EECE 695F Fall 2005

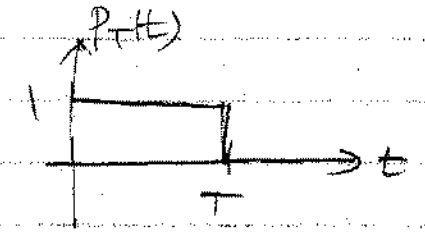
HW #5

1. Given

• Tx

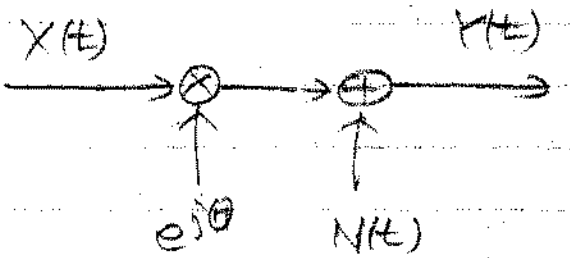
m	X(t)
0	0
1	$A P_T(t)$

where



$P_T(m=0) = 1/2$, $P_T(m=1) = 1/2$.

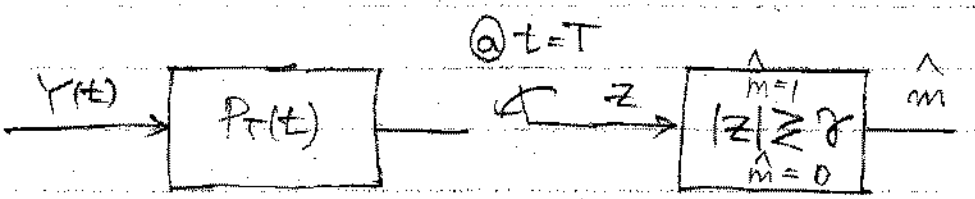
• Channel



where Θ is a uniform r.v. on $\Theta \in [0, 2\pi)$

$N(t)$ is a proper complex white Gaussian noise w/ $S_{NN}(f) = 2\sigma^2$

• Rx



Answer the following questions

(a) Find the conditional probability density function of Z given m=0

(b) Repeat (a) given m=1

(c) Find Pe

(d) Find γ that minimizes Pe

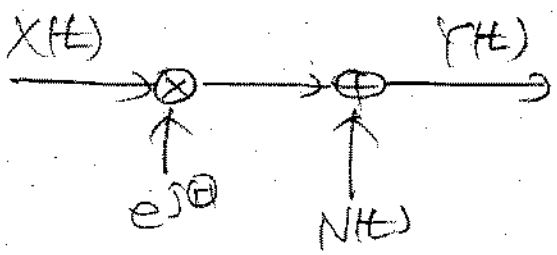
2 Given

• Tx

m	X(t)
0	$A(p_T(t)+p_T(t-T))$
1	$A(p_T(t)-p_T(t-T))$

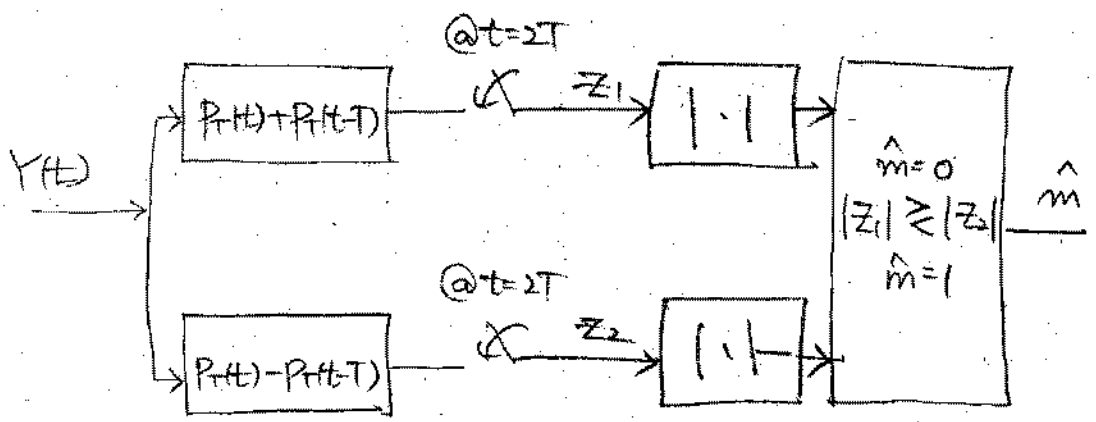
where $Pr(m=0)=1/3$
$Pr(m=1)=2/3$

• Channel



where Θ is a uniform r.v. on $\Theta \in [0, 2\pi)$
$N(t)$ is a proper complex white Gaussian noise with $S_{NN}(f) = 2\sigma^2$

• Rx



Answer the following questions

(a) Find the conditional probability distribution of $\underline{z} \triangleq \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ given $m=0$

(b) Find P_e.

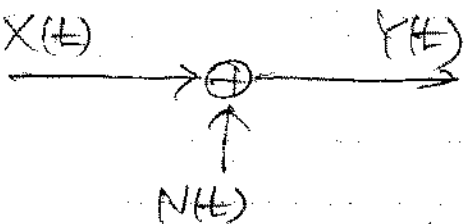
3 Given

- Tx

m	X(t)
0	$A p_T(t)$
1	$-A p_T(t)$

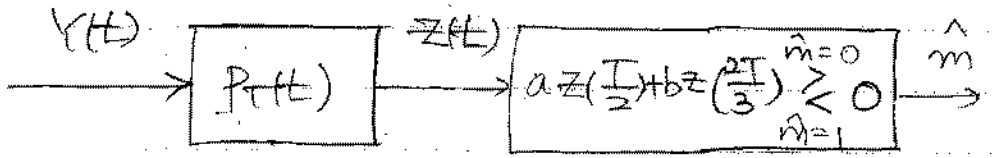
where $Pr(m=0) = 1/2$
$Pr(m=1) = 1/2$

- Channel



where $N(t)$ is a real-valued white Gaussian noise w/ $S_{NN}(f) = N_0/2$

- Rx



Answer the following questions

(a) Find the conditional probability distribution of $\underline{z} = \begin{bmatrix} z(\frac{T}{2}) \\ z(\frac{2T}{3}) \end{bmatrix}$ given $m=0$

(b) Find the P_e in terms of a & b

(c) Find the optimum a and b that minimizes P_e.

4. The transmitted signal $S(t)$ is given by

$$S(t) = \sqrt{P}\left(\sum_{n=-\infty}^{\infty} b[n]\, p_T(t-nT)\right)$$

Where $(b[n])_n$ is a sequence of i.i.d. r.v's w/ $Pr(b[n]=1) = Pr(b[n]=-1) = 1/2$

The channel is given by

$$h(t) = \delta(t) - \frac{1}{2}\delta(t-T)$$

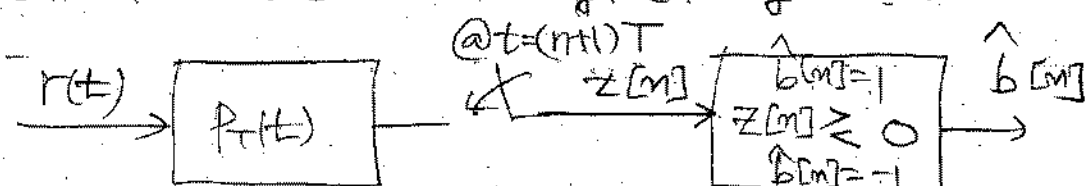
Hence, the received signal $r(t)$ is modeled as

$$r(t) = S(t) * h(t) + N(t)$$

Where $N(t)$ is a real-valued AWGN w/ PSD $S_{NN}(f) = N_0/2$. Answer the following questions.

Subproblems (a) - (c):

When the receiver is given by



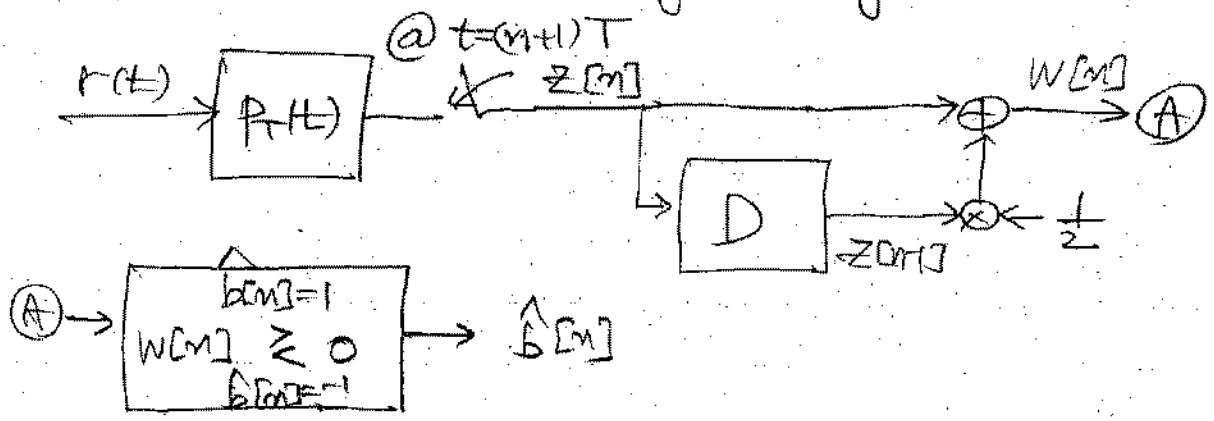
(a) Find the conditional probability distribution of $Z[n]$ given $b[n]$ and $b[n-1]$

(b) Find the conditional probability of decision error for $b[n]$ given $b[n]$ and $b[n-1]$

(c) Find the average probability of decision

error.

subproblems (d)-(f)
When the receiver is given by

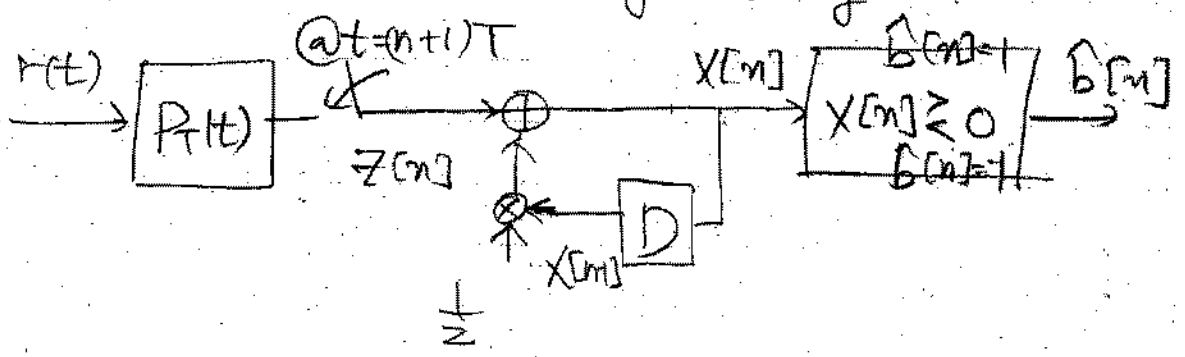


(d) Find the conditional probability distribution of W[n] given b[n] and b[n-2]

(e) Repeat (b) given b[n] and b[n-2]

(f) Repeat (c).

subproblems (g)-(k)
When the receiver is given by



(g) Find the relation among $Z[n]$, $X[n]$, and $X[n-1]$.

(h) Show that $X[n] = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^l Z[n-l]$

(i) Find the conditional probability distribution of $X[n]$ given $b[n]$.

(j) Repeat (b) given $b[n]$

(k) Repeat (c)