EECE695F Fall 2005

HW#1. (Pre-requisites)

(5) 1. (Fourier Transform of complex-valued signal)

Suppose that a complex-valued signal $x(t)$ has its Fourier transform given by

$$X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt \triangleq \mathcal{F}\{x(t)\}$$

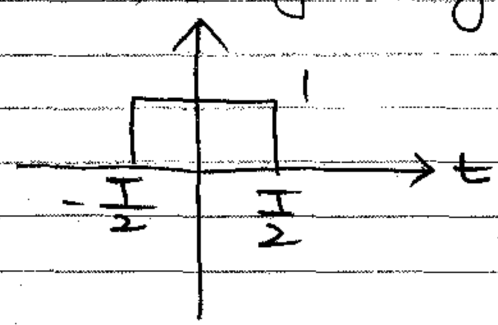
Answer the following questions

(a) Find $\mathcal{F}\{x(-t)\}$ in terms of $X(f)$.

(b) Find $\mathcal{F}\{x(t)^*\}$ " " " $X(f)$

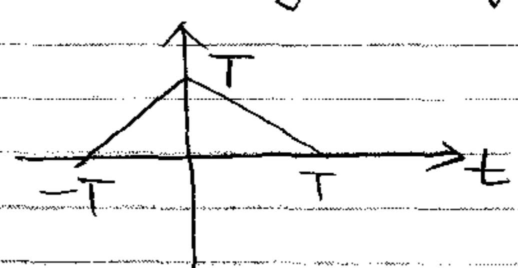
(c) Find $\mathcal{F}\{x(-t)^*\}$ " " " $X(f)$.

(d) When $x(t)$ is given by



find $X(f)$

(e) When $x(t)$ is given by



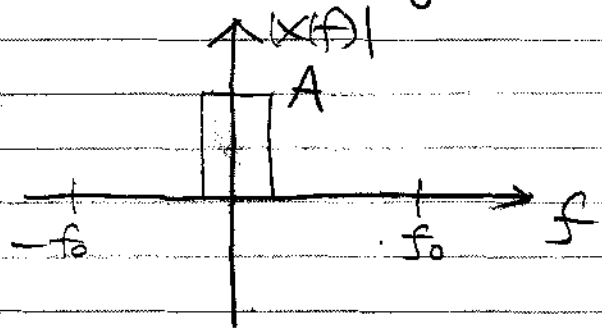
find $X(f)$.

(10) 2 (Digital Audio System)

A band-limited acoustic signal $x(t)$ with bandwidth 10 kHz is sampled at the sampling rate 22 kHz and is stored in a storage device. For convenience, each sample $x[n]$ is assumed un-quantized, i.e.,

$$x[n] \triangleq x\left(\frac{n}{f_0}\right) \quad \text{where } f_0 = 22\,\text{kHz}.$$

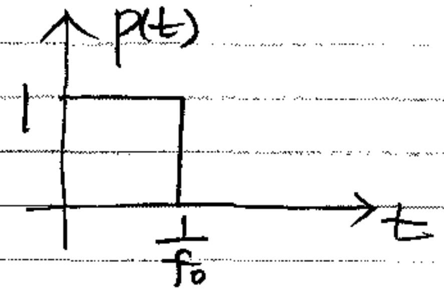
Suppose $|X(f)|$ is given by



Sketch $|Y(f)|$, when $y(t)$ is the D/A converted signal of $x[n]$, defined as

$$y(t) \triangleq \sum_{n=-\infty}^{\infty} x[n]\, p\left(t - \frac{n}{f_0}\right)$$

where



(5) 3. Suppose that a linear system has the impulse response $h[m,n]$ which means that the output at time m to the impulse input at time n is $h[m,n]$.

Show that the input/output relation of this system is given by

$$y[m] = \sum_{n=-\infty}^{\infty} h[m,n]\, x[n]$$

where $(x[m])_{m=-\infty}^{\infty}$ is the input and

$(y[m])_{m=-\infty}^{\infty}$ is the output

(5) 4. Suppose that $\underline{X}$ is a real-valued Gaussian random vector with mean zero and covariance matrix C, which is positive definite. We denote $\underline{X} \sim N(\underline{0}, C)$

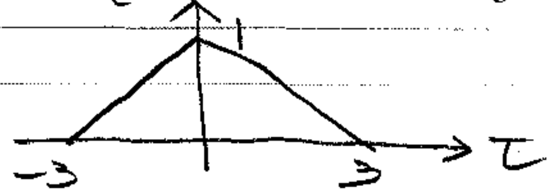
In addition, suppose that $\underline{Y} \sim N(\underline{0}, I)$

Show that $\underline{Z} = \left(\sum_{n=1}^{N} \sqrt{\lambda_n}\, \underline{v}_n \underline{v}_n^T\right) \underline{Y}$

has the same probability distribution as $\underline{X}$ where

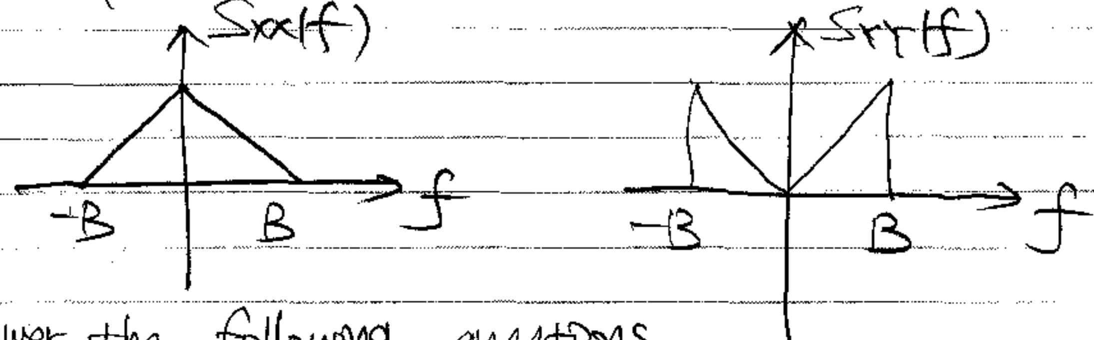
$$C = \sum_{n=1}^{N} \lambda_n \underline{v}_n \underline{v}_n^T \quad \text{and} \quad \underline{v}_n^T \underline{v}_m = \delta_{m,n}$$

(5) 5. Suppose that a real-valued zero-mean WSS random process has the auto-correlation function $R_{XX}(\tau) \triangleq E\{X(t+\tau)X(t)\}$ given by



Find the variance of $Z \triangleq X(0)+X(1)-X(2)$

(10) 6. Two uncorrelated WSS, real-valued, zero-mean random processes $X(t)$ and $Y(t)$ have the power spectral densities given by



Answer the following questions.

(a) Find the PSD of $X(t)+Y(t)$

(b) Find the PSD of $X(t)-Y(t)$

(c) Discuss whether $W(t)=aX(t)+bY(t)$ is WSS or not. $(ab\neq 0)$

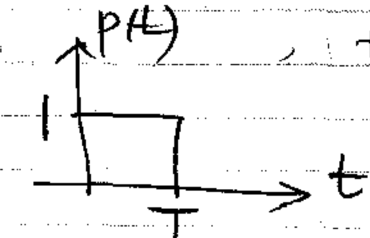
(d) Discuss whether

$$Z(t)=X(t)\cos 2\pi f_c t - Y(t)\sin 2\pi f_c t$$

is WSS or not.

(5) 7. When
$$x(t)=\sum_{n=0}^{N-1} a_n p(t-nT)\cos 2\pi f_c t - \sum_{n=0}^{N-1} b_n p(t-nT)\sin 2\pi f_c t$$

with $a_n, b_n \in \{+1, -1\}$, and $f_c \gg \frac{1}{T}$



Show that

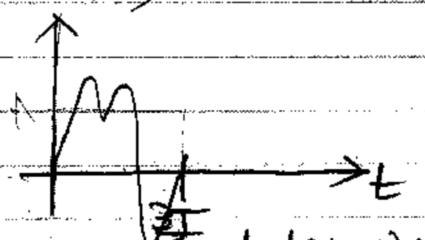
$$\therefore \; x(t) = \mathrm{Re}\left\{ \sum_{n=0}^{N-1} (a_n + j b_n)\, p(t-nT)\, e^{j2\pi f_c t} \right\}$$

where $j = \sqrt{-1}$.

(15) 8. Suppose that the received signal $Y(t)$ is modeled as

$$Y(t) = S_i(t) + N(t), \qquad t \in [0, T)$$

where $S_0(t)$ and $S_1(t) = S_0\left(t - \frac{T}{4}\right)$



are known deterministic signals, $N(t)$ is an AWGN with $E\{N(t)\} = 0$, $E\{N(t+\tau)N(t)\} = N_0\,\delta(\tau)$

and $\Pr(i=0) = \Pr(i=1) = 1/2$.

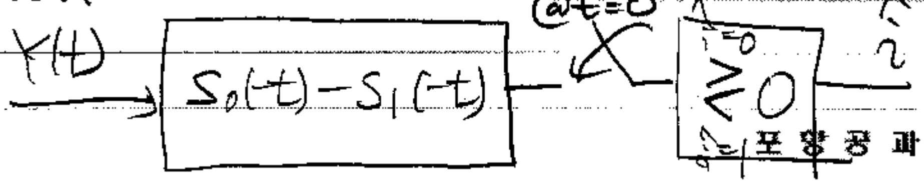
When

$$\frac{\int_0^T S_0(t) S_1(t)\, dt}{\int_0^T S_0(t)^2\, dt} = \rho$$

answer the following questions.

(a) Show that $|\rho| < 1$

(b) Show that



is the receiver that minimizes the average probability of decision error.

(c) Find the minimum average probability of decision error of the receiver in (b).
(Use $Q(x) \triangleq \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{y^2}{2}} dy$.)

(20) 9 (MATLAB programming)

Consider a rectangle $\{(x,y) \mid 0 < x, y < 1\}$.

Over this rectangle, three waves are propagating:

(i) $Z_1(t; x,y) = e^{j(2\pi f_0 t - \lambda x)}$

(ii) $Z_2(t; x,y) = e^{j(2\pi f_0 t - \lambda(-\frac{x}{2} - \frac{\sqrt{3}}{2}y))}$

(iii) $Z_3(t; x,y) = e^{j(2\pi f_0 t - \lambda y)}$

We ~~observe~~ the sum of them.

(a) Plot the graph of $|Z(x,y)|$ as a function of x & y, where

$$Z(x,y) \triangleq \left(Z_1(t;x,y) + Z_2(t;x,y) + Z_3(t;x,y)\right) \times e^{-j2\pi f_0 t}$$

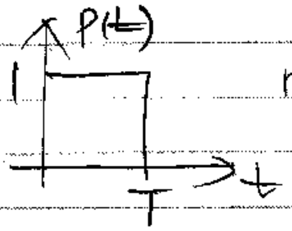
Use $\lambda = \frac{1}{20}$.

(b) Plot the graph of $|Z(x,x)|$ as a function of x.

(20) 10. The received signal is modeled as

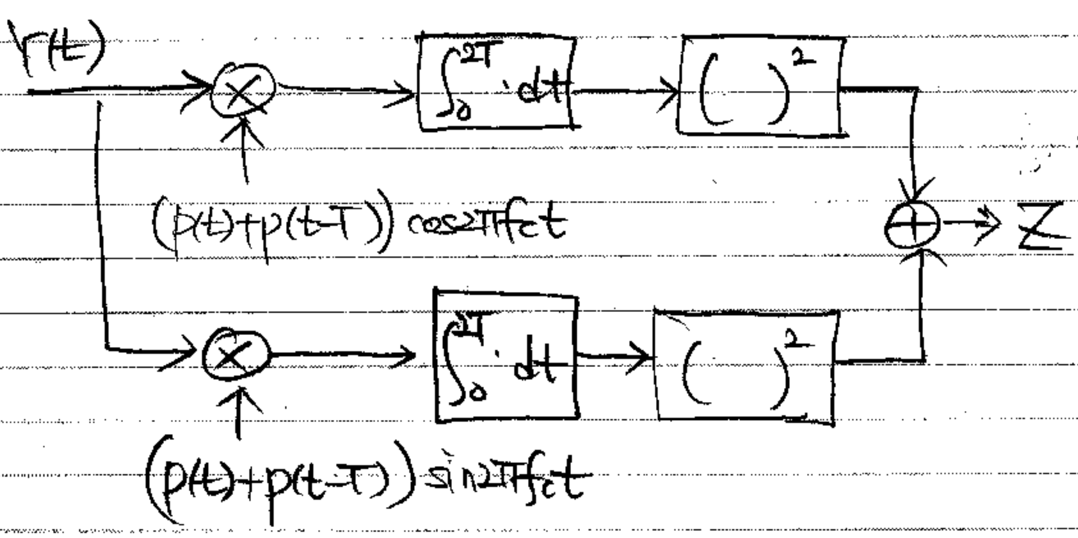
$$Y(t)=\sqrt{2P}\Big(p(t)+b\,p(t-T)\Big)\cos(2\pi f_c t+\theta)+N(t)$$

where $Pr(b=1)=Pr(b=-1)=1/2$, $f_c \gg \frac{1}{T}$,



, $\theta \in [0,2\pi)$ is an unknown but not random, and $N(t)$ is a zero-mean AWGN w/ 2-sided PSD $\frac{N_0}{2}$.

When the receiver is given by



Answer the following questions.

(a) Find the conditional pdfs of Z given $b=1$, and $b=-1$, respectively.

(b) Find the MAP detector that processes Z.