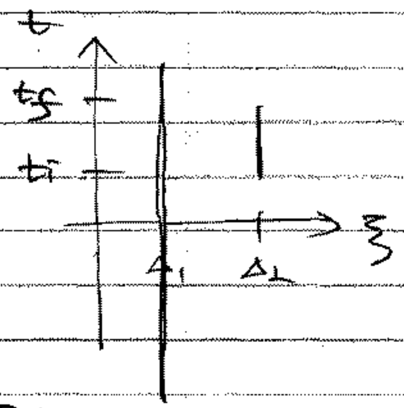
Wireless Channels : Fading Phenomenon.

ⓞ Speed of light, Propagation delay, and Bandwidth

- As we know, the speed of light is $C = 3\times10^8$ m/s. Thus, you may say that the **multipath** scenario may not leads to resolvable multipath components in the received signal.

Remind that one of the example channels had the channel response $(\Delta_1 < \Delta_2)$

$$C(\xi, t) = \underbrace{\alpha_0\,\delta(\xi-\Delta_1)}_{\text{1st arriving path. static}} + \underbrace{\alpha_1\,\delta(\xi-\Delta_2)}_{\text{2nd arriving path}}\underbrace{1_{[t_i, t_f]}(t)}_{\text{come \& go}}$$



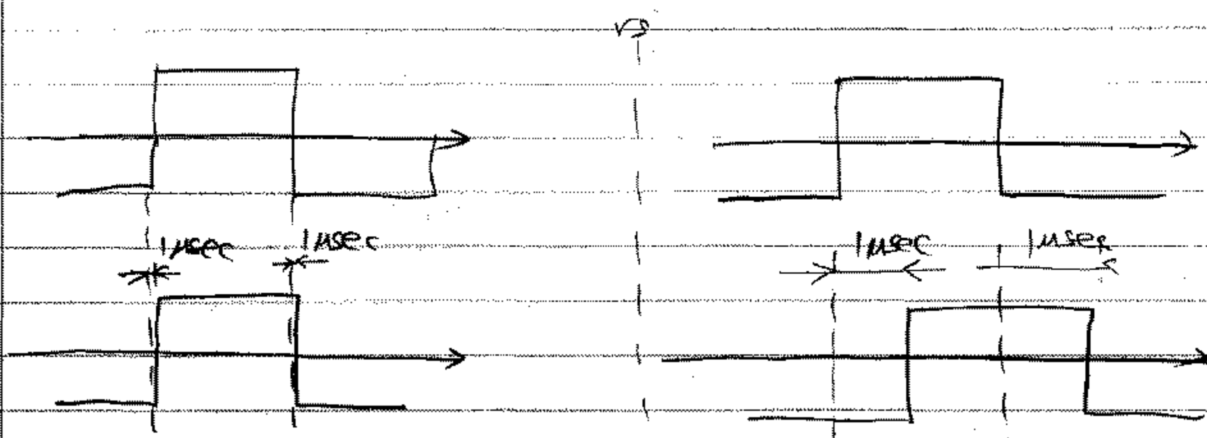
Q1.
The question is, $|\Delta_1 - \Delta_2|$ is significant? i.e. can we just say that $\Delta_1 \simeq \Delta_2$?
A1
The answer is, "It depends on the bandwidth of the transmitted signal"

- Q2. Consider a cellular system where a mobile station receives a LOS signal that travels 1 km and a non-LOS signal that travels

1.3km from the basestation. What is the difference in propagation delay?

A2. $\frac{300m}{3\times10^8 m/s} = 10^{-6} sec = 1 \mu sec$ ← 1 chip delay in IS-95.

So, if the transmitted signal is a linearly modulated signal w/ $T \gg 1\mu sec$ than we can set $\Delta_1 \simeq \Delta_2$, while if $T \lesssim 1\mu sec$, we can not set $\Delta_1 \simeq \Delta_2$



In summary, given the same multipath delay profile of a channel, a wider bandwidth signal suffers more distortion.

Some people categorize wireless systems as narrowband vs. wideband according to the degree of distortion by the multipath channel

already studied

I. narrowband : if excess bandwidth $\simeq 0$
   wideband : if excess bandwidth $\gg 1$

II. narrowband : if delay spread $\ll$ 1/bandwidth
   wideband : if delay spread $\gtrsim$ 1/bandwidth

III. narrowband : if $f_0 \gg BW$
   wide band : if $f_0 > BW$

Two path deterministic channel &
frequency selectivity

- In time domain

$$h(\xi) = \alpha_1 \delta(\xi - \Delta_1) + \alpha_2 \delta(\xi - \Delta_2)$$

- In freq. domain

$$H(f) = \alpha_1 e^{-j2\pi f \Delta_1} + \alpha_2 e^{-j2\pi f \Delta_2}$$

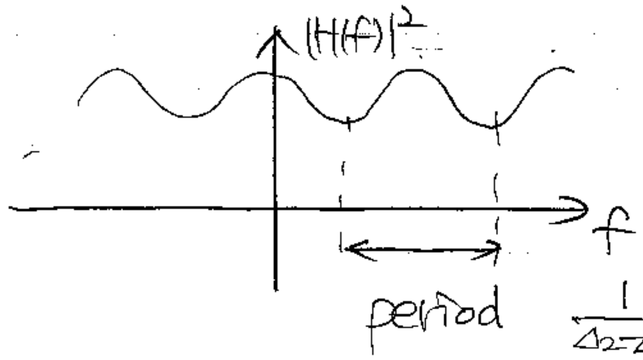
$$\Rightarrow |H(f)| = |\alpha_1 + \alpha_2 e^{-j2\pi f(\Delta_2 - \Delta_1)}|$$

Note that the magnitude response depends only on the time difference $\Delta_2 - \Delta_1$!!

$$|H(f)|^2 = |\alpha_1|^2 + |\alpha_2|^2 + 2\mathrm{Re}\{\alpha_1 \alpha_2^* e^{j2\pi f(\Delta_2 - \Delta_1)}\}$$

$$= |\alpha_1|^2 + |\alpha_2|^2 + 2|\alpha_1||\alpha_2| \mathrm{Re}\{e^{j(2\pi f(\Delta_2 - \Delta_1) + \theta)}\}$$

$$= |\alpha_1|^2 + |\alpha_2|^2 + 2|\alpha_1||\alpha_2| \cos(2\pi f(\Delta_2 - \Delta_1) + \theta)$$



- If the channel input has bandwidth $B \ll \frac{1}{\Delta_2 - \Delta_1}$ then the " looks almost flat

$$\because \quad B(\Delta_2 - \Delta_1) \ll 1 \Rightarrow \text{flat}$$

To the contrary,

$$\underbrace{B(\Delta_2 - \Delta_1)} \gtrsim 1 \Rightarrow \text{freq. selective}$$

In summary, " bandwidth – delay spread product " determines the severity of freq. selectivity.

○ Non-resolvable multipath case

- Consider a wireless channel with channel response

$$c(\xi, t) = \alpha_0 \delta(\xi - \Delta_0) + \alpha_1 \delta(\xi - \Delta_1)$$

and the transmitted signal

$$s(t) = \mathrm{Re}\{ s_\ell(t) e^{j2\pi f_c t} \}.$$

Then, as we learned already, the received signal $r(t)$ is given by

$$r(t) = \int_{-\infty}^{\infty} c(\xi, t) s(t-\xi) d\xi$$

$$= \alpha_0 s(t-\Delta_0) + \alpha_1 s(t-\Delta_1)$$

$$= \alpha_0 \mathrm{Re}\{ s_\ell(t-\Delta_0) e^{j2\pi f_c (t-\Delta_0)} \} + \alpha_1 \mathrm{Re}\{ s_\ell(t-\Delta_1) e^{j2\pi f_c (t-\Delta_1)} \}$$

$$= \mathrm{Re}\{ (\alpha_0 e^{-j2\pi f_c \Delta_0} + \alpha_1 e^{-j2\pi f_c \Delta_1}) s_\ell(t-\Delta) e^{j2\pi f_c t} \}$$

if $|\Delta_0 - \Delta_1| \ll 1/\text{bandwidth of } s_\ell(t)$. $(\Delta \triangleq \Delta_0 \simeq \Delta_1)$

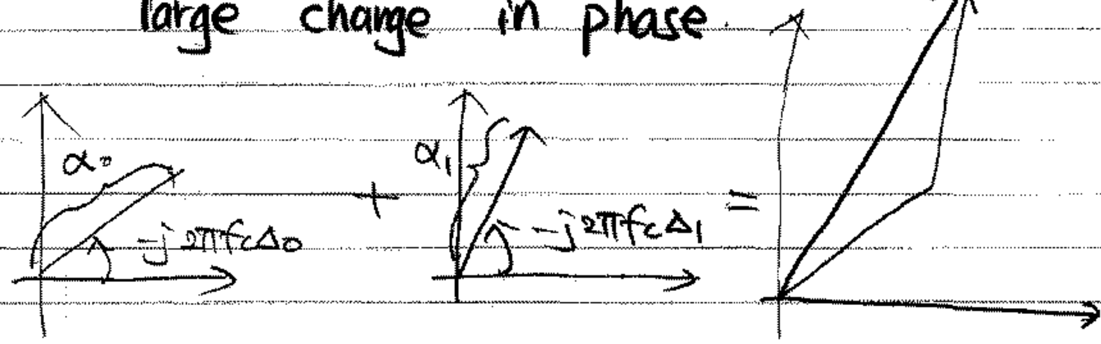
- Although the narrowband signal assumption made it possible to combine two multipath components into a single delay signal, the complex gain

$$\alpha_0 e^{-j2\pi f_c \Delta_0} + \alpha_1 e^{-j2\pi f_c \Delta_1}$$

cannot be combined as $\simeq (\alpha_0 + \alpha_1) e^{-j2\pi f_c \Delta}$ because $f_c \Delta_0$ can be significantly different from $f_c \Delta_1$

ex) $f_c = 1\,\mathrm{GHz}$, $\Delta_0 - \Delta_1 = 10^{-6}\,\mathrm{sec} \Rightarrow f_c\Delta_0 - f_c\Delta_1 = 10^3$

- Note that **small change in delay** can cause **large change in phase**.



- We here in non-resolvable multipath case have a dilemma.

$$\Delta \simeq \Delta_0 \simeq \Delta_1 \simeq \Delta_2 \simeq \cdots \simeq \Delta_{N-1}$$

holds for $s_e(t-\Delta_n) \cong s_e(t-\Delta)$, it does not hold for

$$\{e^{-j2\pi f \Delta_n}\}_{n=0}^{N-1}$$

- In case we have a **very large number of non-resolvable multipath**, the central limit theorem is often resorted to.

The claim is that let's model that α_n's are independent random variables & that $2\pi f_c \Delta_n$'s are i.i.d. uniform random variables on $[0, 2\pi)$

Then, it would be O.K. to approximate

$$\sum_{n=0}^{N-1} \alpha_n e^{-j2\pi f_c \Delta_n}$$

as a **proper-complex Gaussian** random variable α with mean zero.

$$\therefore \quad r(t) = \mathrm{Re}\{\alpha s_e(t-\Delta) e^{j2\pi f_c t}\}$$

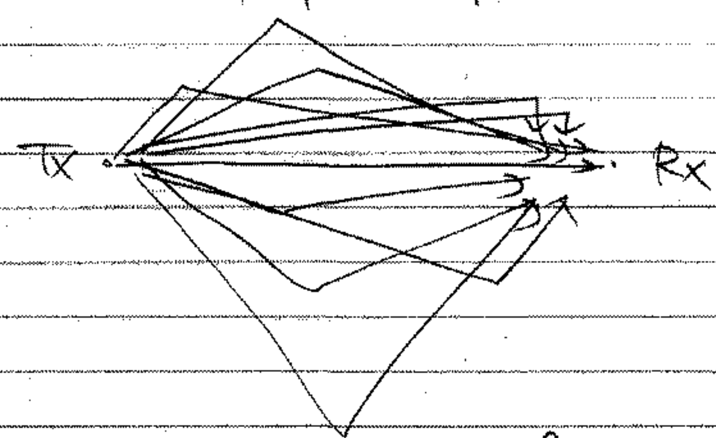
$$\alpha \sim \mathcal{CN}(0, \sigma_\alpha^2)$$

$$= \mathrm{Re}\left\{ \underbrace{|a|}_{\text{a Rayleigh random variable}} e^{j\overbrace{\angle a}^{\text{a uniform random variable}}} s_b(t-\Delta)\, e^{j2\pi f_c t} \right\}$$

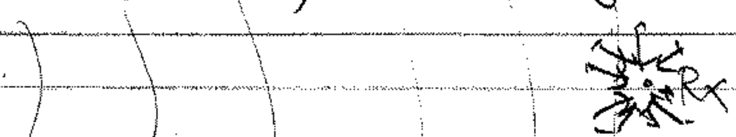
- If a LOS dominates, then Rician approximation is often used.

○ Motion & Fading

- We've just seen that the case with large non-resolvable multipaths enable us to model the complex channel gain as a proper-complex Gaussian r.v.



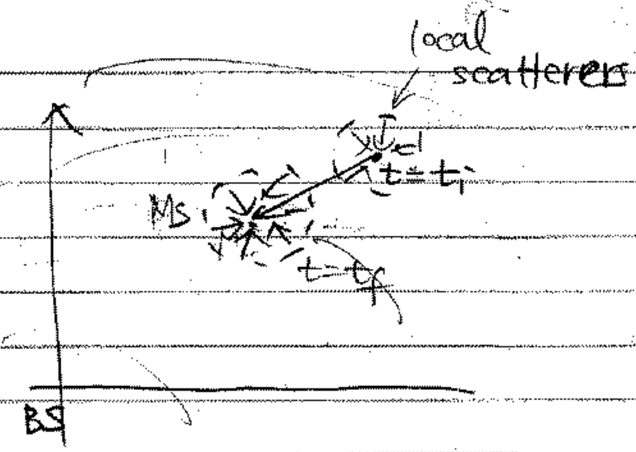
This occurs in wireless channel where many scatterers are very close to the Rx, so that delay spread is very small among multiple paths, which



is the case in cellular downlink. (BS→MS)

- Now, what if the Rx is in **motion**?

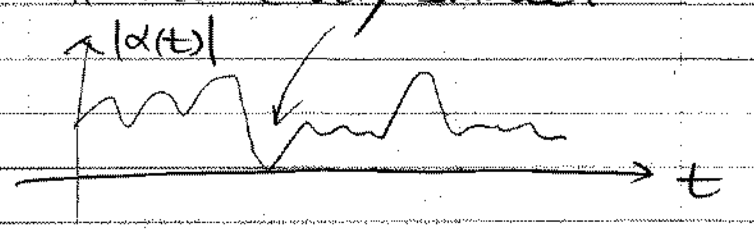
We can easily guess that the complex gain now becomes a random process that **varies as a function of time.**



$\alpha(t_i)$
$\alpha(t_f)$

If a large # of multipaths these may be well approximated as proper-complex WSS random process.

- If the multipath components interfere destructively, $|\alpha(t)|$ will become very small.



"fade-in fade-out" channel → **fading channel**.

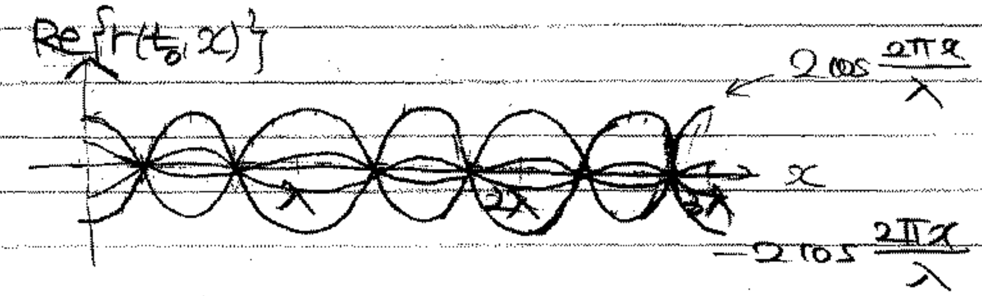
- Channel fading (magnitude fluctuation) is due to the motion of
  - Tx,
  - Rx, or
  - Scatterers

  and at least two different non-resolvable propagation paths.

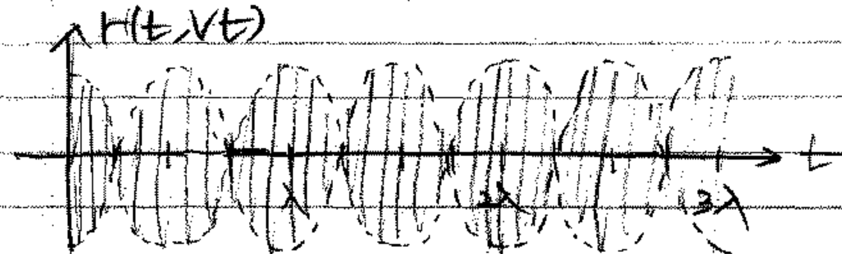
- (ex) Two-path fading channel.
  Suppose the received signal $r(t,x)$ is given by

$$r(t,x) = \exp\{j2\pi(f_c t - \frac{x}{\lambda})\} + \exp\{j2\pi(f_c t + \frac{x}{\lambda})\}$$

$$= \exp(j2\pi f_c t)\, 2\cos(2\pi \frac{x}{\lambda})$$



If the Rx moves slowly from $x=0$ to $x=3\lambda$ then, the received signal will look like



This shows only two paths and a motion is enough to cause channel fading.

multipath channel
- resolvable
- non-resolvable
- hybrid

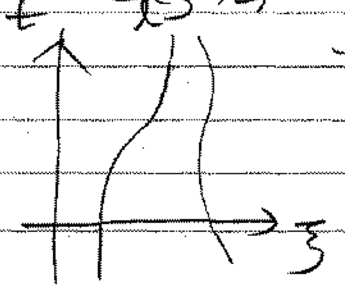
multipath channel
- fading
- non-fading

○ Summary

- Fading phenomenon can be incorporated in $C_l(\xi;t)$ model, e.g. as

$$C_l(\xi;t) = \underbrace{\alpha_0(t)}\,\delta(\xi-\Delta_0(t)) + \underbrace{\alpha_1(t)}\,\delta(\xi-\Delta_1(t))$$

time-varying coefficients due to **multipath fading.**



- The key words are multipaths and motion.

- In the next note, we consider statistical modeling of $C(\xi, t)$.

$$\text{multipath fading channel} \left\langle \begin{array}{l} \boxed{\text{random}} \\ \text{non-random} \end{array} \right.$$