Wireless Channels : Scattering function

○ Doubly spread channel (PZB 8-2.3)

- A **Doubly spread channel** is a WSSUS channel that spreads the signal transmitted through the channel in time and frequency.

- A **singly spread degenerate channel** is a WSSUS channel that spreads the signal transmitted through the channel only in one domain.

○ Relation b/w correlation functions

- $\phi_c(\xi_1, \Delta t)\,\delta(\xi_1 - \xi_2)$ **space-time correlation function**

  $\Updownarrow$ 2-D Fourier transform pair w/ $\xi_1 f_1, \xi_2 f_2$

  $\phi_c(\Delta f; \Delta t)$ **spaced-frequency spaced-time correlation function**

  $\Updownarrow$ 2-D Fourier transform pair w/ $\Delta f \tau, \Delta t \lambda$

  $S(\tau; \lambda)$ **scattering function**

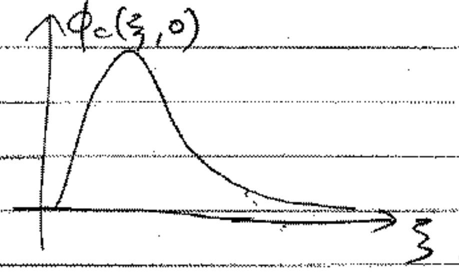
  $\Updownarrow$ 1-D Fourier transform pair w/ $\Delta t \lambda$

  $\phi_c(\tau, \Delta t)$

  1-D Fourier transform pair w/ $\Delta f \tau$

③ Scattering function

(i) The MIP was defined as $\phi_c(\xi, 0)$ and the delay spread was defined as the length of the interval that contains the support of $\phi_c(\xi,0)$.



The related definitions are **the mean delay** and **the mean-square delay spread** defined as

$$m_R = \frac{\int_{-\infty}^{\infty} \xi \phi_c(\xi,0)\, d\xi}{\int_{-\infty}^{\infty} \phi_c(\xi,0)\, d\xi}$$

and

$$L = \frac{\int_{-\infty}^{\infty} \xi^2 \phi_c(\xi,0)\, d\xi}{\int_{-\infty}^{\infty} \phi_c(\xi,0)\, d\xi} - m_R^2$$

respectively.

In terms of the scattering functions,

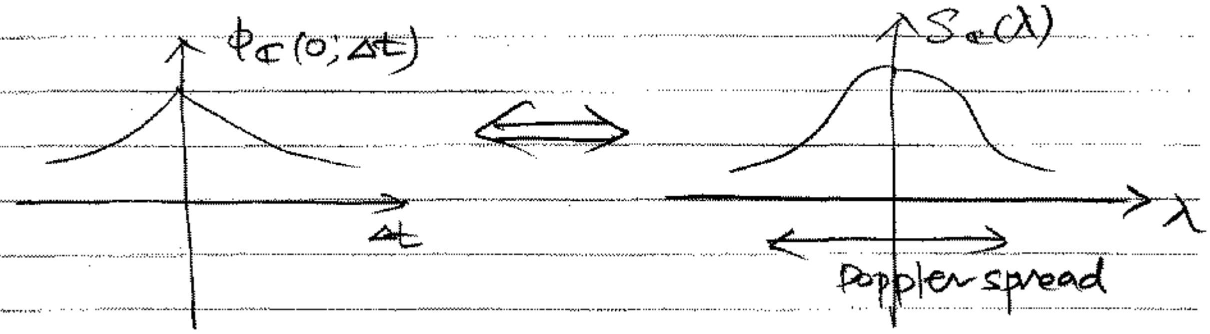
$$m_R = \frac{\int_{-\infty}^{\infty} \xi \int_{-\infty}^{\infty} S(\xi,\lambda) e^{j2\pi \Delta t \lambda} d\lambda \Big|_{\Delta t=0} d\xi}{}$$

$$= \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tau S(\tau,\lambda)\, d\lambda\, d\tau}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\tau,\lambda)\, d\lambda\, d\tau}$$

and

$$L = \frac{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \tau^2 S(\tau,\lambda)\, d\lambda\, d\tau}{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S(\tau,\lambda)\, d\lambda\, d\tau} - m_R^2$$

(ii) The Doppler spread was defined in terms of $S_c(\lambda)$, which is the Fourier transform of $\phi_c(0, \Delta t)$



The related definitions are **the mean Doppler spread** and **the mean-square Doppler spread** defined as

$$m_R = \frac{\int_{-\infty}^{\infty} \lambda S_c(\lambda) d\lambda}{\int_{-\infty}^{\infty} S_c(\lambda) d\lambda}$$

$$= \frac{\int_{-\infty}^{\infty} \lambda \int_{-\infty}^{\infty} \phi_c(\Delta f; \Delta t) e^{-j2\pi\lambda\Delta t} d\Delta t \Big|_{\Delta f=0} d\lambda}{\cdots}$$

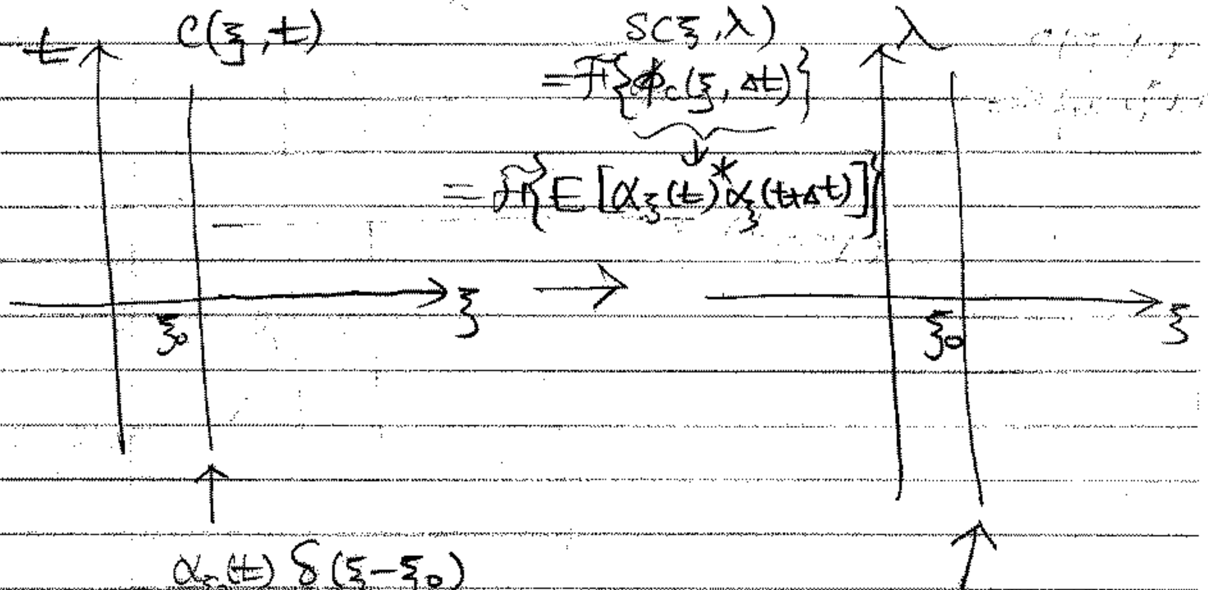
$$= \frac{\int_{-\infty}^{\infty} \lambda \int_{-\infty}^{\infty} S(\tau, \lambda) e^{-j2\pi\tau\Delta f} d\tau \Big|_{\Delta f=0} d\lambda}{\cdots}$$

$$= \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \lambda S(\tau, \lambda) d\tau d\lambda}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\tau, \lambda) d\tau d\lambda}$$

$$B = \frac{\int_{-\infty}^{\infty} \lambda^2 S_c(\lambda) d\lambda}{\int_{-\infty}^{\infty} S_c(\lambda) d\lambda} - m_R^2$$

$$= \frac{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \lambda^2 S(\tau, \lambda) d\lambda d\tau}{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\tau, \lambda) d\lambda d\tau} - m_R^2$$

- Thus, the scattering function $S(\xi,\lambda)$ gives us the information about the relation b/w the propagation delay and associated multiplicative process' PSD.



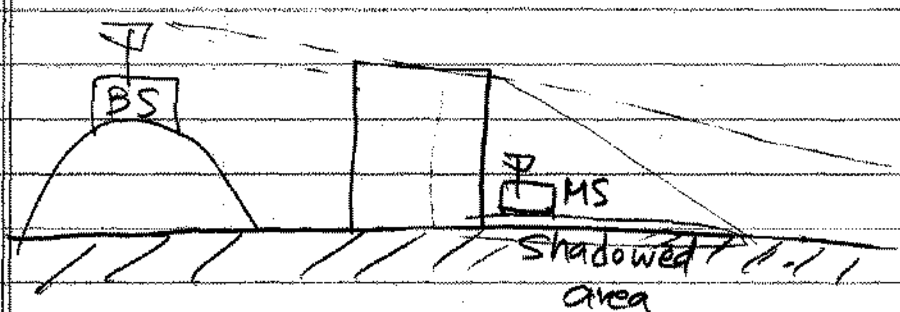
contains the information of the power spectrum of $\alpha_{\xi_0}(t)$.

Since $\alpha_{\xi_0}(t)$ is strictly stationary, $\alpha_{\xi_0}(t)\,\delta(t-\xi_0)$ has the PSD that is the convolution of $S(\xi_0,\lambda)$ and the PSD of $\delta(t-\xi_0)$.

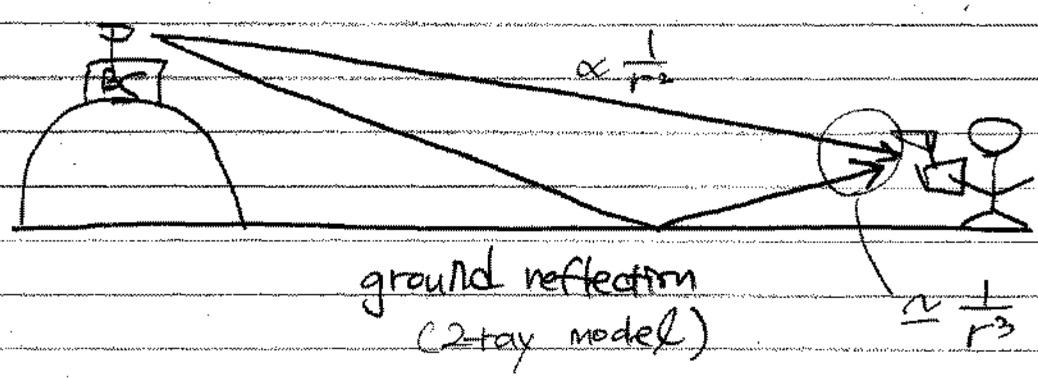
○ T.S. Rappaport, Wireless Communications : Principles & Practice, Prentice Hall

Mobile radio propagation
- large-scale path loss
  - reflection
  - diffraction
  - scattering
  - shadowing
- small-scale fading
  - linear channel model
  - WSSUS channel model

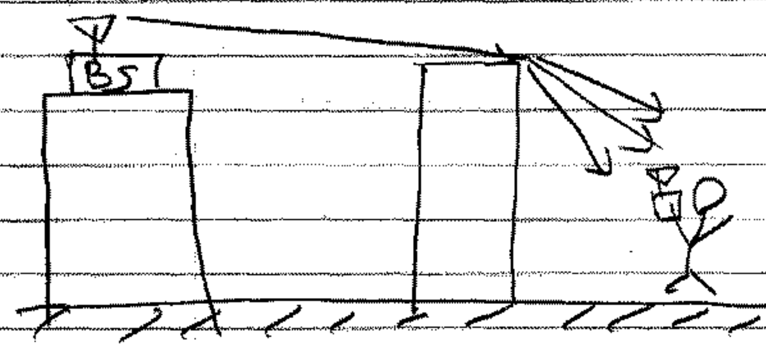
• Shadowing



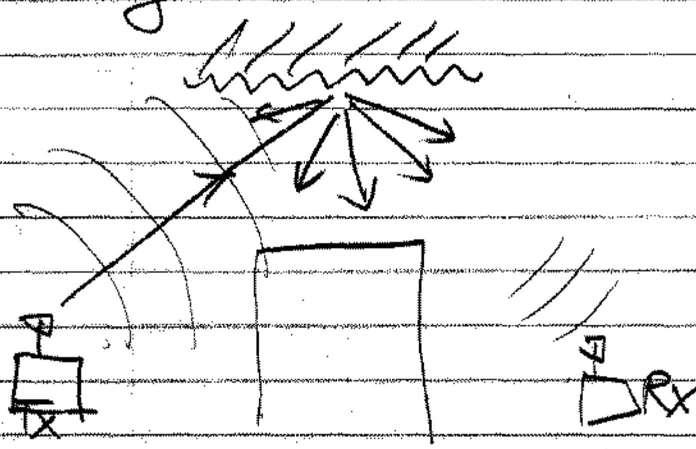
• reflection



- diffraction



- scattering



- There are many large-scale path loss models
  - outdoor propagation models
  - indoor propagation models

- There are many statistical fading channel models
  - Clarke's model for flat fading
  - Saleh – Valenzuela indoor statistical model

- There are
  - channel simulation softwares &
  - channel simulators.

- For a fair evaluation of different systems, many standardization task force teams select a channel model with which comparisons are made.