Review for lecture #1

- In wireless communications, we mainly deal with REAL-valued BANDPASS signals.

- Given a real-valued bandpass signal $x(t)$ with highest signal component at $f = B\,[Hz]$, if we choose $f_c \in [\frac{B}{2}, B]$ and $\theta \in [0, 2\pi)$ then we can find "unique" pair of REAL-valued BASEBAND signals $x_c(t)$ and $x_s(t)$ such that

$$x(t) = x_c(t)\cos(2\pi f_c t + \theta) - x_s(t)\sin(2\pi f_c t + \theta)$$

- $x(t)$ real bandpass

  $x_+(t) = \mathcal{F}^{-1}\{2X(f)u(f)\}$ complex " ← pre-envelope, analytic signal

  $x_e(t) = \mathcal{F}^{-1}\{X_+(f+f_c)e^{-j\theta}\}$ " baseband ← complex envelope of $x(t)$

  $= \underbrace{x_c(t)}_{\text{Inphase component}} + j\underbrace{x_s(t)}_{\text{quadrature component}}$.

- $x(t) = \mathrm{Re}\{x_e(t)e^{j(2\pi f_c t + \theta)}\}$

- $X_c(f)$ = conjugate symmetric part of $X_e(f)$

  $jX_s(f)$ = conjugate anti-symmetric part of $X_e(f)$

- Important tricks

  (i) $e^{j\phi} = \cos\phi + j\sin\phi$

  (ii) $\mathrm{Re}\{z\} = \frac{z + z^*}{2}$, $j\,\mathrm{Im}\{z\} = \frac{z - z^*}{2}$

  (iii) $x(t)$ real $\Leftrightarrow X(f) = X(-f)^*, \forall f$.