Zero-Forcing Decision Feedback Equalizer (ZF-DFE)

o LE vs DFE

So ~~far~~, we ~~learned about~~ two linear equalizers;

- ZF-LE and
- MMSE-LE.

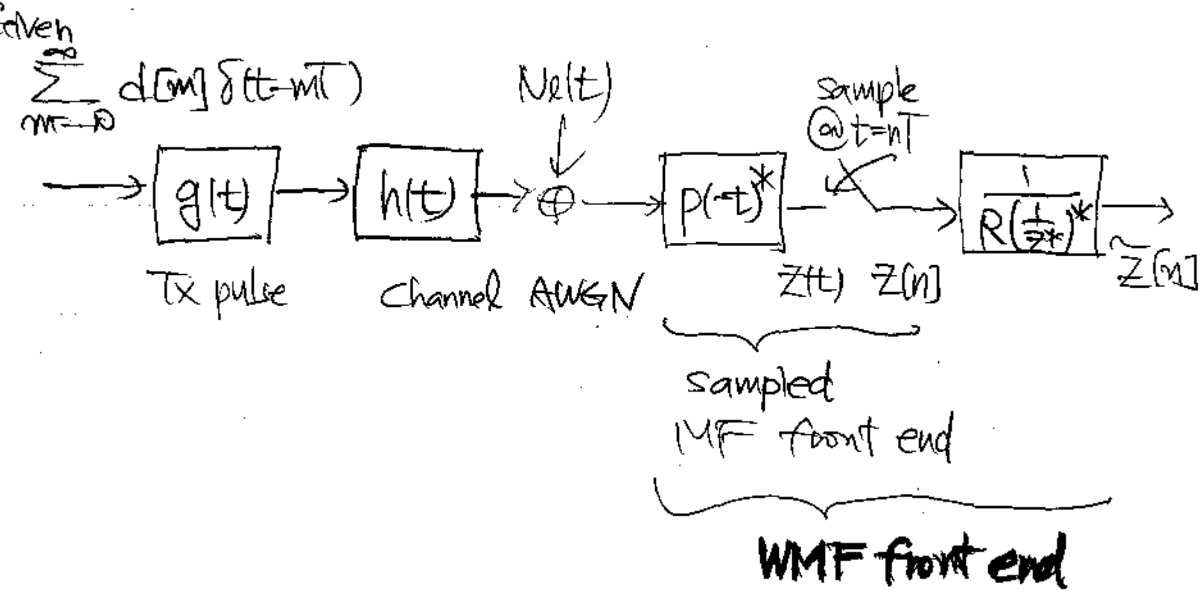
In this handout, we learn about a non-linear equalizer. As most of the non-linear system designs, we slightly modify a well-known linear system by inserting a very simple non-linear block.

There are two approaches in obtaining the ZF-DFE;

- WMF front-end
- ZF-LE

o Derivation of ZF-DFE from WMF front end

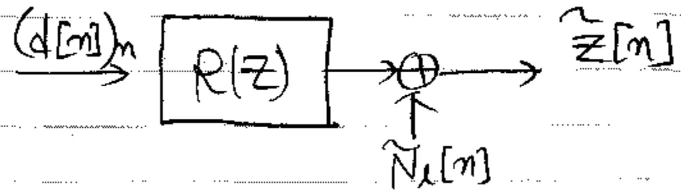
· Given



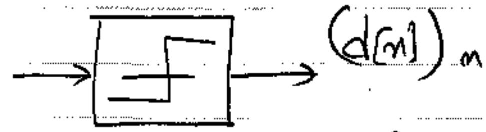
$$\tilde{Z}[n] = \sqrt{2P}\sum_{m=-\infty}^{\infty} d[m]\, r[n-m] + \underbrace{\tilde{N}_e[n]}_{\textbf{AWGN}\text{ sequence}}$$

where $\tilde{p}(nT) \overset{z}{\longleftrightarrow} Q(z) = R(z)\,R\left(\frac{1}{z^*}\right)^*$ with $E\{\tilde{N}_e[m]^*\tilde{N}_e[m+n]\} = 2N_0\,\delta[n]$

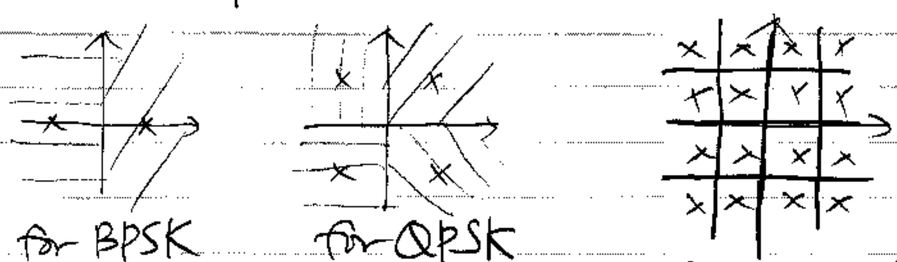
- The end-to-end response can be represented by



- The key assumption in the use of a slicer is that



the slicer output is perfect.



for BPSK   for QPSK   for 16-QAM

- The objective in the design of a DFE is to make the slicer input as "good" as possible, so that the slicer output is close to $(d[n])_n$.

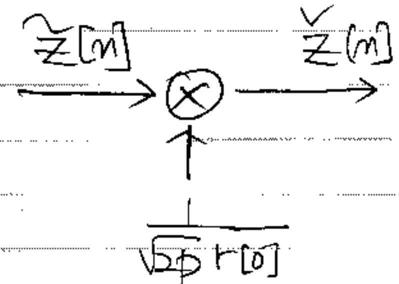
- In ZF-DFE, the goodness is defined as

(i) The slicer input contains no ISI but $d[n]$ only as the signal component. → ZF

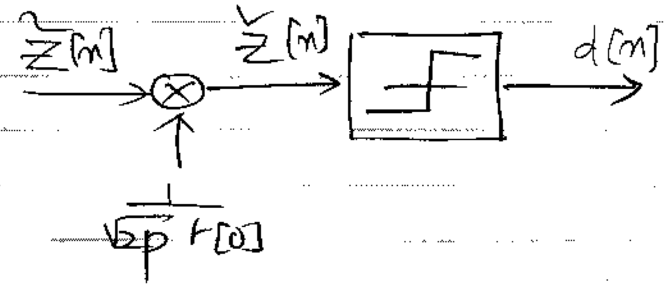
(ii) The slicer input SNR is maximized

(iii) The slicer input noise sequence is white.

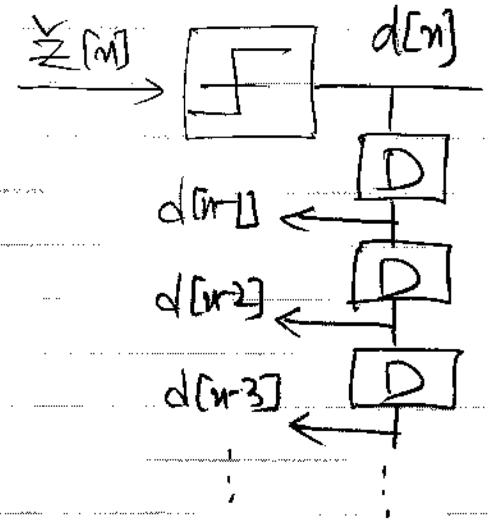
- If we directly feed the slicer with $\tilde{z}[n]$, then (i) is violated. In particular, $d[n]$ even has the coefficient $\sqrt{2p}\, r[0]$. So, we first scale $\tilde{z}[n]$ as



- If we feed the slicer with $\check{z}[n]$, then



the slicer output is assumed to be $d[n]$. By delaying this output we can obtain $d[n-1]$, $d[n-2]$, $d[n-3]$, $\cdots$ as



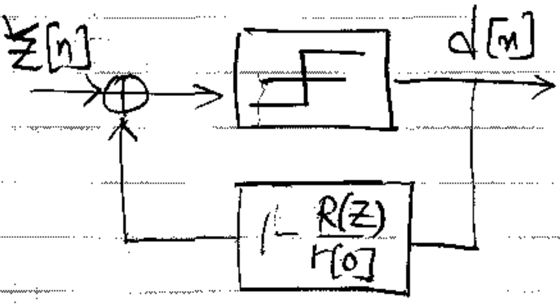
- To meet condition (i), we use these past decisions as

$$\tilde{z}[n] - \frac{r[1]}{r[0]} d[n-1] - \frac{r[2]}{r[0]} d[n-2] - \cdots$$

$$= d[n] + \frac{r[-1]}{r[0]} d[n+1] + \cdots$$

However, the precursors remain if $r[n]$ is not causal. So, we need **causal $r[n]$**

Then,



perfectly cancels the ISI.

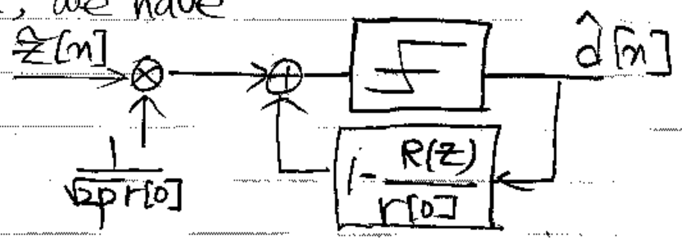
- Now, take a look at the noise part.
The slicer input is now given by

$$d[n] + \frac{\tilde{N}_e[n]}{\sqrt{2p}\, r[0]}$$

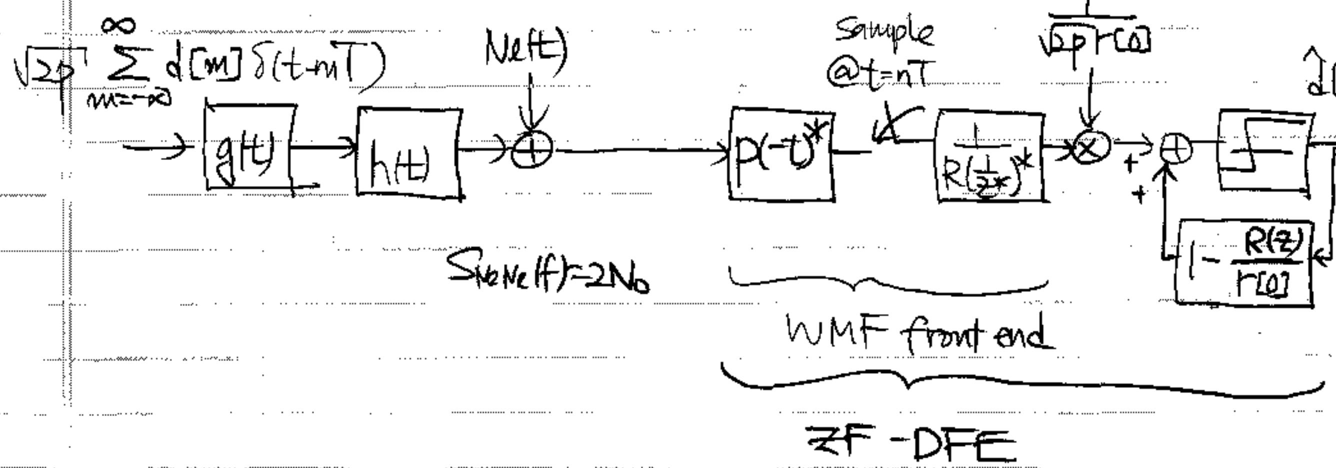
where $(\tilde{N}_e[n])_n$ is **white** with variance $\frac{2N_0}{2p|r[0]|^2}$

So, the input SNR is maximized by choosing the minimum phase $R(Z)$.

- Therefore, we have



Combined w/ the WMF front-end, the ZF-DFE is given by

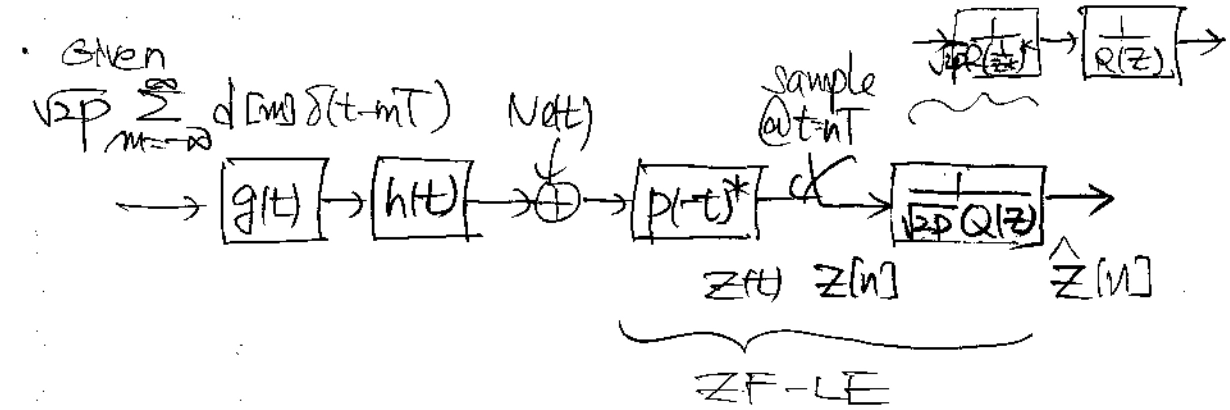


where $R(z)$ must be the minimum phase system that factors $Q(z)$ as $Q(z)=R(z)R(\frac{1}{z^*})^*$

- In practice, a DFE suffers error propagation.

○ Derivation of ZF-DFE from ZF-LE

- Given



$\hat{z}[n] = d[n] + \hat{N}_e[n]$

where $\hat{N}_e[n]$ is a colored noise with PSD $\propto \frac{1}{Q(e^{j2\pi f})}$

- Consider

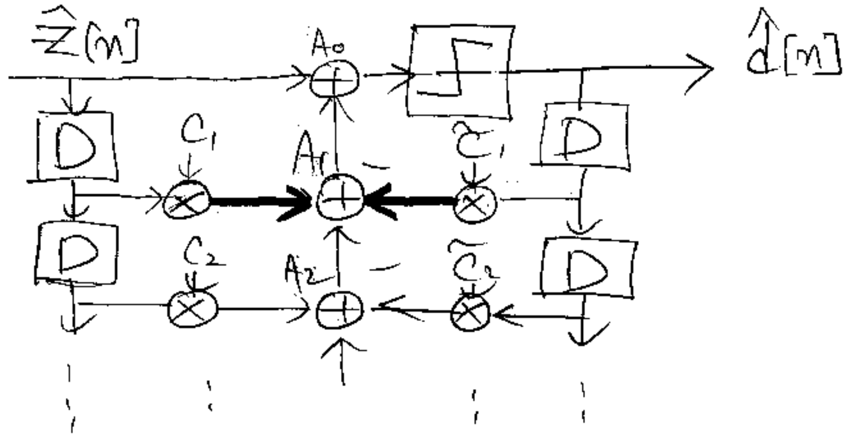
If the decision is perfect, then we may **subtract $\hat{d}[n-1]$ from $\hat{\hat{z}}[n-1]$ to obtain $\hat{N}_e[n-1]$.**

Similarly, we may subtract $\hat{d}[n-2]$ from $\hat{\hat{z}}[n-2]$ to obtain $\hat{N}_e(n-2)$, and so on.

- Since the noise sequence is colored, $\hat{N}_e[n-1]$, $\hat{N}_e[n-2]$, ... contain the information on $\hat{N}_e[n]$.

Thus,
we may subtract the predicted value of $\hat{N}_e[n]$ from $\hat{\hat{z}}[n]$ to have a better input to the slicer.

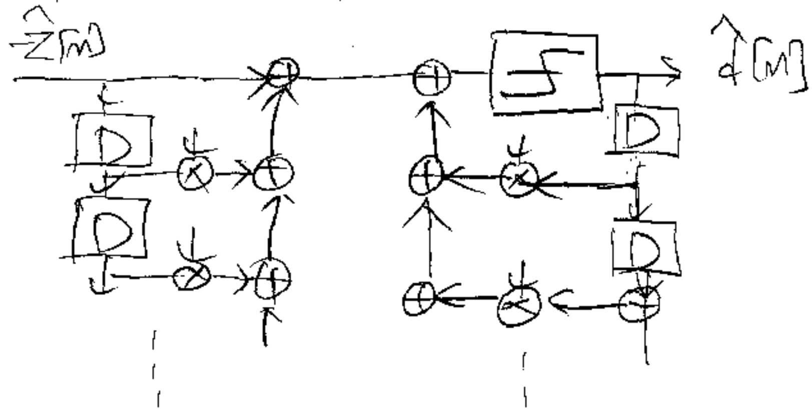
- Therefore, we want a system like

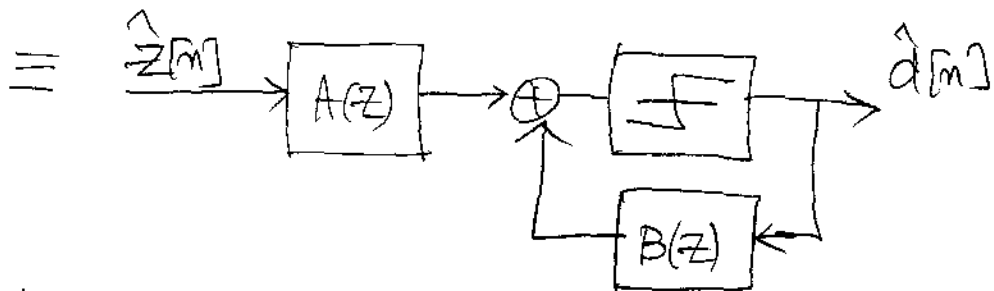


where the adder A_i does **subtraction of $\tilde{c}_i \hat{d}[n-i]$ from** $c_i \hat{\hat{z}}[n-i] = c_i \hat{d}[n-i] + c_i \hat{N}[n-i]$ and

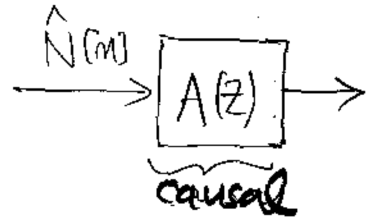
at the same time does subtraction of the best predicted value of $N[n]$ from $\hat{\hat{z}}[n-i]$

- This system is equivalent to

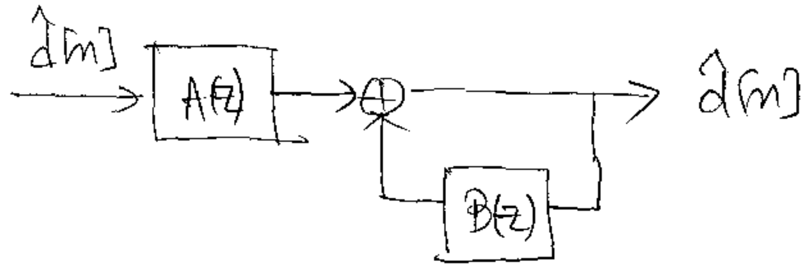




where
we want $A(z)$ to be the optimal causal predictor of $\hat{N}[n]$ when used as



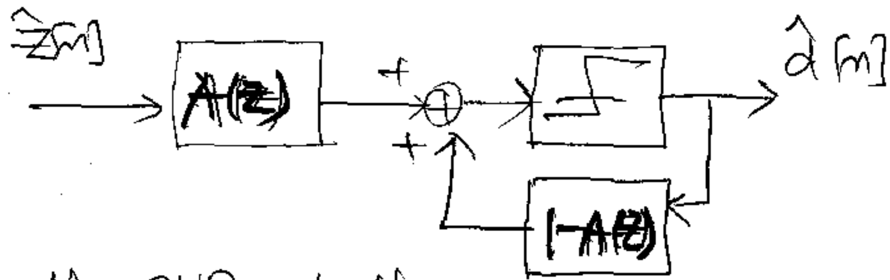
and
we want $B(z)$ to be the optimal canceller of the so-called postcursor ISI when used as



Thus, $B(z) = 1 - \underbrace{A(z)}_{\text{causal}}$ and $A(z)$ is monic.

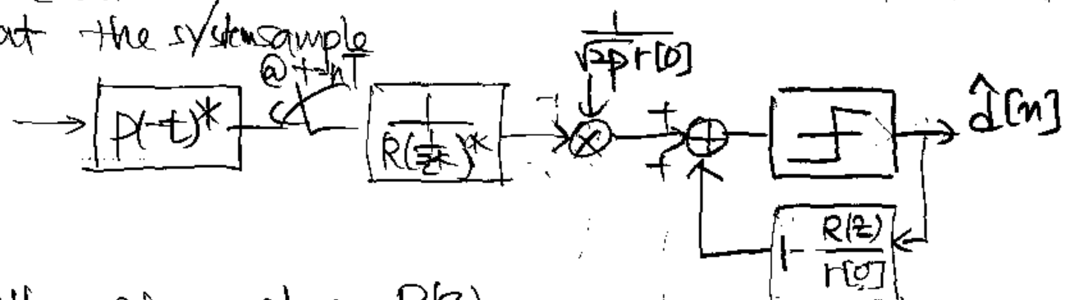
causal

- Therefore, we want a system



where the SNR at the slicer input is maximized.

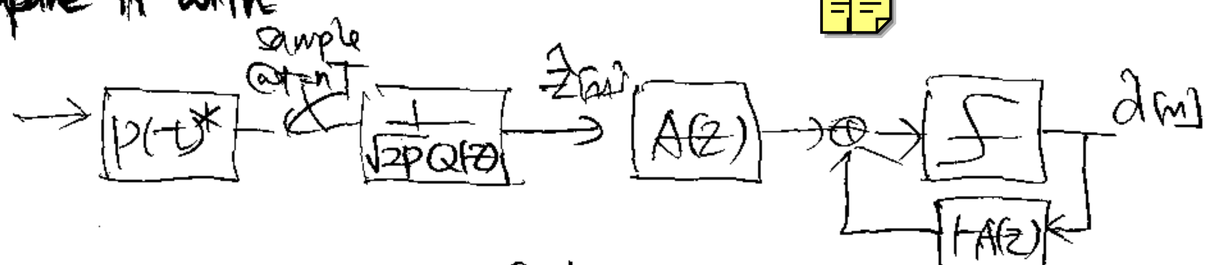
We've already seen from the derivation of the ZF-DFE that the system



with minimum phase $R(z)$
maximizes the slicer input SNR and at the same time < zero forces the input and / whitens the noise input

to the slicer.

Compare it with



Then, by inspection, we find

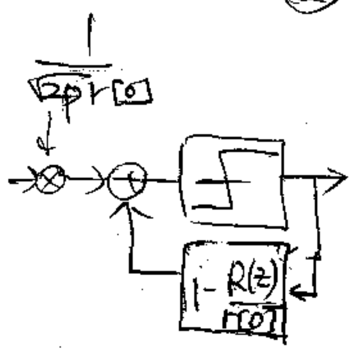
$$A(z) = \frac{R(z)}{r[0]} \leftarrow \text{monic}$$

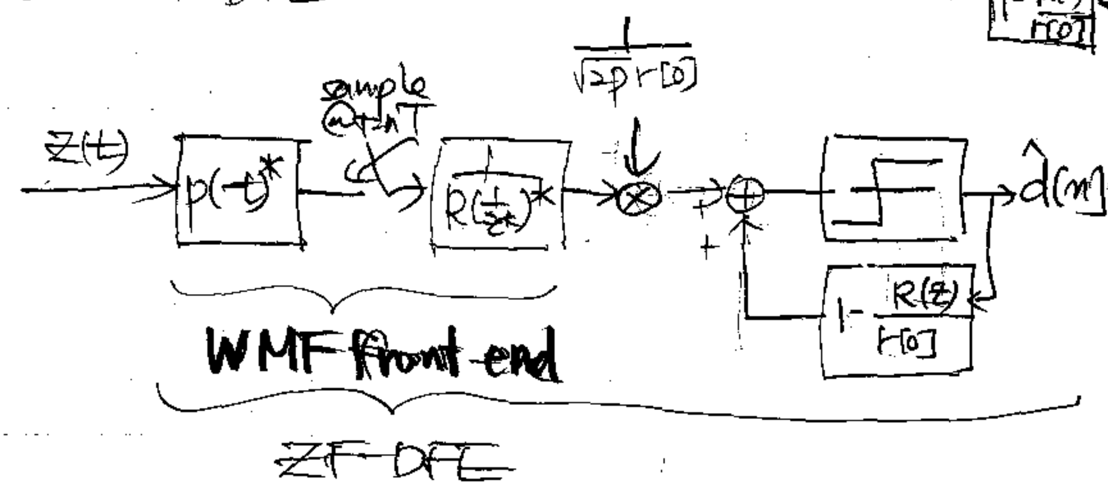
↙ for this see LMMSE prediction. (innovation of ... is white)

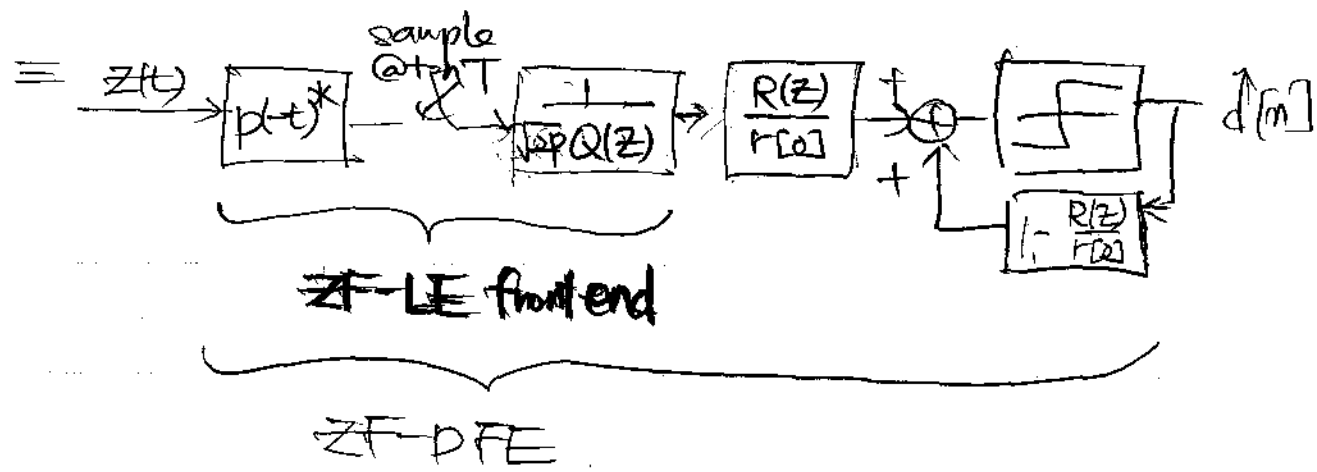
whitens the noise (i.e., optimally predicts the noise) and maximizes the input SNR to the slicer w/ ZF.

(of course, p5 figure already shows $\hat{c}_i = -c_i, \forall i$.)

o Summary

- The WMF front end followed by  does ZF-DFE

- 



References

1. Barry, Lee, and Messerschmitt, Digital Communication, 3rd ed. pp. 355 – 365
2. Fischer, Precoding and Signal Shaping for Digital Transmission, pp. 49 – 71