Summary of Handout #11

○ PSD of a real-valued 2nd-order random process

- Def

$$S_{XX}(f) = \mathcal{F}\left[A\{R_{XX}(t, t+\tau)\}\right]$$

- Main Result

$$X(t) \longrightarrow \boxed{h(t)} \longrightarrow Y(t) \quad \Rightarrow S_{YY}(f) = S_{XX}(f)\,|H(f)|^2$$

○ PSD of a real-baseband linearly modulated signal

- $X(t) = \sum_{n=-\infty}^{\infty} I_n\, g(t-nT)$

  with $\phi(m) \triangleq E\{I_n I_{n+m}\} \overset{DTFT}{\longleftrightarrow} \Phi(f)$

- $$\underbrace{S_{XX}(f)}_{\text{Watt/Hz}} = \underbrace{\frac{1}{T}}_{\text{1/sec}}\, \Phi(fT)\, \underbrace{|G(f)|^2}_{\substack{\text{energy spectral}\\ \text{density}\\ \text{Joule/Hz}}}$$

○ PSD of a real-bandpass linearly modulated signal

- $Y(t) = \mathrm{Re}\{X(t)\, e^{j2\pi f_c t}\}$

  where $X(t) = \sum_{n=-\infty}^{\infty} I_n\, g(t-nT)$

  with $\phi(m) = E\{I_n^* I_{n+m}\} \overset{DTFT}{\longleftrightarrow} \Phi(f)$

- $$S_{YY}(f) = \frac{1}{4}\{S_{XX}(f-f_c) + S_{XX}(-f-f_c)\}$$

where $S_{XX}(f) \triangleq \mathcal{F}\left[A\{E[X(t)^* X(t+\tau)]\}\right]$

$$= \frac{1}{T}\Phi(fT)|G(f)|^2$$

PSD of a complex-baseband linearly modulated signal

∘ Example

$$Y(t) = \mathrm{Re}\{\sqrt{2P}\sum_{m=-\infty}^{\infty} d[m]\, p(t-mT)\, e^{j2\pi f_c t}\}$$

where $E\{|d[m]|^2\}=1$ & $\{d[m]\}_m$ an uncorrelated seq.
$\|p(t)\|^2 = T$

$$\Rightarrow S_{XX}(f) = \frac{2P}{T}|P(f)|^2$$

$$\Rightarrow S_{YY}(f) = \frac{P}{2T}\{|P(f-f_c)|^2 + |P(-f-f_c)|^2\}$$