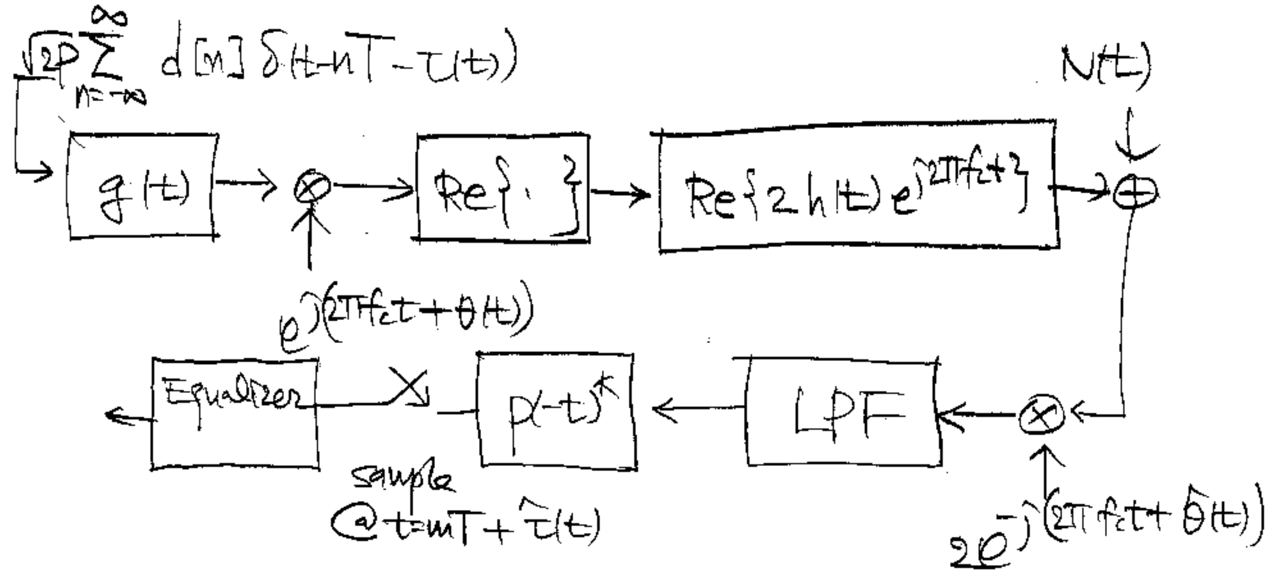
Synchronization

(Based on J. M. Cioffi, *Digital Communications*, Chapter 6 & Barry, Lee, Messerschmitt, *Digital Communication* 3rd ed. Chapters 14–16)

○ Assumptions made so far

- block diagram



- Assumptions in Tx
  - (i) The source **message clock** is synchronized to the symbol clock.
  - (ii) The **symbol clock** and **carrier frequency** are locked.
  - (iii) $\tau(t)$ = constant, $\forall t$, $\theta(t)$ = constant, $\forall t$.

- Assumption in Rx
  - (iv) $\hat{\theta}(t)=\theta(t)$, $\forall t$ $\hat{\tau}(t)=\tau(t)$, $\forall t$

- Assumption in Channel
  - (v) The channel is well modeled by an LTI system.

○ Realities

- Often, **pilot** or **training signals**, symbols, or tones are embedded in the transmitted signal.

or not.

- Whether pilot is used or not. [for an easy lock at Tx.

[(ii) **Symbol rate** & carrier **frequency** are often chosen
L(v) If the channel is time-varying, however, then the symbol rate & the carrier frequency of the received signal can **not** be **locked**. Why?

This is **due to channel fading** that causes **Doppler shift** of carrier frequency.

$\Rightarrow$ We cannot obtain the symbol rate from the recovered carrier frequency.

[(iii) $\tau(t) \neq$ const, $\theta(t) \neq$ const
L(iv) $\hat{\theta}(t)$ cannot be equal to $\theta(t)$, $\forall t$
$\hat{\tau}(t)$ cannot be " " $\tau(t)$, $\forall t$

$\Rightarrow$ Either $\theta(t)$ is **tracked** by $\hat{\theta}(t)$
or high-precision **crystal oscillators** at both Tx & Rx **are used**.
Either $\tau(t)$ is tracked by $\hat{\tau}(t)$
or " " " " " , combined with oversampling are used ($\Rightarrow$ fractionally spaced equalizers (FSE))

o Two main approaches

- Tracking of $\tau(t)$ & $\theta(t)$ by $\hat{\tau}(t)$ & $\hat{\theta}(t)$.
For this, we need to design estimators for $\tau(t)$ & $\theta(t)$, given the observation of the received signal corrupted by a noise & other uncertainties from the channel, etc.
/ex/ narrow band single-user systems
- Using high-precision crystal oscillators at both Tx & Rx /ex/ multi-user systems.
- Etc.

○ What to recover

when filtered by an LTV,

- Symbol clock ← symbol timing synchronization
- Carrier frequency ← carrier recovery

the **symbol clock is not much affected** but the **carrier phase** is.

- Frame ← frame synchronization in data transmission
- Network Clock ← pointers and Add/Delete methods.

○ The term "phase"

The term phase is used
not only for a sinusoidal waveform
but also for any periodic signal.

/ex/. 1/ the magnitude and the phase of a complex sinusoidal waveform $\underline{A}\, e^{j(2\pi f_0 t + \theta)}$

$\underline{A}$ ← mag; $(2\pi f_0 t + \theta)$ ← **phase angle**

2. **timing-phase** error of the symbol clock in Rx

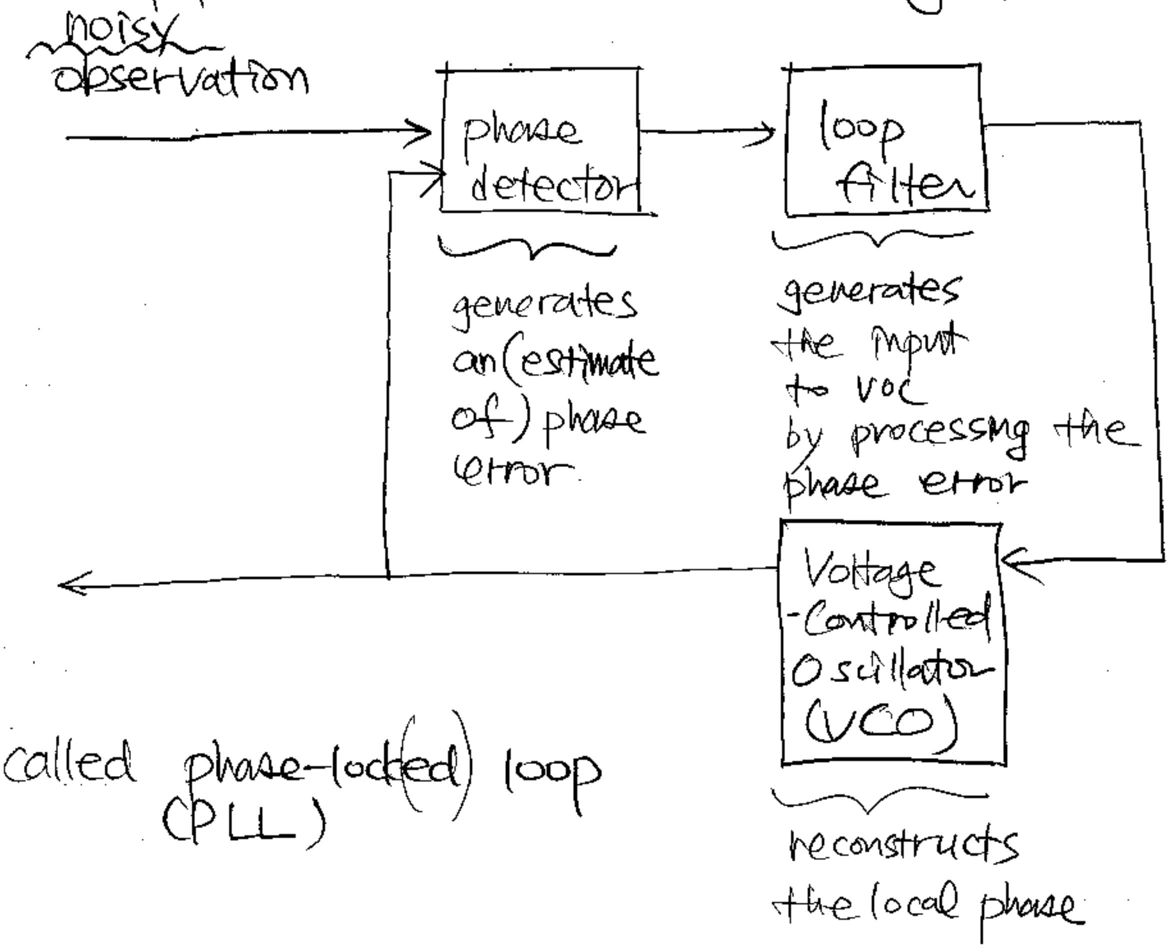
3. **code-phase** at a time of a periodically transmitted m-sequence

⇒ There are various types of "phase" recovery.

○ Two main approaches for tracking

- continuous time / discrete time
- open-loop / decision-directed

○ Most popular structure for tracking phase angle.



called phase-locked loop (PLL)

- If reduce the delay in loop filter, then vulnerable to noise,

→ trade-off!

- why PLL?

The PLL underlies most synchronization techniques.

○ PLL

analysis is hard.

system: linear / non-linear

non-linear ← PLL

PLL: transient behavior / steady-state or in-lock behavior

we are interested in this!

PLL: CT / DT

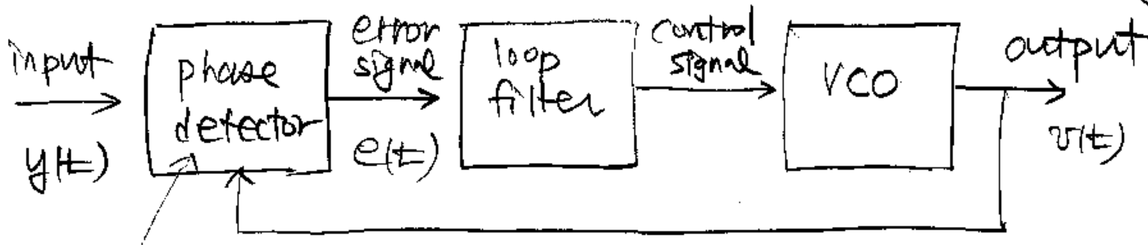
○ CT PLL (BLM, *Digital Commun.* Ch.14.1)

mainly based on

- Assumptions

Even if the freq. of input is not ω_v, we can write in this form

Input : $y(t) = A_y \cos(\omega_v t + \theta(t))$ + noise



VCO output : $v(t) = A_v \cos(\omega_v t + \hat{\theta}(t))$

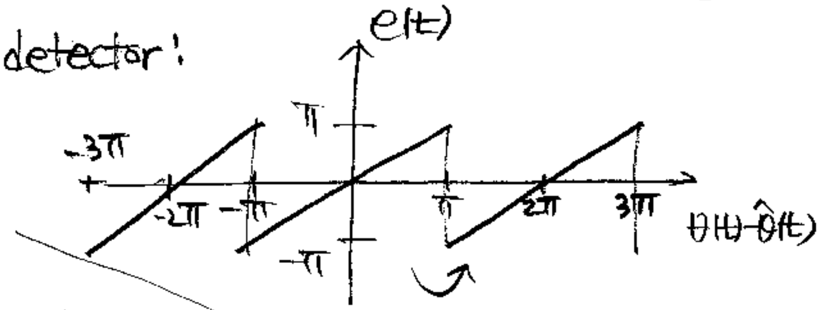
ω_v: nominal / freerunning frequency of the VCO.

- phase detector

any gain is included in the loop filter.

Output of ideal phase detector : $e(t) = \theta(t) - \hat{\theta}(t)$

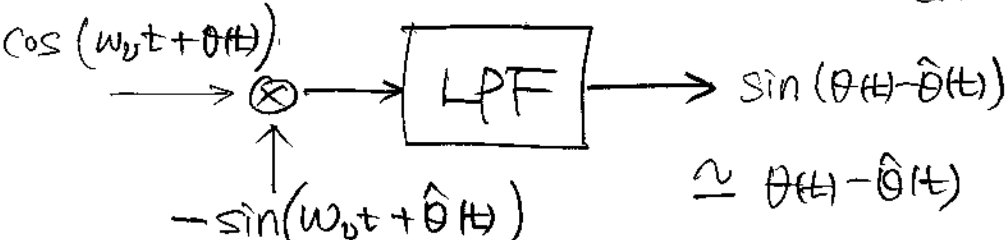
- sawtooth / modulo-2π phase detector:



cycle slip occurs.

- Maximum Likelihood phase estimator: a series of approximations leading to a quantity $\propto$ phase error.

- demodulating phase detector:

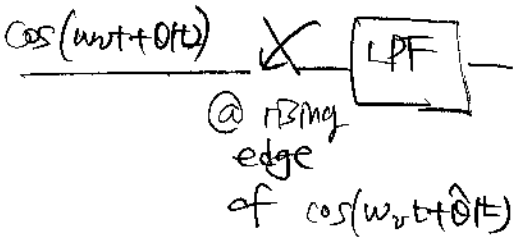


$$\simeq \theta(t)-\hat{\theta}(t)$$

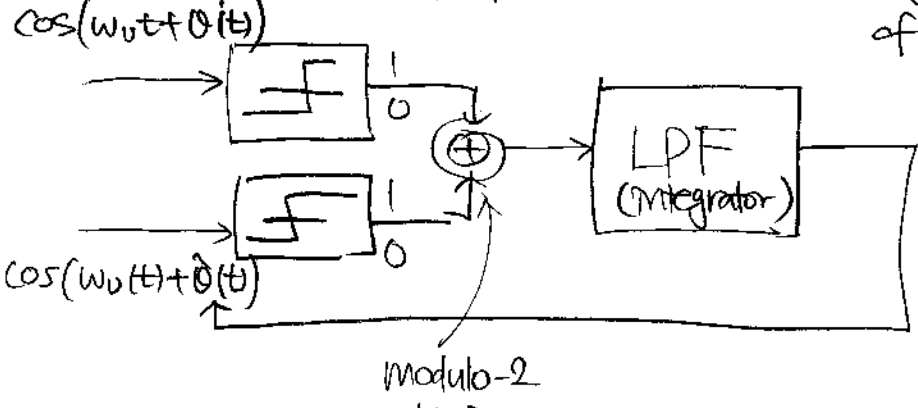
$$-\sin(\omega_0 t+\hat{\theta}(t))\cos(\omega_0 t+\theta(t))$$

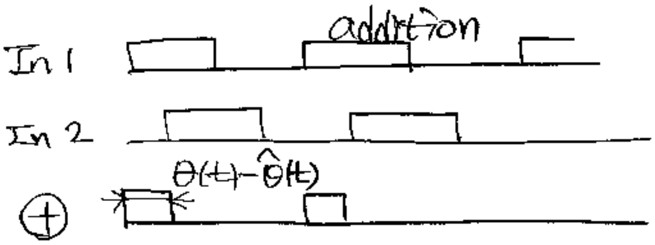
$$= -\sin(2\omega_0 t+\hat{\theta}(t)+\theta(t)) + \sin(\theta(t)-\hat{\theta}(t))$$

- sampling phase detector:



- binary phase detector:





— VCO

— Ideal VCO : output $v(t) = A_o \cos(\omega_o t + \hat{\theta}(t))$

$$\frac{d\hat{\theta}(t)}{dt} = K_{vco} \cdot \underbrace{e(t)}_{\text{input}} \quad \cdots (*)$$

Analog VCO physics is beyond the scope.

input = output of phase detector

— loop filter

1st order PLL has the loop filter that has
$L(s) = K_L$

2nd order PLL has " " " " "
$$L(s) = K_L \left(\frac{s+K_1}{s+K_2} \right) \text{ or }$$
$$= \alpha + \frac{\beta}{s}$$

Note that, by (*), $\hat{\theta}(t) = K_{vco} \cdot L(t) * e(t)$

$$\Rightarrow \frac{\hat{\theta}(s)}{\theta(s)} = \frac{K_{vco} \cdot L(s)}{s + K_{vco} \cdot L(s)}$$

A PLL is phase locked if $\theta(t) - \hat{\theta}(t) = \text{const}$.
" " " perfectly phase locked if " $= 0$

· The 1st order PLL may not be perfectly locked.
· α & β must be chosen to make phase locked (perfectly.)
· Analysis of 1st-order & 2nd-order PLL is

omitted here. See references

o DT PLL

Similar to CT PLL except that we need to use z-transform instead of Laplace transform