Review of Handout #20

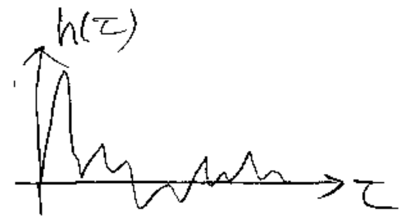
○ LTI system's impulse response

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(t-\tau)\, x(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

$$\simeq \sum_i \underbrace{h(\tau_i)}_{\text{weight on the delayed version}}\ \underbrace{x(t-\tau_i)}_{\text{delayed version of input } x(t).}\ \Delta i$$



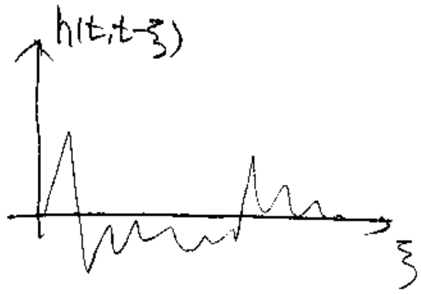
○ LTV system's impulse response

$$x(t) \longrightarrow \boxed{h(t,\tau)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(t,\tau)\, x(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \underbrace{h(t, t-\xi)}_{\text{weight on the delayed version}}\ \underbrace{x(t-\xi)}_{\text{delayed version of input } x(t)}\, d\xi$$

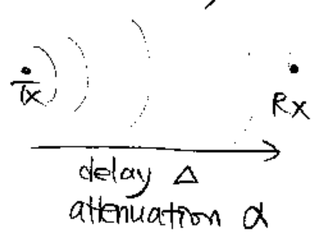
$$\triangleq c(\xi, t)$$



○ $h(t,\tau)$ is causal $\Rightarrow$ $c(\xi, t) = 0 \quad \forall \xi < 0$

$$h(t,\tau) = \alpha\,\delta(t-\tau-\Delta)$$

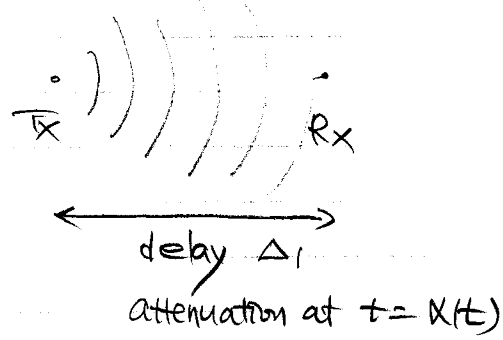
$$\Leftrightarrow \quad c(\xi, t) = \alpha\,\delta(\xi - \Delta)$$



- variable gain

$$h(t,\tau) = \alpha_1(t)\,\delta(t-\tau-\Delta_1)$$

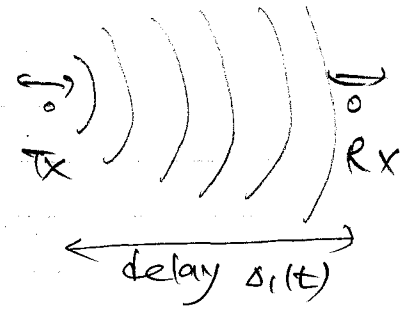
$$\Leftrightarrow c(\xi,t) = \alpha_1(t)\,\delta(\xi-\Delta_1)$$



- variable delay

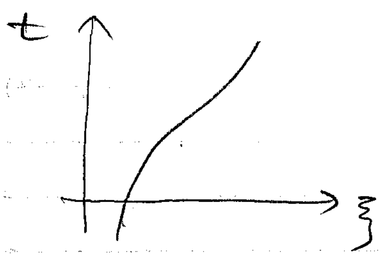
$$h(t,\tau) = \alpha_1\,\delta(t-\tau-\Delta_1(t))$$

$$\Leftrightarrow c(\xi,t) = \alpha_1\,\delta(\xi-\Delta_1(t))$$



- variable gain, variable delay

$$c(\xi,t) = \alpha_1(t)\,\delta(\xi-\Delta_1(t))$$



○ LTI approximation to slowly varying LTV channel

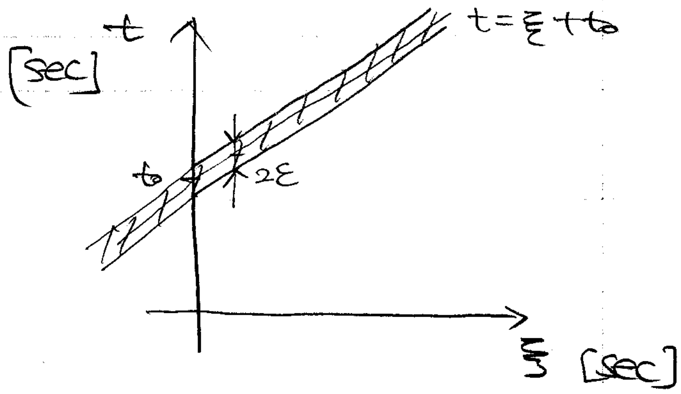
- The general input-output relation of a LTV system is given by

$$y(t)=\int_{-\infty}^{\infty} c(\xi,t)\, x(t-\xi)\, d\xi$$

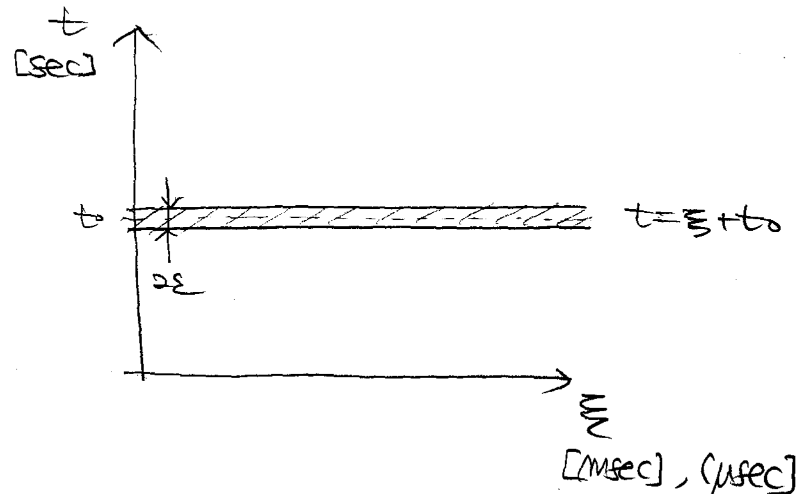
where $c(\xi,t) \triangleq h(t, t-\xi)$ with $h(t,\tau)$ being the response of the system at time t to the impulse input applied at time τ.

- Now, suppose that input $x(t)$ has approximate support $t_0-\varepsilon < t < t_0+\varepsilon$. Then, $x(t-\xi)$ can have non-zero value on $t_0-\varepsilon < t-\xi < t_0+\varepsilon$.

which means $c(\xi,t)$ on $\xi+t_0-\varepsilon < t < \xi+t_0+\varepsilon$ only affects the output.
This area is shown below as



- Since the system is slowly varying, the scale of t is, e.g., [sec] while that of ξ is [msec] or [μsec].
So, in these different scales, this area corresponds to the shaded area below, e.g.,



which means

"If the channel varies slowly relative to the input then an LTV system can be approximated to an LTI system with impulse response $c(\xi, t)$ as a function of ξ.

$$\therefore\ y(t) = \int_{-\infty}^{\infty} c(\xi, t)\, x(t-\xi)\, d\xi$$

$$= \int_{-\infty}^{\infty} h_t(\xi)\, x(t-\xi)\, d\xi$$

because $c(\xi, t)$ of an LTI system has $c(\xi; t_0) = c(\xi; t_1)$, $\forall t_0 \neq t_1$. $\Rightarrow$ $c(\xi, t_0) = h(\xi)$.