EECE 695F Fall 2006

HW #12

1. The complex envelope of a received signal is modeled as

$$r(t) = \int_{-\infty}^{\infty} c(\xi, t)\, x(t-\xi)\, d\xi$$

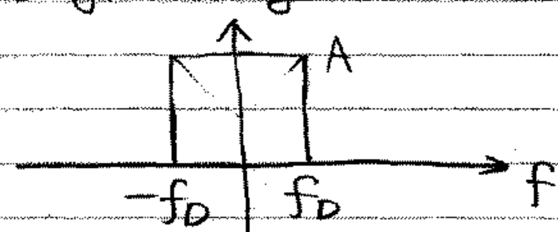
where $c(\xi, t) = \alpha(t)\,\delta(\xi - \Delta)$,

$$x(t) = \sqrt{2P} \sum_{m=-\infty}^{\infty} d[m]\, p(t - mT),$$

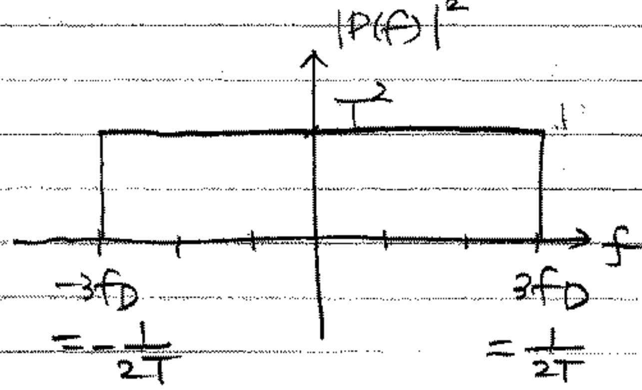
with $\alpha(t)$ being a proper-complex zero-mean random process having $E[\alpha(t)^* \alpha(t+\tau)] = \mathcal{F}^{-1}\{S_{\alpha\alpha}(f)\}$

where

$S_{\alpha\alpha}(f)$ is given by



, $p(t)$ having the energy spectral density



, $\{d[m]\}_{m=-\infty}^{\infty}$ is a sequence of i.i.d proper-complex random variables with $E\{d[m]\}=0$, $E\{|d[m]|^2\}=1$.

Answer the following questions

(5) (a) Find $r(t)$ in terms of $\alpha(t)$, Δ, and $x(t)$.

(5) (b) Find the PSD of $\alpha(t)$

(15) (c) Find the PSD of $r(t)$. Fully justify your answer.

2. Suppose that we have a function $g: \mathbb{R}^2 \to \mathbb{R}$, say $g(x_1, x_2)$

The Fourier transform with respect to x_1 is defined as

$$\mathcal{F}_1\{g\} = G_1(f_1, x_2) \triangleq \int_{-\infty}^{\infty} g(x_1, x_2)\, e^{-j2\pi f_1 x_1}\, dx_1$$

and the Fourier transform with respect to x_2 is defined as

$$\mathcal{F}_2\{g\} = G_2(x_1, f_2) \triangleq \int_{-\infty}^{\infty} g(x_1, x_2)\, e^{-j2\pi f_2 x_2}\, dx_2$$

The inverse transforms $\mathcal{F}_1^{-1}$ and $\mathcal{F}_2^{-1}$ are similarly defined.

When we define the 2-D Fourier transform as

$$\mathcal{F}\{g\} = G(f_1, f_2) \triangleq \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x_1, x_2)\, e^{-j2\pi(f_1 x_1 + f_2 x_2)}\, dx_1\, dx_2$$

and its inverse transform as

$$g(x_1, x_2) = \mathcal{F}^{-1}\{G\} \triangleq \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(f_1, f_2)\, e^{j2\pi(x_1 f_1 + x_2 f_2)}\, df_1\, df_2$$

Answer the following questions.

(5) (a) Show that $\mathcal{F}_2[\mathcal{F}_1\{g\}] = \mathcal{F}_1[\mathcal{F}_2\{g\}] = \mathcal{F}\{g\}$

(Assume that Fubini's theorem holds.)

(5) (b) Show that

$$\mathcal{F}_1^{-1}\{\mathcal{F}\{g\}\} = \mathcal{F}_2\{g\}.$$

and

$$\mathcal{F}_2^{-1}\{\mathcal{F}\{g\}\} = \mathcal{F}_1\{g\}.$$

(10) (c) Show that

$$\int_{-\infty}^{\infty} g(x_1, x_2)\,dx_1 = \mathcal{F}_2^{-1}\{G(0, f_2)\} = G_1(0, x_2)$$

and that

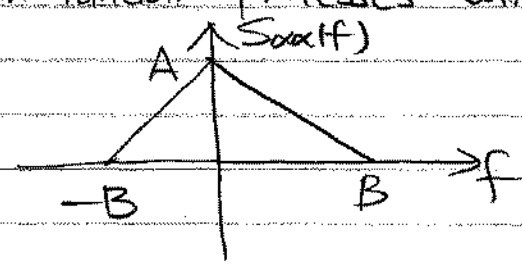
$$\int_{-\infty}^{\infty} g(x_1, x_2)\,dx_2 = \mathcal{F}_1^{-1}\{G(f_1, 0)\} = G_2(x_1, 0)$$

3. When the complex channel response $C(\xi, t)$ is given by

$$C(\xi, t) = C_0\,\alpha_0(t)\,\delta(\xi - \Delta_0) + C_1\,\alpha_1(t)\,\delta(\xi - \Delta_1)$$

Answer the following questions. (We assume that

$\alpha_i(t)$ are i.i.d. proper-complex zero-mean Gaussian random processes with $S_{\alpha\alpha}(f)$



and that $0 < \Delta_1 = 2\Delta_0$.

(5) (a) Find the MIP and sketch it.

(5) (b) Find the coherence bandwidth when $C_0 \neq 0$, $C_1 = 0$ and sketch it.

(5) (c) Find the spaced-frequency spaced-time correlation function $\Phi_C(\Delta f, \Delta t)$.

(5) (d) Find the scattering function. $S(\tau, \lambda)$

(5) (e) Find the spaced-time correlation function and sketch it.

(5) (f) Find the mean Doppler shift.

(5) (g) Find the mean-square Doppler spread.