM block transmission rate.

- The PSD of this signal model is given by

$$S_{ZZ}(f) = \frac{|P(f)|^2}{T_b} \sum_{k=1}^{K} |a_k|^2 |W_C(f - f_k)|^2 .$$

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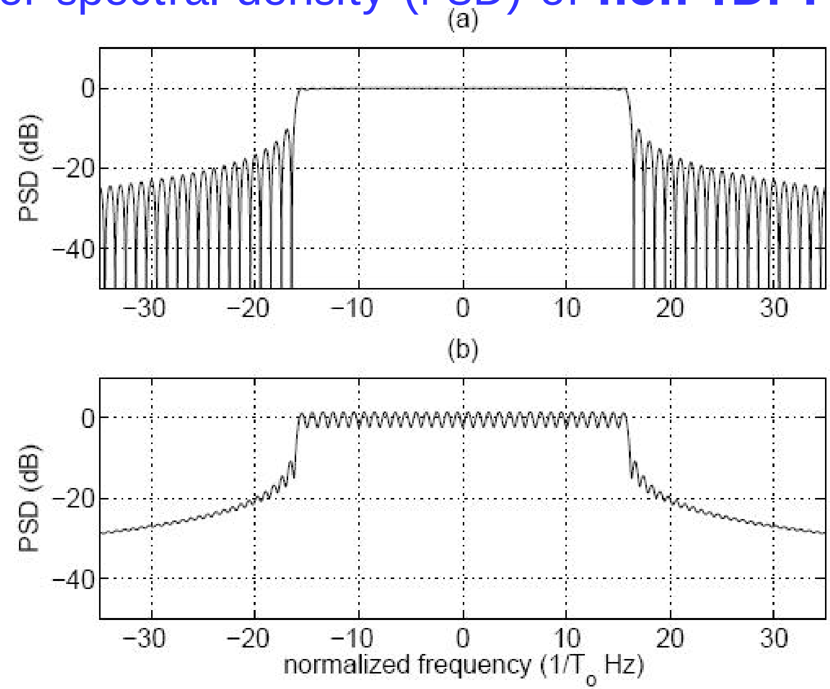
Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **non-IDFT-based** signal model (cont.)



- Parameter

  $a_k = 1, \ \forall k$

  $K = 32$

- Rectangular windowing

  (a) : $T_b = T_0$

  (b) : $T_b = 1.25 T_0$

  Window duration= T_b

- (a) : Fourier transform of the rectangular window function is a sinc function.

  ➔ PSD is almost flat except at the edges.

  (At the edge, the PSD decays approximately at the rate $1/f^2$.)

- (b) : $\sum_{k=-\infty}^{\infty} |W_C(f - f_k)|^2$ is a periodic function with period $1/T_0$.

  ➔ PSD is not flat in the mid-bands.

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Review of Non-IDFT-Based Signal Model and its PSD (cont.)

❖ OFDM signal model in IEEE 801.11.a

- Baseband signal frame : $r_{PACKET}(t)=r_{PREAMBLE}(t)+r_{SIGNAL}(t-t_{SIGNAL})+r_{DATA}(t-t_{DATA})$
  - $t_{SIGNAL}=16ms$, $t_{Data}=20ms$
- Subframe : $r_{SUBFRAME}(t)=w_{T,SUBFRAME}(t)\sum_{k=-N_{ST}/2}^{N_{ST}/2} C_k e^{j2pkD_f(t-T_{GUARD})}$
  - $w_{T,SUBFRAME}(t)$: Continuous-time windowing function with duration t_{TR}
  - C_k : Data, pilots, or training symbols
  - $D_f=1/T_{FFT}$: Subcarrier frequency spacing ($=1/T_0$ in our models)
  - T_{GUARD} : Guard time to create the cyclic prefix

    ➔ For the short training seq. ($=0\ \mu s$), long training seq. ($=T_{GI2}$), data OFDM symbols ($=T_{GI}$)
- Windowing function may have the time duration extended over than one OFDM period, T_{FFT}, and the transition time overlapped via an adjacent OFDM symbol windowing. (Also in our signal model)

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OFDM signal model in IEEE 801.11.a (cont.)

- Timing-related parameter

Parameter	Value
N_{SD} : Number of data subcarriers	48
N_{SP} : Number of pilots subcarriers	4
N_{ST} : Number of total subcarriers	52 (N_{SD} + N_{SP})
D_f : Subcarrier frequency spacing	0.3125 MHz (=20 MHz/64)
T_{FFT} : IFFT/FFT period	3.2 μs ($1/D_f$)
$T_{PREAMBLE}$: PLCP Preamble duration	16 μs ($T_{SHORT} + T_{LONG}$)
T_{SIGNAL} : Duration of the SIGNAL BPSK-OFDM symbol	4.0 μs ($T_{GI} + T_{FFT}$)
T_{GI} : GI duration	0.8 μs ($T_{FFT}/4$)
T_{GI2} : Training symbol GI duration	1.6 μs ($T_{FFT}/2$)
T_{SYM} : Symbol interval	4 μs ($T_{GI} + T_{FFT}$)
T_{SHORT} : Short training sequence duration	8 μs ($10T_{FFT}/4$)
T_{LONG} : Long training sequence duraion	8 μs ($T_{GI2} + 2T_{FFT}$)

- OFDM signal model in IEEE 801.16.e and WiBro
  - OFDM transmit signal

$$s(t) = \mathrm{Re}\left\{ \sum_{\substack{k=-N_{ST}/2 \\ k \neq 0}}^{N_{ST}/2} C_k e^{j2pk\Delta_f (t-t_g)} \right\}, 0 \le t \le T_s$$

- C_k : Complex number of data symbol (QAM)

-$D_f = F_s / N_{FFT}$: Subcarrier frequency spacing

- N_{FFT} : Smallest power by 2 greater than number of used subcarriers

- F_s : Smallest power by 2 greater than number of used subcarriers

- T_g : Guard time to create the cyclic prefix

IDFT-Based Signal Model and its PSD

1. **IDFT-based discrete-time** signal model

- Problem of the non-IDFT signal model : Its straightforward implementation requires $|k|$ local oscillators that are tuned to $|k|$ different subcarrier frequencies where $k = \{k : a_k \neq 0\}$, i.e., $|k|$ is the number of non-null subcarriers.
  - ➔ Idea : Using the K times sampling of the signal $e^{j2pf_k t}$ in the interval

    $0 \le t < T_0$ at the rate K/T_0 for $k=1,2,\ldots,K$
  - ➔ Orthogonal vectors that are proportional to the column vectors of the K-point IDFT matrix
- The most common discrete-time signal model for a single vector of OFDM symbols is

$$\underline{Y} = S\,\underline{X}$$

where $\boldsymbol{S}$ is the K-point IDFT matrix whose (k, l)th entry is given by $s_k[l] = \frac{1}{\sqrt{K}} e^{j\frac{2pkl}{K}}$, $\underline{X}$ is the vector consisting of K data symbols $\{X_k\}_{k=1}^{K}$ and $\underline{Y}$ is the vector of OFDM symbol.

IDFT-Based Signal Model and its PSD (cont.)

1. **IDFT-based discrete-time** signal model (cont.)

- This procedure can be rewritten as

$$\underline{Y} = S\,\underline{X} = X_1 \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot 1}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot 1}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot 1}{K}} \end{bmatrix} + X_2 \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot 2}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot 2}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot 2}{K}} \end{bmatrix} + \cdots + X_K \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi 1\cdot K}{K}} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi 2\cdot K}{K}} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi K\cdot K}{K}} \end{bmatrix}$$

- Operation characteristics
  - The data symbols $\{X_k\}_{k=1}^{K}$ are orthogonally multiplexed.
  - The energy of different data symbol is centered at different subcarrier frequencies.

IDFT-Based Signal Model and its PSD (cont.)

1. **IDFT-based discrete-time** signal model (cont.)

- For a transmission over a continuous-time channel, the OFDM symbol is D/A-converted then up-converted using a local oscillator whose frequency is tuned to the center frequency of the transmitted bandpass signal.

  ➔ performed efficiently without multiple local oscillators

- In many case, IDFT multiplication can be implemented by a computationally efficient inverse Fast Fourier Transform (IFFT) instead of IDFT, then multiplication requires only $O(N \log N)$ operations reduced from $O(N^2)$

- Not for a single OFDM symbol but for a sequence of OFDM symbols, what is the signal model considering all of analog wideband filter, D/A converter, digital interpolation filter, cyclic prefix (or postfix), windowing sequence?

  ➔ Needs some modifications to IDFT-based continuous-time signal model

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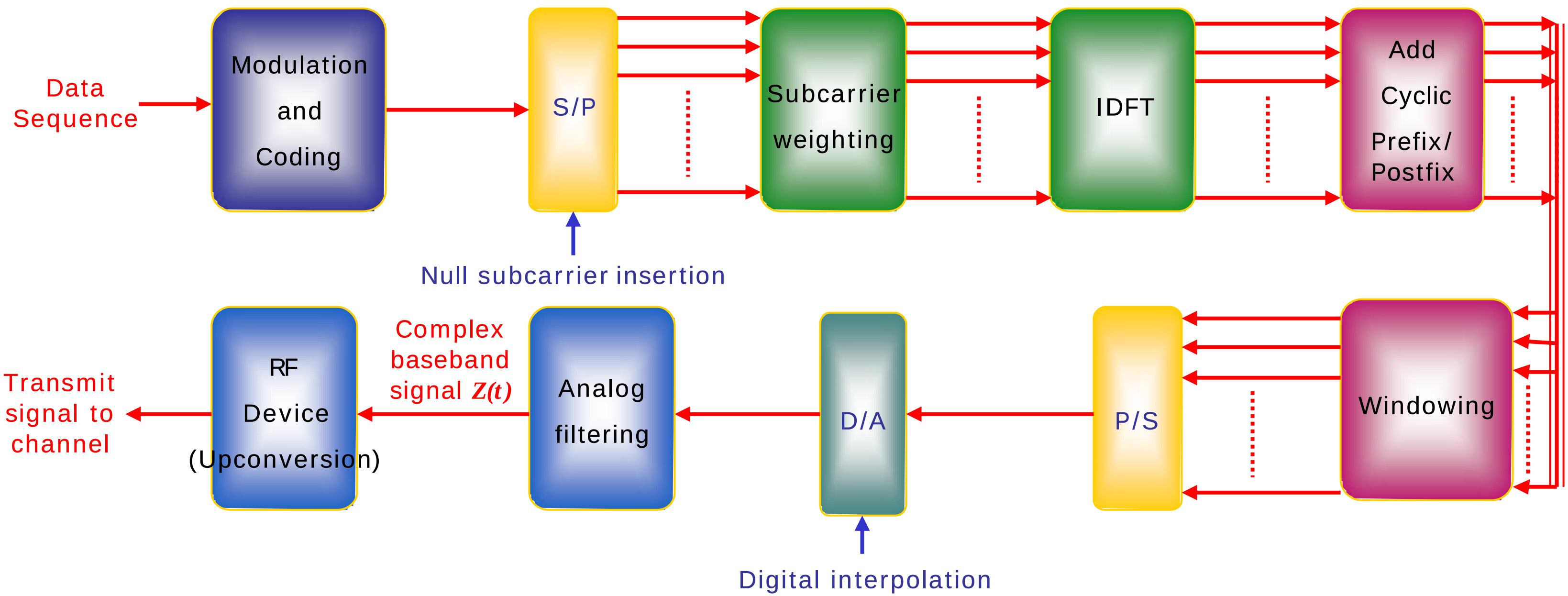
IDFT-Based Signal Model and its PSD (cont.)

2. **IDFT-based continuous-time** signal model

- Revised system structure for IDFT-based transmitter block diagram for OFDM



- IDFT matrix point : K

- Length of cyclic prefix : $L+M$, cyclic postfix : M

($L \geq$ channel length, M : # of overlapped entries of the vectors after P/S)

- Length of windowing sequence : $K+L+2M$

IDFT-Based Signal Model and its PSD (cont.)

❖ 2. **IDFT-based continuous-time** signal model (cont.)

- *D/A converter* : would be functions such as the sample-and-hold

  - If the D/A input is clocked in every T_c [sec], then D/A converter can be viewed as a linear modulator employing a rectangular pulse with duration T_c.

  ➔ The D/A converter combined with the analog filter can be viewed as a linear modulator employing a pulse, say $p(t)$ with the pulse transmission rate $1/T_c$ (also when D/A uses a digital interpolation that operates oversampling).

- *Windowing sequence* : Usually designed to properly weight the first M and the last M symbols,

  ➔ The proper shape of the roll-off of the windowing sequence reduces the inter-channel interference (ICI).

- *Null subcarriers* : Not to use an analog filter with a sharp transition from passband to stopband, some carriers whose the index k 's are around $k \approx K/2$ being nulled out.

  - The useful subcarriers will be less than K, but still K-point IDFT is performed with data symbols.

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IDFT-Based Signal Model and its PSD (cont.)

2. **IDFT-based continuous-time** signal model (cont.)

- General form of the complex baseband equivalent for IDFT-based continuous-time signal model is,

$$Z(t) = \sum_{k=1}^{K} \sum_{m=-\infty}^{\infty} a_k X_k[m] s_k(t - mT_b).$$

- a_k is the subcarrier weighting factor.

- $X_k[m]$ is the mth symbol for the kth subcarrier.

- T_b is the OFDM symol block period given by $T_b = (K + L + M)T_c$.

- T_c is the transmit pulse time duration.

(OFDM subcarrier spacing is $1/T_0$ [Hz] , so $KT_c = T_0$)

- $s_k(t)$ is the kth subcarrier waveform.

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IDFT-Based Signal Model and its PSD (cont.)

2. **IDFT-based continuous-time** signal model (cont.)

- $s_k(t)$ is defined by considering the windowing sequence $w_D[l]$, IDFT matrix element $s_k[l]$, and transmit pulse $p(t)$. So, $s_k(t)$ is given by

$$s_k(t) = \sum_{l=1}^{K+L+2M} a_k[l]\, p(t - lT_c)$$

where $a_k[l] = w_D[l] s_k[l]$ for $k = 1,2,3,\ldots,K$, and $l = 1,2,3,\ldots,K+L+2M$.
($s_k[l]$ is from IDFT matrix (k, l)th element and is defined as $s_k[l] = \frac{1}{\sqrt{K}} e^{j\frac{2pkl}{K}}$, but now up to $l = K+L+2M$)

- $p(t)$ is the equivalent transmit pulse including all the effects of the D/A converter, digital interpolation filter, and analog wideband filter.

- This modified signal model looks similar to a symbol synchronous DS-CDMA signal model where $s_k(t)$ serves as the signature waveform and $a_k[l]$ serves as the signature sequence, both the kth users.

IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **IDFT-based continuous** signal model

- Fourier transform of the kth subcarrier waveform $s_k(t)$

$$S_k(f)=\frac{1}{\sqrt{K}}P(f)\sum_{k=1}^{K+L+2M} w_D[l]\exp\left(-j2p\left(f-\frac{k}{T_0}\right)\right)=\frac{1}{\sqrt{K}}P(f)W_D\left(f-\frac{k}{T_0}\right)$$

- Using above result, PSD of this signal model is obtained as

$$S_{ZZ}(f)=\frac{|P(f)|^2}{(K+L+M)T_0}\sum_{k=1}^{K}|a_k|^2\left|W_D(f-\frac{k}{T_0})\right|^2$$

where $W_D(f)$ is the discrete-time Fourier transform of the windowing sequence $\{w_D[l]\}_{l=1}^{K+L+2M}$ defined as $W_D(f)=\sum_{l=1}^{K+L+2M} w_D[l]e^{-j2pfT_c l}$.

IDFT-Based Signal Model and its PSD (cont.)

❖ Power spectral density (PSD) of **IDFT-based continuous** signal model (cont.)

Non-IDFT-based : $$S_{ZZ}(f) = \frac{|P(f)|^2}{T_b} \sum_{k=1}^{K} |a_k|^2 |W_C(f - f_k)|^2$$

IDFT-based : $$S_{ZZ}(f) = \frac{|P(f)|^2}{(K+L+M)T_0} \sum_{k=1}^{K} |a_k|^2 \left| W_D(f - \frac{k}{T_0}) \right|^2$$

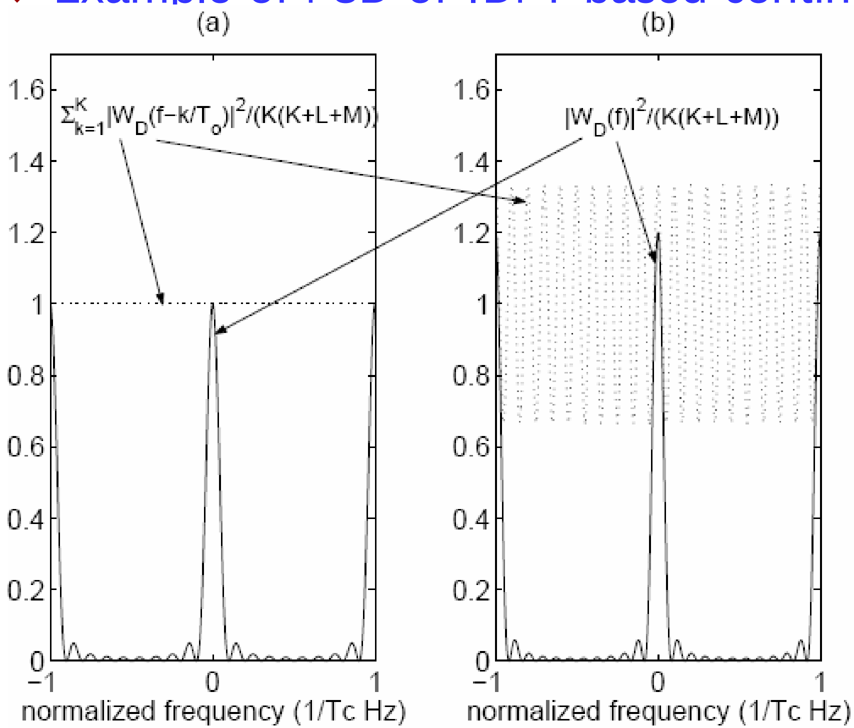
- Significant difference from the non-IDFT-based model is that the Fourier transform of the windowing sequence $\{w_D[l]\}_{l=1}^{K+L+2M}$ is used instead of that of the continuous windowing function $w_c(t)$
- [Note] Two properties of $W_D(f)$

  (1) $W_D(f)$ is periodic with a period $1/T_c$

  (2) $W_D(f - k/T_0)$ is periodic in k with a period K under the assumption $KT_c = T_0$

IDFT-Based Signal Model and its PSD (cont.)

❖ Example of PSD of IDFT-based continuous signal model



- Parameter

Rectangular windowing sequence for

(a) : $K = 10, L + M = 0$

(b) : $K = 10, L + M = 2$

- [Note] $|W_D(f)|^2 = \left|\frac{sin(pf(K+L+2M)T_c)}{sin(pfT_c)}\right|$

- Increase in (K+L+2M) ➔ Decrease in the width of the main lobe and increase in the amount of in-band energy contained in the interval $-\frac{1}{2T_0} \le f \le \frac{1}{2T_0}$
- Periodic with period $1/T_c$

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IDFT-Based Signal Model and its PSD (cont.)

❖ New insight into PSD of OFDM signals

- Definition

  If the aggregate transmit pulse $p(t)$ has a positive excess bandwidth, then, due to the periodicity of $W_D(f)$, a logical subcarrier can have the signal energy concentrated at $f = \frac{k + lK}{T_0}$ for some integers l, that is, a logical subcarrier can have more than one physical subcarriers.

  ➔ *Nominal subcarrier* : Physical subcarrier with the greatest energy, and

  Ghost subcarriers : Other physical subcarriers except the *nominal subcarrier*

- For some OFDM logical subcarriers, we may observe two physical subcarriers, the nominal subcarriers and ghost carrier, using IDFT-based continuous signal model.

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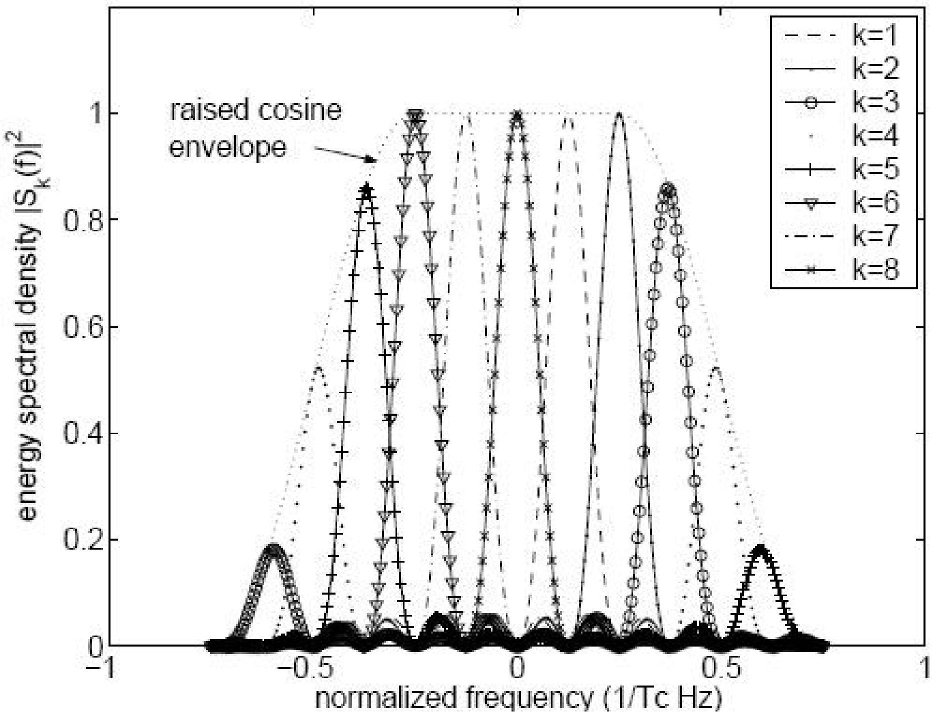
IDFT-Based Signal Model and its PSD (cont.)

❖ Example : Energy spectral density of the subcarrier waveforms



- $p(t)$ is the SRRC pulse with β=0.5, *K*=8, *L*=0, *M*=0, *no null subcarrier*
- The subcarriers with indexes k=3,4, and 5 have the significant portion of the energy split into two physical subcarriers located $1/T_c$ [Hz] apart.

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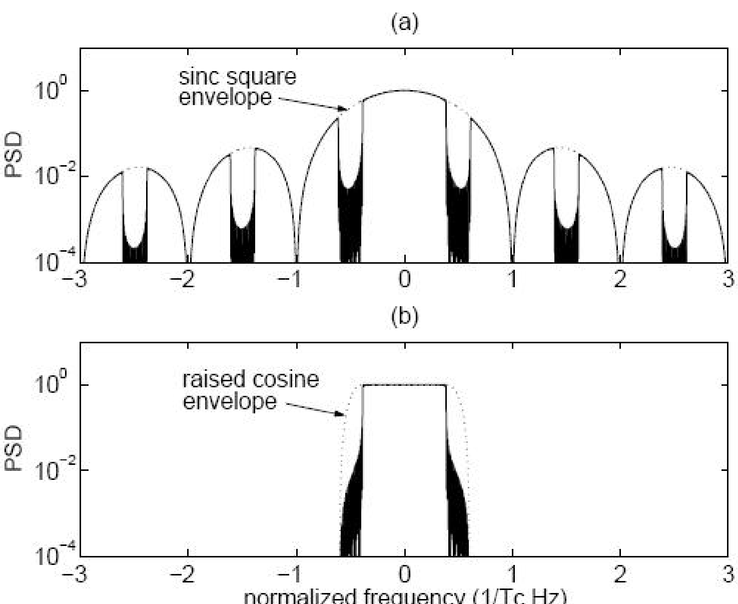
IDFT-Based Signal Model and its PSD (cont.)

❖ Example : Virtual roll-off effect by using proper null-subcarrier

- Very fast roll-off in the PSD may be needed, but designing such a filter is costly.

  ➔ Because we need a high-rate D/A converter, high order wideband transmit filter

- Efficient solution : By nulling subcarriers with indexes $k \approx K/2$



- PSD $S_{ZZ}(f)$ of complex baseband signal $Z(t)$
- Parameter
  - *K=128, L=0, M=0, 29 null-subcarriers*
  - *Sample-and-hold in D/A converter*
- (a) : No wideband analog filtering

  (b) : Use of an analog filter to make p(t) the SRRC pulse with β=0.2

➔ Possible to make a transition band very small

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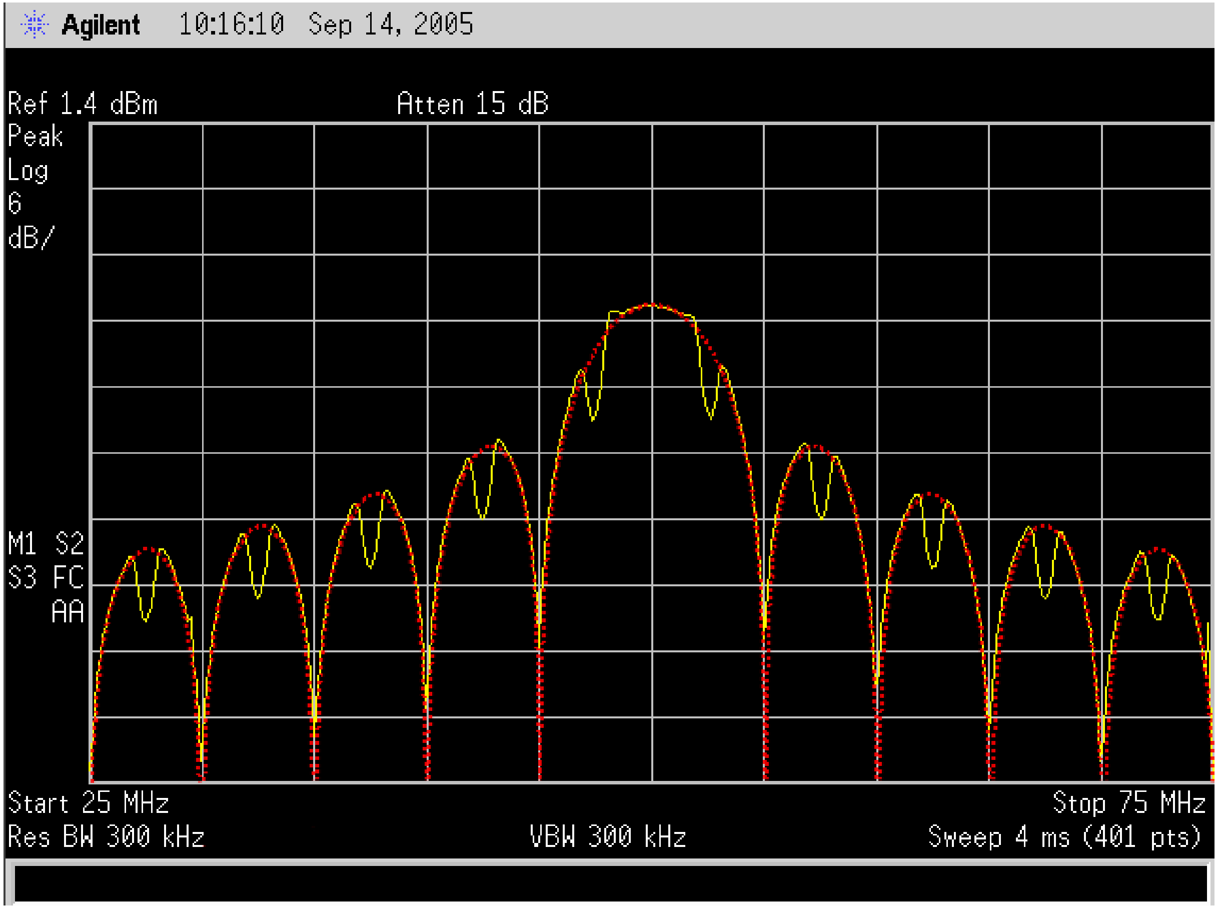
IDFT-Based Signal Model and its PSD (cont.)



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IDFT-Based Signal Model and its PSD (cont.)

- In WiBro standard, the signal model is given by

$$s(t) = \mathrm{Re}\left\{ \sum_{\substack{k=-N_{ST}/2 \\ k\neq 0}}^{N_{ST}/2} C_k e^{j2pk\Delta_f (t-t_g)} \right\}, 0 \le t \le T_s$$

  - A rectangular windowing function without any window overlapping is used.
  - Non-IDFT-based signal model is used.

- To use IFFT in the transmitter, we need the following or an equivalent :
  - A rectangular windowing sequence without any window overlapping
  - A sample-and-hold D/A converter with inverse-sinc shaped transmit filter in the frequency band for non-null subcarriers

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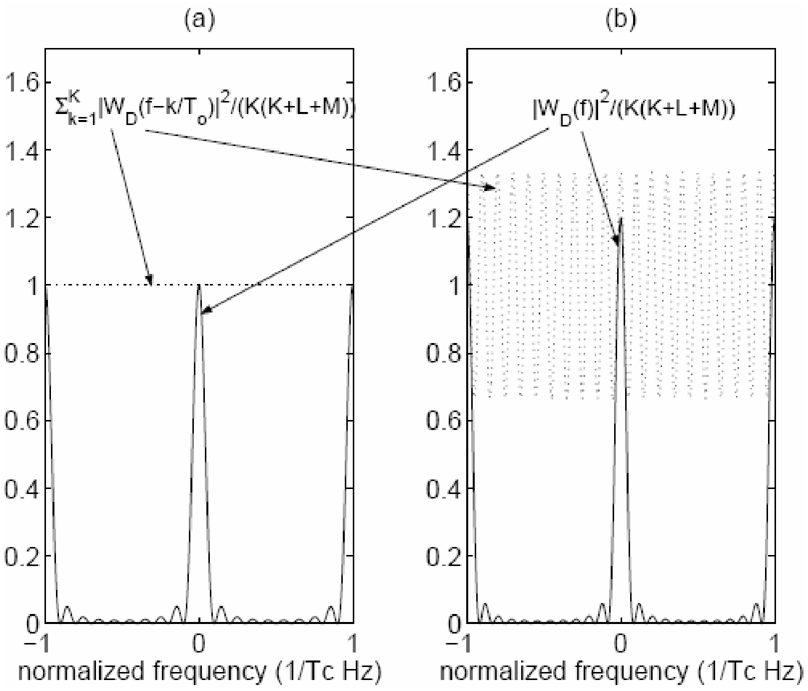
IDFT-Based Signal Model and its PSD (cont.)



$$|W_D(f)|^2 = \left|\frac{\sin(pf(K+L)T_c)}{\sin(pfT_c)}\right|^2 \approx \left|\frac{\sin(pf(K+L)T_c)}{pfT_c}\right|^2$$

for *large* $K+L$ *and*
for $f \approx 0$

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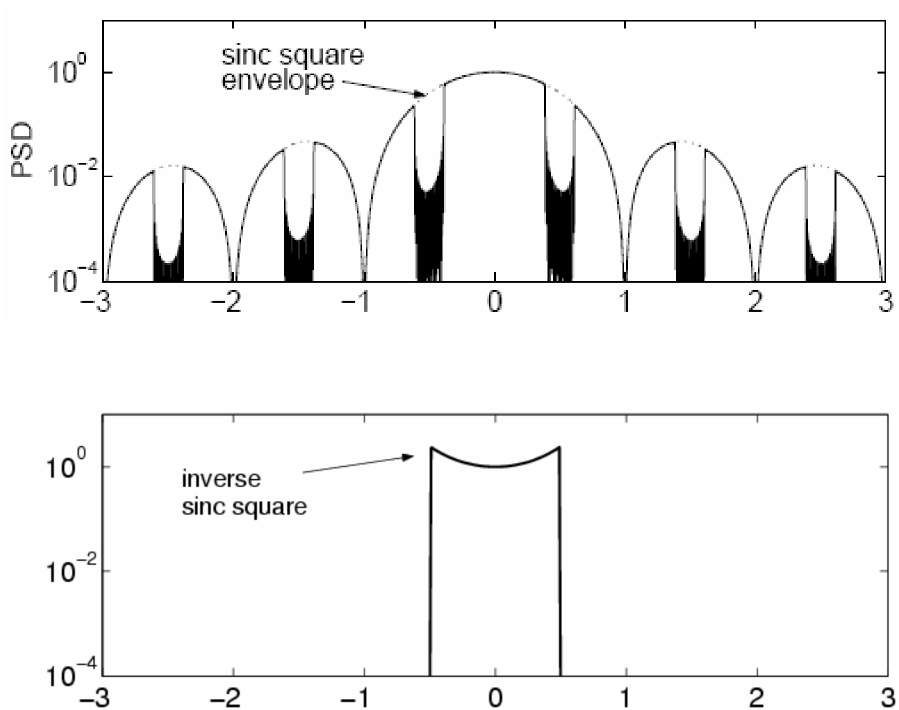

- The transmit filter must invert the sinc square envelope to equally distribute the power among non-null subcarriers.
- The transition from the passband to stopband must lie over the null-subcarrier frequency band.

Conclusions

- D/A conversion is a necessary procedure in modern digital audio systems and in digital communication systems.

$$Y_c(f)=P(f)X_d(fT)$$

- Upsampling combined with interpolation is often used in commercial DACs and in modern radio transceiver.

$$Y_d(f)=X_d(fN)$$

- IFFT-based OFDM signaling also requires D/A conversion.

$$S_{ZZ}(f)=\frac{|P(f)|^2}{(K+L+M)T_0}\sum_{k=1}^{K}|a_k|^2\left|W_D\left(f-\frac{k}{T_0}\right)\right|^2$$

  - To ease the burden of designing a good interpolation filter, null subcarriers are used.
  - The interpolation filter must have the transition from passband to stopband in the null subcarriers' frequency band.

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