Review of Handout #13

- The Most popular OFDM signal model

$$X(t) = \sum_{k=1}^{K} X_k e^{j2\pi f_k t}, \quad 0 \le t < T_0$$

where

$$f_{k+1} - f_k = \frac{1}{T_0}, \quad \forall k$$

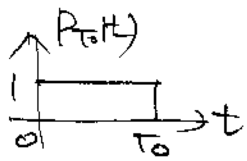
← To keep the orthogonality

$\Rightarrow$ No PSD because $X(t)$ is not a power signal

- Slight modification of the most popular model

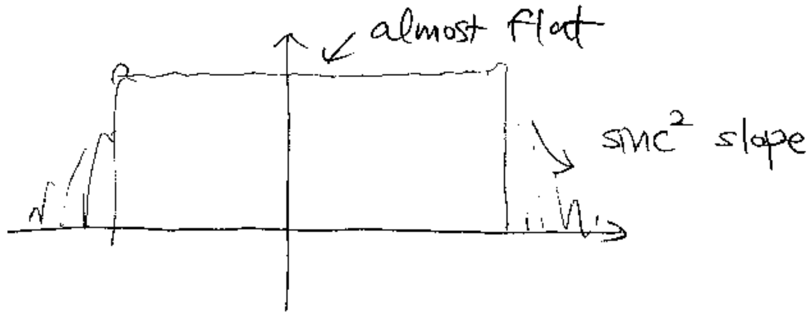
$$X(t) = \sum_{k=1}^{K} \Big(\sum_{m=-\infty}^{\infty} \underbrace{X_k[m]}_{\text{assume uncorrelated w/ unit power}} P_{T_0}(t - mT_0) \Big) e^{j2\pi f_k t}, \quad -\infty < t < \infty$$

where



and $f_{k+1} - f_k = \frac{1}{T_0}, \ \forall k$

$$\Rightarrow S_{XX}(f) = \sum_{k=1}^{K} \frac{1}{T_0} \left| P_{T_0}(f - f_k) \right|^2 \propto \sum_{k=1}^{K} \frac{1}{T_0} \operatorname{sinc}^2\big((f - f_k)T_0\big)$$

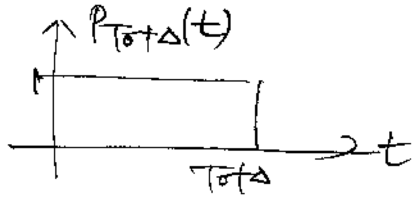


- More modification to adopt CP (Cyclic Prefix)

$$x(t) = \sum_{k=1}^{K} \left(\sum_{m=-\infty}^{\infty} X_k[m]\, P_{T_0+\Delta}\big(t - m(T_0+\Delta)\big) \right) e^{j2\pi f_k t},$$

$-\infty < t < \infty$

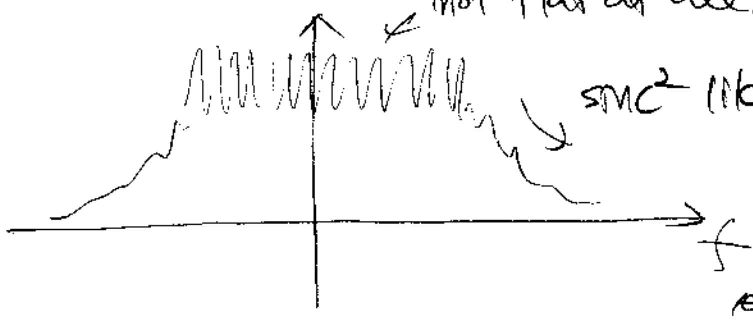
where



and $f_{k+1} - f_k = \frac{1}{T_0}, \ \forall k$

$$\Rightarrow S_{xx}(f) \propto \sum_{k=1}^{K} \frac{1}{T_0} \operatorname{sinc}^2\big((f-f_k)(T_0+\Delta)\big)$$

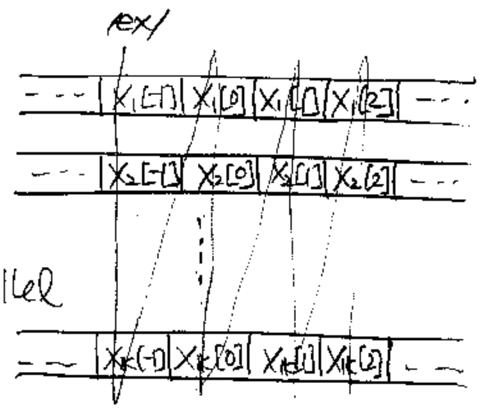
not flat at all.



∘ IFFT-based OFDM

| ... | d[-1] | d[0] | d[1] | d[2] | ... |

→



serial to parallel

$$\cdots, \begin{bmatrix} X_1[-1] \\ X_2[-1] \\ \vdots \\ X_K[-1] \end{bmatrix}, \begin{bmatrix} X_1[0] \\ X_2[0] \\ \vdots \\ X_K[0] \end{bmatrix}, \cdots$$

$\downarrow$ IFFT

$$\underline{Y}[m] = \mathrm{IFFT}\ \underline{X}[m]$$

$$= X_1 \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}0\cdot 0} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}0\cdot 1} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}0\cdot (K-1)} \end{bmatrix} + \cdots + X_K \begin{bmatrix} \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(K-1)\cdot 0} \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(K-1)\cdot 1} \\ \vdots \\ \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(K-1)(K-1)} \end{bmatrix}$$

$\downarrow$ Add CP

$$\begin{bmatrix} \left.\begin{matrix} Y_{K-L+1}[m] \\ \vdots \\ Y_K[m] \end{matrix}\right\} L \\ \left.\begin{matrix} Y_1[m] \\ Y_2[m] \\ \vdots \\ \boxed{\begin{matrix} Y_{K-L+1}[m] \\ \vdots \\ Y_K[m] \end{matrix}} \end{matrix}\right\} K \end{bmatrix}$$

$\downarrow$ Parallel to Serial Conversion

$$\cdots, \begin{bmatrix} \hat{Y}_1[-1] \\ \hat{Y}_2[-1] \\ \vdots \\ \hat{Y}_{K+L}[-1] \end{bmatrix}, \begin{bmatrix} \hat{Y}_1[0] \\ \hat{Y}_2[0] \\ \vdots \\ \hat{Y}_{K+L}[0] \end{bmatrix}, \begin{bmatrix} \hat{Y}_1[1] \\ \hat{Y}_2[1] \\ \vdots \\ \hat{Y}_{K+L}[1] \end{bmatrix}, \cdots \rightarrow \boxed{\hat{Y}_1[-1]}\boxed{\hat{Y}_2[-1]}\cdots\boxed{\hat{Y}_{K+L}[-1]}\boxed{\hat{Y}_1[0]}\cdots$$

$\downarrow$ linear modulation

$$- \quad x(t) = \sum_{m=-\infty}^{\infty} \left(\sum_{k=1}^{K+L} Y_k[m]\, p(t-(k+(K+L))T_c) \right)$$

$$= \sum_{m=-\infty}^{\infty} \sum_{k=1}^{K+L} \left(\sum_{k'=1}^{K} X_{k'}[m] \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(k-L-1)(k'-1)} \times p(t-(k+(K+L)m)T_c) \right)$$

$$= \sum_{m=-\infty}^{\infty} \sum_{k'=1}^{K} X_{k'}[m] \underbrace{\left(\sum_{k=1}^{K+L} \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(k-L-1)(k'-1)} \times p(t-(k+(K+L)m)T_c) \right)}_{\triangleq\, S_{k'}(t-m(K+L)T_c)}$$

$$= \sum_{k'=1}^{K} \underbrace{\left(\sum_{m=-\infty}^{\infty} X_{k'}[m]\, S_{k'}(t-mT_b) \right)}$$

where $T_b = (K+L)T_c$

linearly modulated signal w/ Tx pulse $S_{k'}(t)$ & symbol Time T_b.

$$\Rightarrow S_{xx}(f) = \sum_{k'=1}^{K} \frac{1}{T_b} \left| S_{k'}(f) \right|^2$$

assuming uncorrelated data seq. w/ unit power

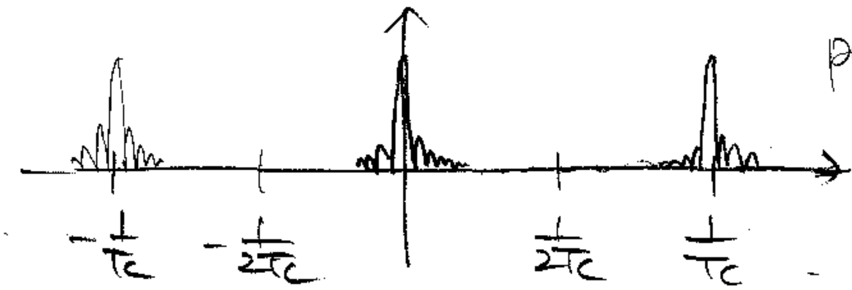
$$= \sum_{k'=1}^{K} \frac{1}{T_b} |P(f)|^2 \left| \sum_{k=1}^{K+L} \frac{1}{\sqrt{K}} e^{j\frac{2\pi}{K}(k-L-1)(k'-1)} \times e^{-j2\pi f k T_c} \right|^2$$

$$\times\, e^{-j2\pi k\left(fT_c - \frac{(k'-1)}{K}\right)}$$

$$\triangleq \sum_{k'=1}^{K} \frac{1}{T_b} |P(f)|^2\, W_D\!\left(f - \frac{(k'-1)}{KT_c}\right)$$

DTFT of a finite-support DT signal evaluated at fT_c

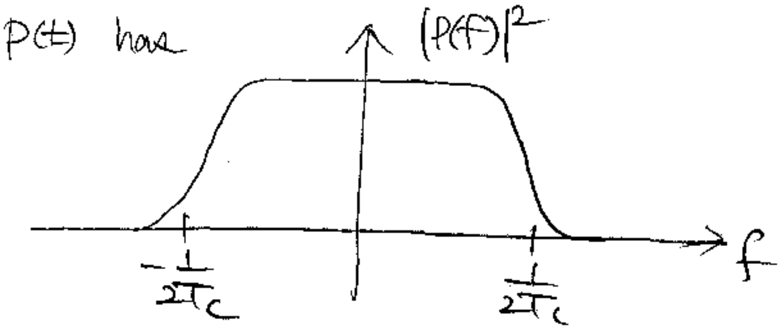
where
$$W_0(f) \triangleq \left|\sum_{k=1}^{K+L} \frac{1}{\sqrt{K}} e^{-j2\pi f k T_c}\right|^2$$
$$= \left|\frac{\sin(\pi f (K+L) T_c)}{\sin(\pi f T_c)}\right|^2$$



periodic w/ period $\frac{1}{T_c}$

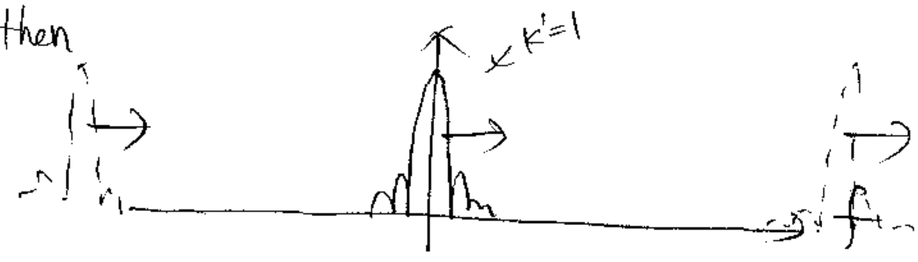
○ Null sub-carriers

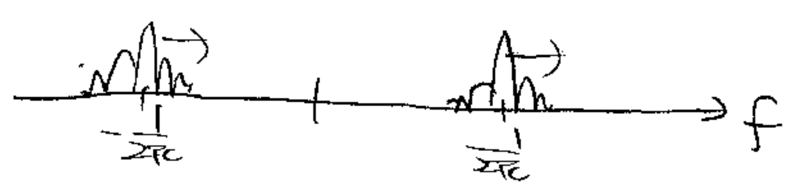
If $p(t)$ have



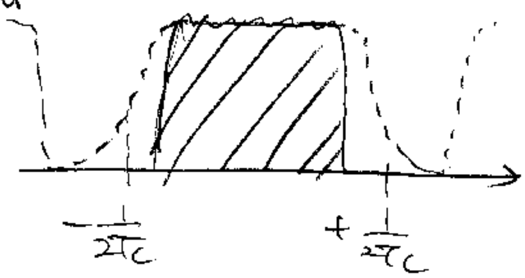
and uncorrelated $X(m)$,

then

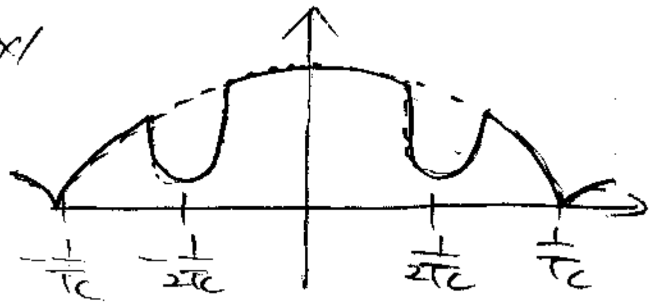




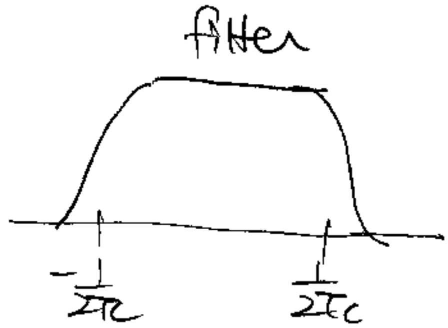
To make this spectrum as similar as that of modified w/ rectangular..., we put null sub-carriers so that



(ex)



→



⇒ K : # of ~~FFT~~ points ⇒ K−N subcarriers

N : # of null subcarriers.