Wireless Channels

Overview

In this note, we study wireless channels. Especially, we study the linear time-varying channel model and related concepts.

We start from the review of linear system theory.

Review of linear system theory

- ~~Def.~~

A **linear system** is a triplet $(\mathcal{D}, L, \mathcal{R})$ where

$\mathcal{D}$ is the set of admissible **inputs**,
$\mathcal{R}$ is the set of possible **outputs**, and
L is a mapping rule from $\mathcal{D}$ to $\mathcal{R}$

such that

$$x \in \mathcal{D},\ y \in \mathcal{D},\ a \in \mathbb{R} \ (\text{or } \mathbb{C}), \text{ and } b \in \mathbb{R} \ (\text{or } \mathbb{C})$$

$$\Rightarrow L(ax+by) = aL(x) + bL(y)$$

, which is called the linearity or **superposition property**.

- Consider a linear system with $\mathcal{D}$ and $\mathcal{R}$ both being the set of discrete-time signals with complex-valued entries.

Let $\{x[n]\}_{n\in\mathbb{Z}}$ be the input signal and
let $\{y[n]\}_{n\in\mathbb{Z}}$ " " output signal.

By using the Dirac delta functions, $x[n]$ can be written as

$$x[n] = \sum_{m=-\infty}^{\infty} x[m]\,\delta[n-m]$$

$$= \cdots x[-1]\,\delta[n+1] + x[0]\,\delta[n] + \cdots$$

Note that $x[m]$ is not a function of n. $\delta[n-m]$ is the function of n. Thus,

$$L(x[n]) = L\left(\sum_{m=-\infty}^{\infty} x[m]\delta[n-m]\right)$$

$$= \sum_{m=-\infty}^{\infty} x[m]\, L(\delta[n-m])$$

- Def. The **impulse response** $h[n,m]$ of a discrete-time input / discrete-time output linear system is defined as

$$\mathbf{h[n,m]} \triangleq \underbrace{L(\delta[n-m])}_{}$$

the **response** of the system **at** time **n** **to** the **impulse** input **at** time **m**

- Therefore, we obtain the input/output relation

$$y[n] = \sum_{m=-\infty}^{\infty} h[n,m]\, x[m]$$

Similarly, we obtain **response at t to impulse at τ.**

$$y(t) = \int_{-\infty}^{\infty} \overbrace{\mathbf{h(t,\tau)}} x(\tau)\, d\tau$$

for a continuous-time input / continuous-time output linear system.

- Def.

A **linear time-invariant (LTI) system** is a linear system such that

$$y(t) = L(x(t)) \Rightarrow y(t-t_0) = L(x(t-t_0)) \quad \forall t_0$$
$$y[n] = L(x[n]) \Rightarrow y[n-n_0] = L(x[n-n_0]) \quad \forall n_0$$

- Now, consider an impulse input at time 0. Then,

$$h(t,0) = L(\delta(t)) \qquad h[n,0] = L(\delta[n])$$

Since the system is LTI,

$$h(t-t_0, 0) = L(\delta(t-t_0)) = h(t, t_0) \qquad h[n-n_0, 0] = L(\delta[n-n_0]) = h[n, n_0]$$

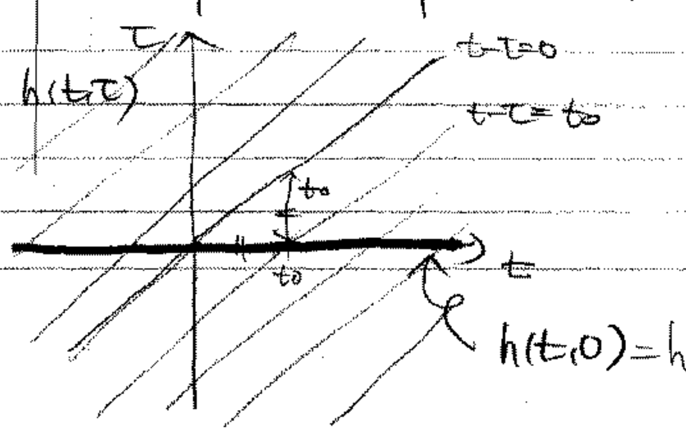
$$\therefore\ h(t-\tau, 0) = h(t,\tau) \triangleq h(t-\tau) \qquad \therefore\ h[n-m, 0] = h[n,m] \triangleq h[n-m].$$

This leads to the well known **convolution integral & sum** as

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

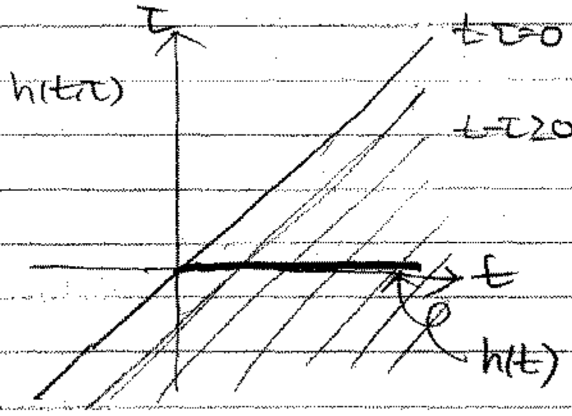
$$y[n] = \sum_{m=-\infty}^{\infty} h[n-m]\, x[m].$$

- Given an impulse response $h(t,\tau)$, if $h(t,\tau)$ does not change on each line $t-\tau=\xi$, then it is an impulse response of an LTI system.



$$h(t,\tau) = h(t+t_0, \tau+t_0) = h(t-\tau, 0)$$

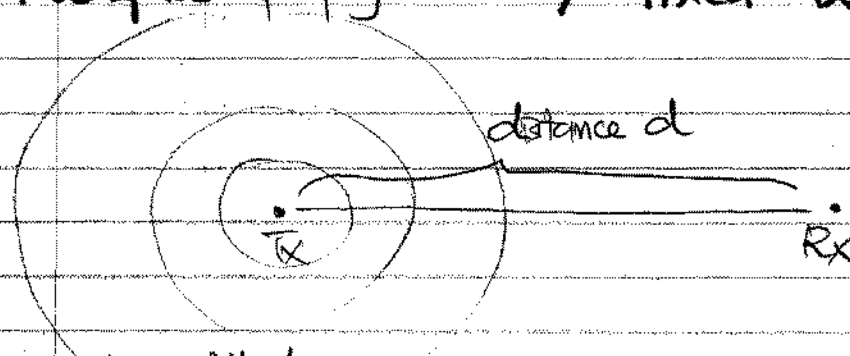
- Furthermore, if an LTI system is causal, then $h(t,\tau)$ has nonzero value only on $t-\tau \geq 0$, i.e.,



○ Wireless channels as linear systems.

- Consider a wireless channel through which a modulated EM wave propagates. Also consider a Tx that emits the wave and an Rx that receives the wave.

(i) **Free space propagation w/ fixed Tx & Rx**



The transmitted signal $s(t) = \mathrm{Re}\{s_0(t)e^{j2\pi f_0 t}\}$ will be received at Rx with a gain α_0 and

a delay $\Delta_0 \left(= \frac{d}{c} > 0\right)$ (c: speed of wave.)

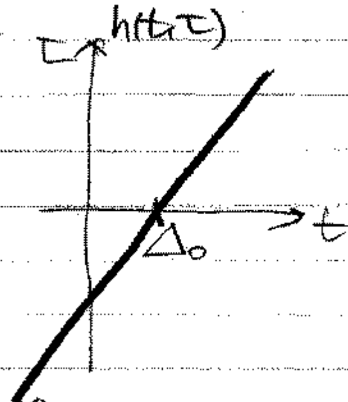
If the Tx signal is $s(t-t_0)$ then the Rx signal becomes $\alpha_0 s(t-t_0-\Delta_0)$.

In addition, due to the property of the EM wave, the superposition principle holds

$\therefore$ **This** wireless **channel is** well **modeled as** an **LTI** system **with** impulse response

$$h(t,\tau) = \alpha_0 \delta(t-\tau-\Delta_0)$$
$$= h(t-\tau)$$

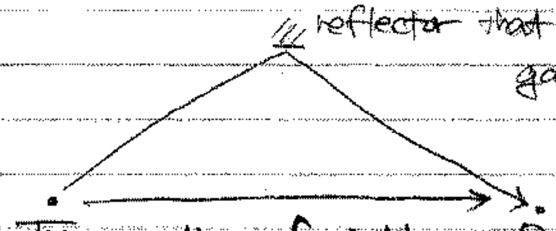
$\Leftrightarrow$ causal, LTI



impulse wall with height α_0 on $t-\tau=\Delta_0 (>0)$

(ii) Free space propagation w/ fixed Tx & Rx and a reflector

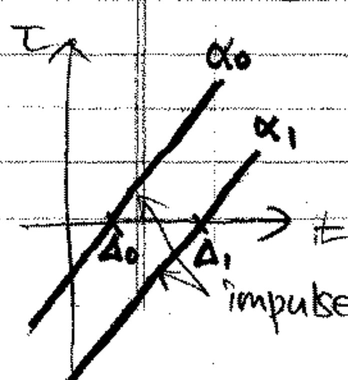
reflector that introduces gain α_1 and delay Δ_2 $(>\Delta_1)$



line-of-sight **(LOS)** propagation path

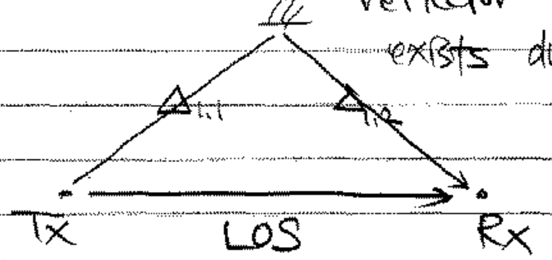
This channel is well modeled as an LTI system with

$$h(t,\tau) = \alpha_0 \delta(t-\tau-\Delta_0) + \alpha_1 \delta(t-\tau-\Delta_1)$$
$$= h(t-\tau)$$



impulse walls. $\Leftrightarrow$ causal, LTI

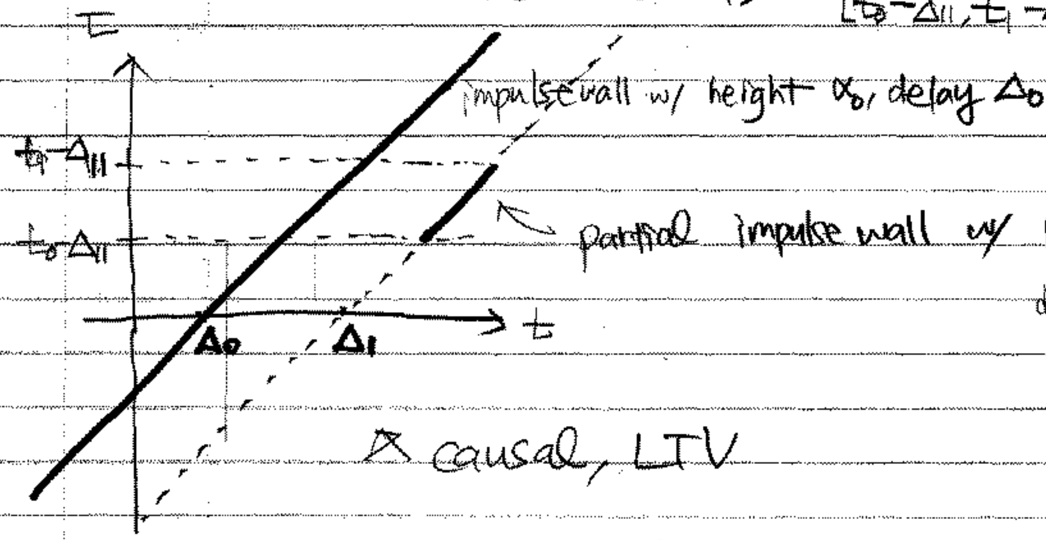
(iii) Free space propagation w/ fixed Tx & Rx and a temporary reflector



Only the Tx signal that existed during $t \in [t_0-\Delta_{11}, t_1-\Delta_{11}]$ can be reflected by the reflector.

This channel is well modeled as an LTV system with
(linear time-varying)

$$h(t,\tau) = \alpha_0 \delta(t-\tau-\Delta_0)$$
$$+ \alpha_1 \delta(t-\tau-\Delta_1) \, 1_{[t_0-\Delta_{11}, t_1-\Delta_{11}]}(\tau)$$



- As we see in (iii), if (scatterers / reflectors) come and go, a wireless channel can no longer be modeled as a causal **LTI** system but a causal **LTV** system.

o Review of convolution integral

• LTV: $x(t) \rightarrow \boxed{h(t,\tau)} \rightarrow y(t) \Leftrightarrow y(t)=\int_{-\infty}^{\infty} h(t,\tau)\,x(\tau)\,d\tau$

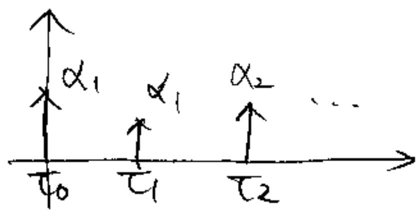
• In addition, if LTI then $h(t-\tau) \triangleq h(t-\tau,0) \Rightarrow$

$$y(t)=\int_{-\infty}^{\infty} h(t-\tau)\,x(\tau)\,d\tau \quad \cdots\cdots (*)$$

$$=\int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau \quad \cdots\cdots (**)$$

• Take a careful look at (**). (**) says the output y at time t is determined by $h(\tau)$.

If $h(\tau)=\sum_{i=1}^{\infty}\alpha_i\,\delta(\tau-\tau_i)$, then we have



which best illustrates the, in this case, specular multipath components.

Thus, (**) is better than (*) in letting us understand the effect of channel.

o Reparameterization of $h(t,\tau)$.

• Now, let's find out the equivalent representation to (**)

$$y(t)=\int_{-\infty}^{\infty} h(t;\tau)\,x(\tau)\,d\tau \qquad \tau = t-\xi$$

$$=\int_{-\infty}^{\infty} h(t;t-\xi)\,x(t-\xi)\,d\xi \qquad h(t;t-\xi) \triangleq h_t(\xi)$$

$$=\int_{-\infty}^{\infty} h_t(\xi)\,x(t-\xi)\,d\xi$$

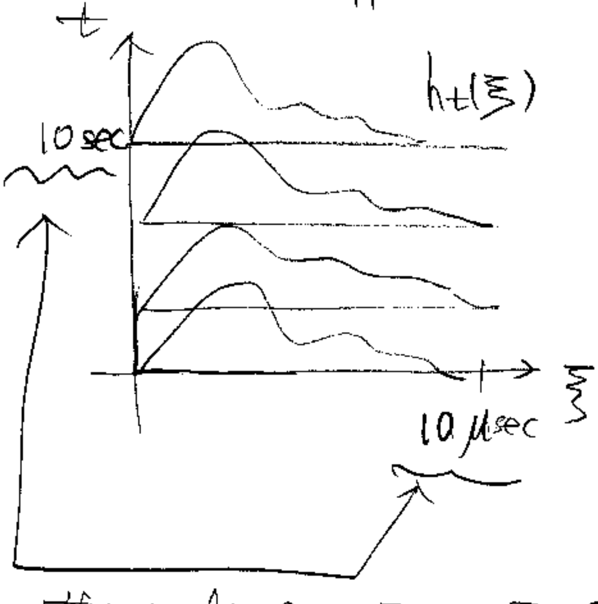
So, the current output at t views $h_t(\xi)$ as the impulse response.

- Consider very slowly time-varying channel relative to the signal duration of input. Then, the output $y(t)$ can be approximated as

$$y(t) \simeq \int_{-\infty}^{\infty} h_t(\xi)\,x(t-\xi)\,d\xi$$

can be interpreted as "approximate impulse response at time t"

ex)



the scale in ξ axis is much smaller than the channel variation
impulse response

- Instead of $h_t(\xi)$, we also use $C(\xi,t)$ in what follows.

○ Representation of impulse response $h(t,\tau)$ using $c(\xi,t)$

- Recall the impulse response of the case (iii),

$$h(t,\tau) = \alpha_0\delta(t-\tau-\Delta_0) + \alpha_1\delta(t-\tau-\Delta_1)\, h_1(\tau)$$

$$\triangleq 1_{[t_0-\Delta_{11},\, t_f-\Delta_{11}]}$$

By the relation b/w input & output, the channel output $y(t)$ to the input $x(t)$ is given by

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, x(\tau)\, d\tau$$

cumbersome.

$$= \int_{-\infty}^{\infty} \Big(\alpha_0\,\delta(t-\tau-\Delta_0) + \alpha_1\,\delta(t-\tau-\Delta_1)\, h_1(\tau)\Big)\, x(\tau)\, d\tau$$

$t-\tau=\xi$

$$= \int_{-\infty}^{\infty} h(t, t-\xi)\, x(t-\xi)\, d\xi$$

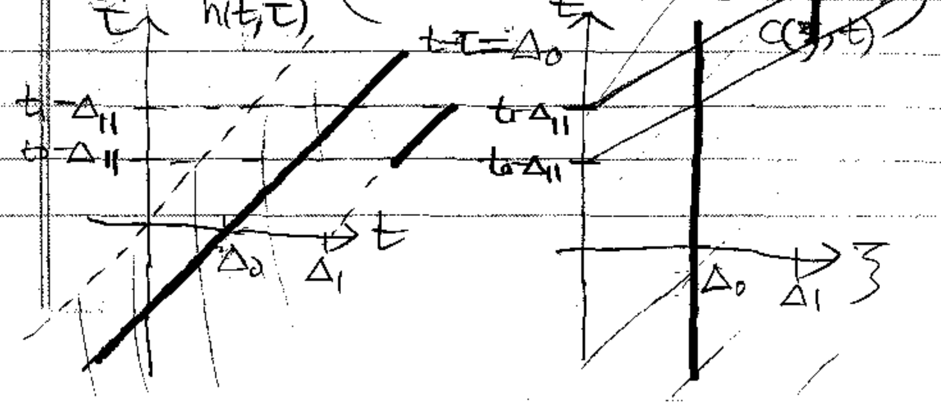
$$= \int_{-\infty}^{\infty} \Big(\alpha_0\delta(\xi-\Delta_0) + \alpha_1\,\delta(\xi-\Delta_1)\, \underbrace{h_1(t-\xi)}_{1_{[t_0+\xi-\Delta_{11},\; t_f+\xi-\Delta_{11}]}(t)}\Big)\, x(t-\xi)\, d\xi$$

$$\triangleq \int_{-\infty}^{\infty} c(\xi,t)\, \underbrace{x(t-\xi)\, d\xi}_{\text{input that affects } y(t) \text{ w/ prop. delay } \xi}$$

propagation delay b/w Tx & Rx

$\therefore\ c(\xi,t) = h(t, t-\xi)$ $\quad t_0-\Delta_{11} \le t-\xi \le t_f-\Delta_{12}$

$$\Big(h(t,\tau) = c(t-\tau, \tau)\Big)$$



- If time-invariant, then

$$c(\xi, t) = c(\xi, t_0) \quad \forall t_0$$
$$= c(\xi)$$

$$\therefore \; y(t) = \int_{-\infty}^{\infty} \underbrace{c(\xi)}_{\text{propagation delay}} x(t-\xi)\, d\xi$$

$$= \int_{0}^{\infty} c(\xi)\, x(t-\xi)\, d\xi \quad (\because \text{causal})$$

- For channel modeling, we adopt the impulse response in the form of

$$c(t-\tau, t)$$

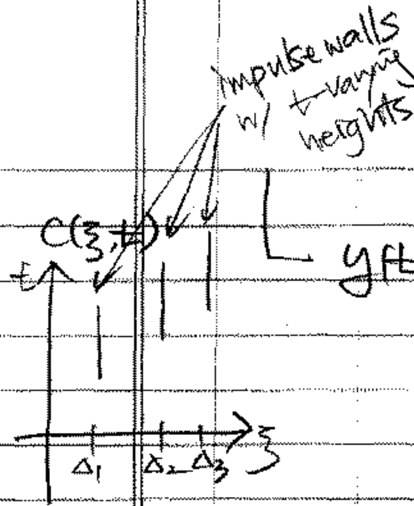
which is the response at time t by the impulse at τ.

As seen in the above, this is because $c(\xi; t)$ vividly shows the contribution of the reflectors that cause propagation delay ξ to the output at t.

○ Examples

(i) Time-varying specular multipath channel w/ fixed **multipath delay profile**

$$c(\xi, t) = \sum_{n=1}^{N} \underbrace{\alpha_n(t)}_{\text{a complex valued signal}} \delta(\xi - \underbrace{\Delta_n}_{\text{delay of the } n\text{th path}})$$

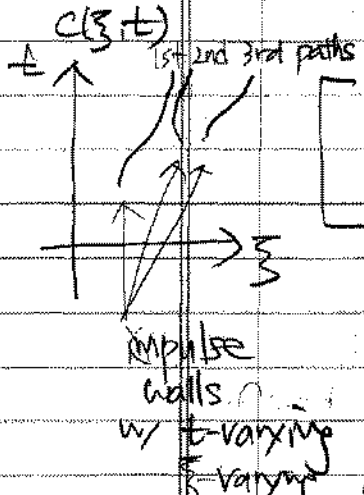


$$y(t) = \int_{-\infty}^{\infty} c(\xi,t)\, x(t-\xi)\, d\xi$$

$$= \int_{-\infty}^{\infty} \sum_{n=1}^{N} \alpha_n(t)\, \delta(\xi-\Delta_n)\, x(t-\xi)\, d\xi$$

$$= \sum_{n=1}^{N} \alpha_n(t)\, x(t-\Delta_n) \qquad \because \int_{-\infty}^{\infty} \delta(\xi-\Delta) f(\xi)\, d\xi = f(\Delta)$$

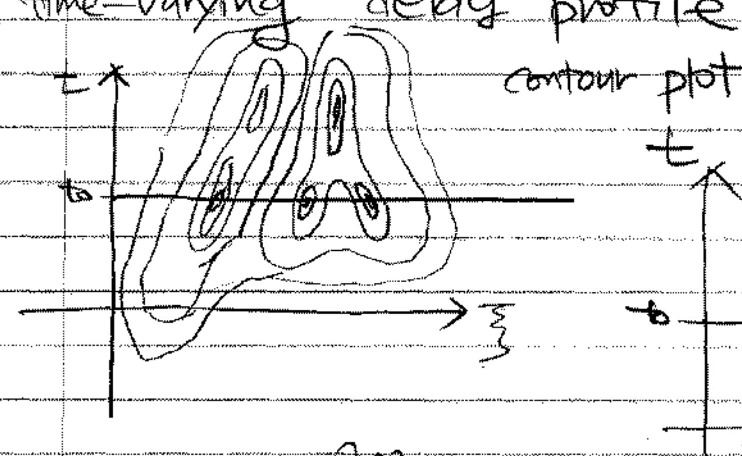
(ii) Time-varying specular multipath channel w/ time-varying delay profile

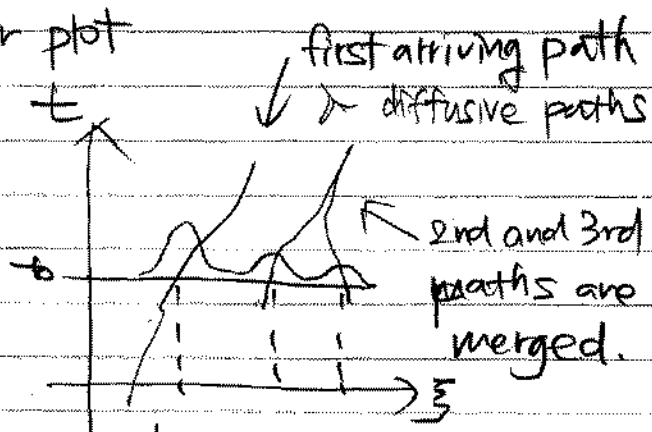


$$c(\xi,t) = \sum_{n=1}^{N} \alpha_n(t)\, \delta(\xi - \Delta_n(t))$$

$$y(t) = \sum_{n=1}^{N} \alpha_n(t)\, x(t-\Delta_n(t))$$

(iii) Time-varying dispersive multipath channel w/ time-varying delay profile





$$y(t_0) = \int_{-\infty}^{\infty} \underbrace{c(\xi,t_0)}_{\substack{\text{instantaneous impulse} \\ \text{response of the channel}}} x(t_0-\xi)\, d\xi$$

○ References

[1] J.W. Mark & W. Zhuang, Wireless Communications & Networking, Prentice Hall pp. 16–18

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[3] J.G. Proakis, Digital Communications, ~~2nd Ed.~~ McGraw Hill, pp. 758 – 761