○ References

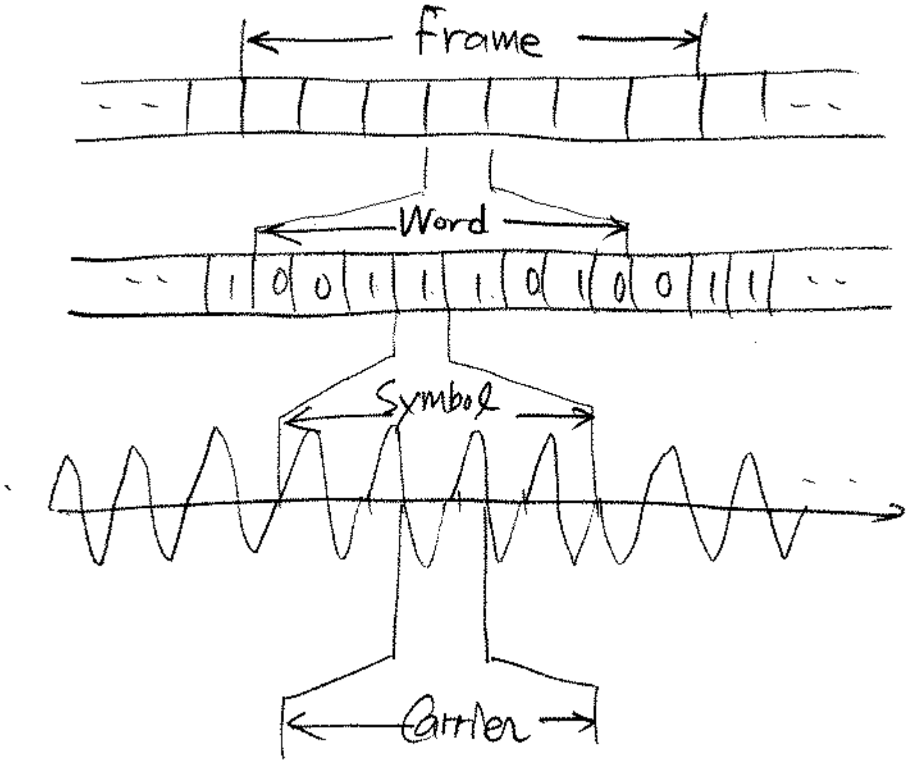
(B): S. Benedetto & E. Biglieri, Principles of Digital Transmission with Wireless Applications. Chapter 9.

(P): J. G. Proakis, Digital Communications, 3rd ed., Chapter 6.

(LM): J. R. Barry, E. A. Lee, and D. G. Messerschmitt, Digital Communication, 3rd ed., Chapters 14, 15, 16.

(C): J. M. Cioffi, Digital Communications, Chapter 6.

○ Different Synchronization Levels in Digital Commun. (B)



○ What to recover

We only consider recovery of
- symbol clock
- symbol timing
- carrier freq.
- carrier phase.

○ Acquisition & Tracking : (B)

- Acquisition time or acquisition phase :
  an initial period of time to synchronize the oscillators

- Tracking phase : the operation of keeping the phase difference between the two oscillators.

  e.g. coarse timing acquisition followed by tracking

○ Regeneration of spectral lines : (B)

- No power efficient modulation scheme contains a spectral line (a Dirac delta function) in its power spectrum, in order not to waste any power

  e.g. DSB-WC is power inefficient in that a carrier tone is transmitted.

- Spectral line is useful in recovering carrier frequency and symbol clock. Thus, any carrier or clock synchronizer will be composed of two conceptually distinct parts :

  (1) a suitable nonlinear circuit to regenerate a carrier or clock frequency from the received signal

(2) a narrowband device (typically a tuned filter or a phase-locked loop) to separate the regenerated carrier or clock from background disturbances

Example(1)$'$ BPSK with rectangular pulse & carrier

$$v(t) = \sum_n a_n p_T(t-nT) \cos(2\pi f_0 t + \theta(t))$$

$$v(t) \longrightarrow \boxed{(\cdot)^2} \longrightarrow y(t) = \frac{1}{2}\Big\{1 + \underbrace{\cos(4\pi f_0 t + 2\theta(t))}\Big\}$$

squaring loop

A spectral line @ frequency $2f_0$, when $\theta(t)$ ignored, is regenerated!

Example (1)$''$ baseband BPSK with non-rectangular pulse.

$$r_D(t) = \sum_n a_n h(t-nT)$$

By Poisson Sum Formula

$$E\{|r_D(t)|^2\} = E\{|a_n|^2\} \sum_n |h(t-nT)|^2$$

$$= \frac{E\{|a_n|^2\}}{T} \sum_\ell \mu_\ell \exp\left(j\frac{2\pi\ell t}{T}\right)$$

where $\mu_\ell \triangleq \int_{-\infty}^{\infty} H^*(f-\frac{\ell}{T}) H(f)\, df$

This means if the BW of $h(t)$ is less than $1/T$, then $E\{|r_D(t)|^2\}$ consists of a constant and a cosine term with period equal to the symbol period.

Therefore, the spectrum of $E\{|r_b(t)|^2\}$ contains the spectral lines with frequencie $\pm\frac{1}{T}$ Hz.

In both cases (1)' and (1)'', nonlinear square-law rectifiers are used to restore the desired spectral lines.

Tuned filters or PLL's may follow to track the restored frequency.

○ Order of frequency & timing recoveries

The above example implies that, given a real bandpass signal

(1) the carrier frequency is first recovered by using a squaring loop & PLL,

(2) the signal is downconverted to the baseband by demodulating using recovered carrier frequency,

and then finally

(3) the symbol clock is recovered.

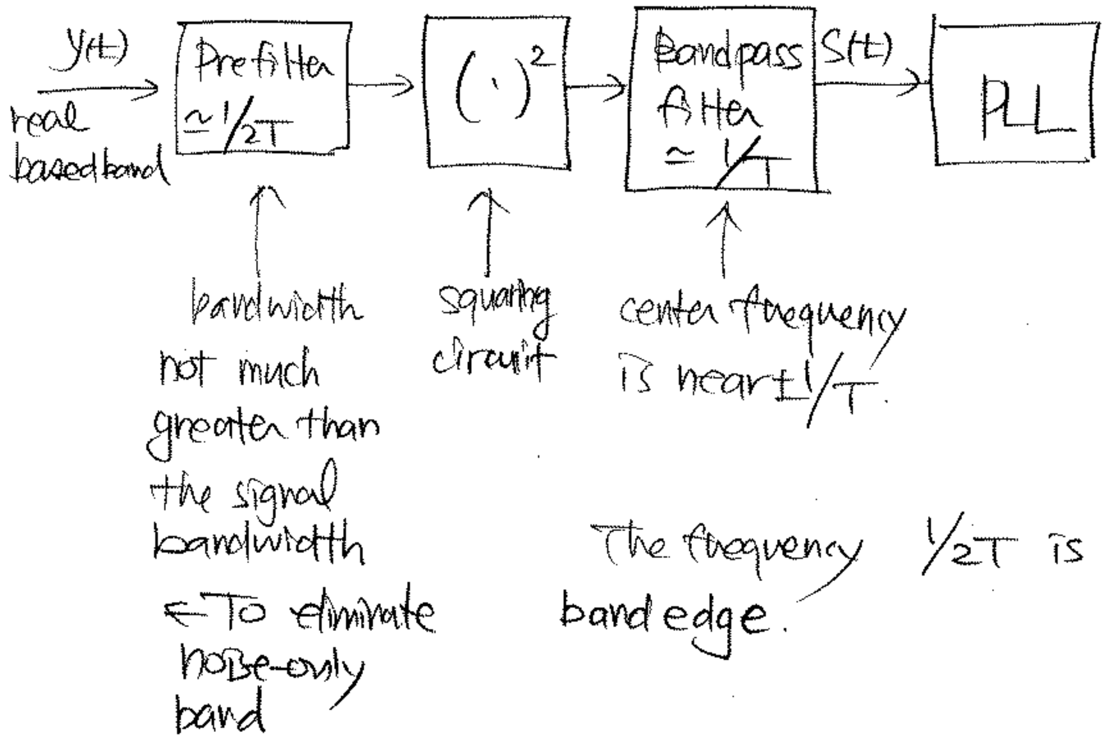
This may be true for very high SNR case. However, there are many other cases without high enough SNR. In these cases, the nonlinear circuits perform very poor in regenerating spectral lines. So, this order of recoveries may not work.

Even in high SNR, symbol clock can be first recovered

○ Square-law timing recovery & envelope timing recovery : ⓒ

- Square-law timing-recovery method

  (i) the simplest
  (ii) the most widely used.
  (iii) If squaring is not enough, then even functions including a fourth-power circuit can be used.



The frequency $1/2T$ is called band edge.

- Envelope timing recovery method.

  (i) a bandpass signal version.
  (ii) $\boxed{(\cdot)^2} \rightarrow \;\rightarrow \boxed{|\cdot|^2} \rightarrow$

(iii) a priori knowledge of carrier frequency.
(iv) often also called a squaring-law method.

○ Bandedge timing recovery w/ carrier recovery! ©

If bandpass, then simple squaring-law method, or precisely speaking, the envelope timing recovery does not work.

To avoid prior carrier recovery, a bandedge timing recovery method is proposed. See ©.

○ High SNR vs medium, low SNR

- In **high SNR**, above methods may work. So, carrier frequency may be recovered first, then the timing may be recovered by squaring or envelope timing recovery.

Or timing could be first recovered.

- However, in **medium or low SNR**, the above methods cannot generate reliable carrier/clock frequencies.

- In practice, a pilot or training sequence is used to recover the timing phase first!!!

under the use of high-precision oscillators to generate approximate carrier frequency & symbol clock

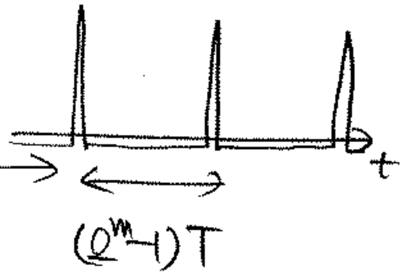
- Using the recovered timing phase & symbols from the pilot or training sequence, the decision-directed carrier frequency, carrier phase, symbol clock, and timing phase recovery subsystems are initialized.

/ex/ Suppose $m(t) = \sum_{n=0}^{2^m-1} a_n p(t-nT)$

is a period of m-sequence modulated signal.

Then,

$\sum_K m(t-k(2^m-1)T) \rightarrow \boxed{m(-t)} \rightarrow$



the sequence (of one period) matched filter output exhibits periodic correlation peaks, which can be used to recover timing phase.

Extension to bandpass case with small carrier offset is straightforward.

- Thus, in what follows, we are more interested in decision-directed methods.

○ Decision - Directed Timing Recovery : ©

- minimization of the mean-square error over the sampling timing phase

- between the equalizer (if any) output and the decision

- $J(\tau) = E\{|\hat{x}_k - z(kT+\tau)|^2\}$

where
$z(kT+\tau)$ is the equalizer (LE or DFE) or other receiver output at sampling timing k corresponding to sampling phase τ.

- The update uses a stochastic-gradient method.

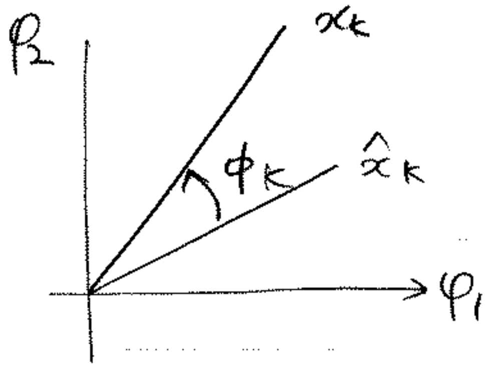
- Actually, τ is $\tau(t)$. Thus, by keeping updating, we track not only the phase but also the clock frequency and the phase.

- This requires, of course, the assumption of high-precision local oscillator to generate the norminal timing clock & initialization by using a pilot or training sequence.

○ Decision - Directed Carrier Recovery : ©

- Decision-directed carrier recovery is more commonly encountered than open loop carrier recovery such as the square-law carrier-recovery.

- As always, the key is to derive the error signal for phase locking.



$x_k = a_k + j b_k$, $\hat{x}_k = \hat{a}_k + j \hat{b}_k$

$$\Rightarrow \frac{x_k}{\hat{x}_k} = \frac{|x_k|}{|\hat{x}_k|} e^{j\phi_k}$$

$$\Rightarrow \phi_k = \arctan \frac{\hat{a}_k b_k - a_k \hat{b}_k}{a_k \hat{a}_k + b_k \hat{b}_k}$$

$$\simeq \arctan \frac{1}{\mathcal{E}_x} (\hat{a}_k b_k - a_k \hat{b}_k)$$

$$\approx \frac{1}{\mathcal{E}_x} (\hat{a}_k b_k - a_k \hat{b}_k)$$

$$\propto (\hat{a}_k b_k - a_k \hat{b}_k)$$

with the approximation increasingly accurate for small phase offset.

- A look-up table for arcsin or arctan can also be used.

○ Pilot ⟨Timing / Carrier⟩ recovery : ©

- pilot vs training signal :

  Usually, pilot means pilot tone.

- In pilot timing recovery, the transmitter inserts a sinusoid of frequency equal to ℓ/p times the desired symbol rate.

- In pilot carrier " , " "
  " " " " " " " "
  " " " carrier frequency.

- The PLL can be used to recover the ⟨symbol rate / carrier frequency⟩ at the receiver.

- Typically, pilots are inserted at unused frequencies in transmission.

- The effect of jitter and noise are largely eliminated because the PLL sees no data dependent jitter at the pilot frequency if the receiver filter preceding the PLL is sufficiently narrow.