~~EECE695f~~ Fall 2006

HW #11

(5) 1. Let $\underline{x} \in \mathbb{R}^N$, $A \in \mathbb{R}^{N\times N}$, $\underline{b} \in \mathbb{R}^N$. Answer the following questions.

(a) Show that

$$\underline{x}^T A \underline{x} = \underline{x}^T \frac{A+A^T}{2} \underline{x}$$

(b) Show that

$$\begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \vdots \\ \frac{\partial f}{\partial x_N} \end{bmatrix} = 2A\underline{x}$$

when $f(\underline{x}) = \underline{x}^T A \underline{x}$

(c) Show that

$$\begin{bmatrix} \frac{\partial g}{\partial x_1} \\ \frac{\partial g}{\partial x_2} \\ \vdots \\ \frac{\partial g}{\partial x_N} \end{bmatrix} = \underline{b}$$

when $g(\underline{x}) = \underline{b}^T \underline{x} = \underline{x}^T \underline{b}$

(5) 2. Let $\underline{y} \in \mathbb{C}^N$, $B \in \mathbb{C}^{N\times N}$, $\underline{b} \in \mathbb{C}^N$. Answer the following questions

(a) Show that

$$\underline{y}^H B \underline{y} = \underline{y}^H \left(\frac{B+B^H}{2} \right) \underline{y}$$

(b) Show that

$$\begin{bmatrix} \frac{\partial f}{\partial y_{1,r}} + j \frac{\partial f}{\partial y_{1,i}} \\ \frac{\partial f}{\partial y_{2,r}} + j \frac{\partial f}{\partial y_{2,i}} \\ \vdots \\ \frac{\partial f}{\partial y_{N,r}} + j \frac{\partial f}{\partial y_{N,i}} \end{bmatrix} = 2B\underline{y}$$

when $f(\underline{y}) = \underline{y}^H B \underline{y}$, $\underline{y} = \begin{bmatrix} y_{1,r} + j y_{1,i} \\ \vdots \\ y_{N,r} + j y_{N,i} \end{bmatrix}$, $j = \sqrt{-1}$.

(c) Show that

$$\begin{bmatrix} \frac{\partial g}{\partial y_{1,r}} + j \frac{\partial g}{\partial y_{1,i}} \\ \frac{\partial g}{\partial y_{2,r}} + j \frac{\partial g}{\partial y_{2,i}} \\ \vdots \\ \frac{\partial g}{\partial y_{N,r}} + j \frac{\partial g}{\partial y_{N,i}} \end{bmatrix} = \underline{b}$$

when $g(\underline{y}) = \mathrm{Re}\{\underline{b}^H \underline{y}\} = \mathrm{Re}\{\underline{y}^H \underline{b}\}$

(10) 3. Suppose that the observation is modeled as

$$\underline{Y} = \sum_{k=1}^{K} \sqrt{P_k}\, \underline{S}_k b_k + \underline{N}$$

where $\{\underline{S}_k\}_{k=1}^{K}$ are known deterministic vectors with $\|\underline{S}_k\| = 1$, $\{b_k\}_{k=1}^{K}$ are i.i.d. proper-complex random variables with $E\{b_k\} = 0$, $E\{|b_k|^2\} = 1$, and $\underline{N} \sim CN(\underline{0}, \sigma^2 I)$

Answer the following questions.

(a) Formulate the LMMSE estimation problem to estimate b_1.

(b) Rewrite the objective function in (a) in terms of $\{p_k\}_{k=1}^{K}$, $\{s_k\}_{k=1}^{K}$, and σ^2.

(c) By differentiating the objective function in (b), find the LMMSE estimator.

(d) By using the orthogonality principle, find the LMMSE estimator.

(5) 4. Find a function $f: [0,1] \to [0,1]$ that maximizes

$$\int_0^1 |f(x) - x| \, dx$$

(5) 5. When $X(t)$ is a WSS random process with PSD $S_{XX}(f)$, find the PSD of the sampled sequence $\{X(nT)\}_{n=-\infty}^{\infty}$.

(5) 6. Show that

$$(A + \underline{p}\,\underline{g}^T)^{-1} = A^{-1} - \frac{A^{-1}\underline{p}\,\underline{g}^T A^{-1}}{1 + \underline{g}^T A^{-1}\underline{p}}$$

when $A \in \mathbb{R}^{N\times N}$ is invertible and $\underline{p}, \underline{g} \in \mathbb{R}^N$.

What if $A \in \mathbb{C}^{N\times N}$, $\underline{p}, \underline{g} \in \mathbb{C}^N$ and we want to invert $(A + \underline{p}\,\underline{g}^H)^{-1}$?