◎ Frame Synchronization : ©

- the **boundary of long symbols or packets** may be unknown.
- For synchronization, **some known** ⟨pattern or characteristic⟩ of transmitted waveform **is required** to be searched.
- The objective is again **to derive a (timing) phase error**.

○ Correlation Method or autocorrelation method

- From estimation theory, it is well known that if

$$Y(t) = S(t-\tau) + N(t)$$

where $S(t)$ is known and
$N(t)$ is white Gaussian,
then the ML estimate of τ is

$$\hat{\tau}_{ML} = \arg\max_{\tau} \int Y(t) S(t-\tau)\, dt$$

or equivalently

$$\hat{\tau}_{ML} = \arg\max_{\tau} \; Y(t) * S(-t) \Big|_{t=\tau}$$

which correlates $Y(t)$ with $S(t-\tau_0)$ to find τ_0 where the correlation peak is located.

- In reality, we have an unknown channel that filters the known signal $S(t)$. Moreover, there

may exist interference that makes the overall noise non-Gaussian and/or non-white.
= colored.

- There are 3 fundamental issues : ©
  1. How to choose $s(t)$?
  2. How the Channel affects $s(t)$?
  3. What is the noise characteristic?

where $s(t)$ should be designed taking 2 & 3 into consideration.

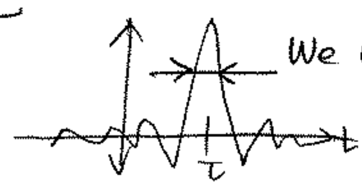
○ Synchronization Patterns : ©

- a synchronization pattern or a synch sequence } is some known transmitted signal.

- Typically, a synch pattern is chosen that creates a large, narrow peak in autocorrelation function.

$$Y(t) * s(-t) = \underbrace{s(t-\tau) * s(-t)} + N(t) * s(-t)$$

$s(t-\tau) \rightarrow \boxed{s(-t)} \rightarrow$



However, this is not the only criterion. It is rather a rule of thumb.

- Consider a sequence of i.i.d. binary random variables $\{a_k\}_{k=-\infty}^{\infty}$ with $Pr(a_k=1)=Pr(a_k=-1)=1/2$. Then,

$$E\{a_k a_{k'}\} = \delta_{k,k'} \Rightarrow E\{\sum_{k=1}^{N} a_{k+l} a_{k'+l}\}$$

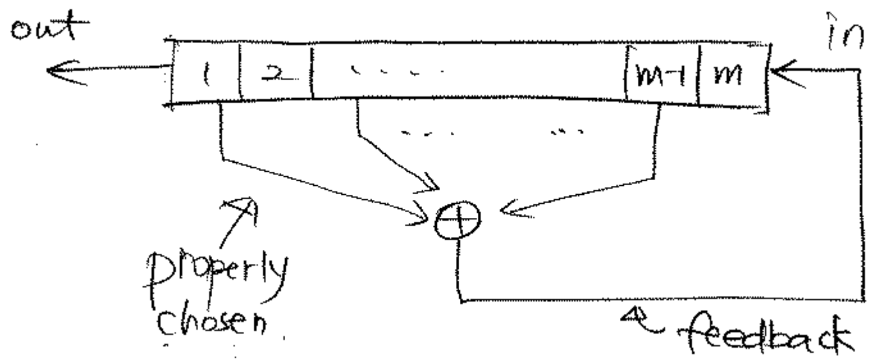
$$= N\delta_{k,k'}$$

A white sequence is a good role model. But we need a deterministic sequence.

→ Various sequence designs.

(i) m-sequence or maximal- (maximum) length shift-register code (sequence) (P)

- one of pseudo-noise (PN) sequences
- Length 2^m-1 bits are generated by an m-stage shift register with linear feedback.



- The sequence is periodic with period $n=2^m-1$
- Each period contains 2^{m-1} ones and $2^{m-1}-1$ zeros.

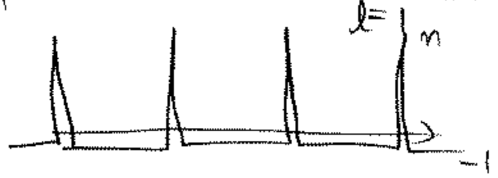
Similar to random sequence with large n.

Other similar properties to random sequences.

Autocorrelation function (bipolar case) — periodic

$$\sum_{l=1}^{n} a_{k+l} a_{k'+l} = \begin{cases} n & \text{if } k=k' \\ -1 & \text{if } k\neq k' \end{cases}$$

Similar to random sequence.



- There are other PN sequences such as Gold codes and Kasami codes.

(ii) Chirp sequence : (C)

- $x_k = e^{j2\pi \frac{k^2}{M}}$, $k = 0, 1, 2, \cdots, M-1$

- If $k \in \mathbb{Z}$ then a periodic sequence with period M is generated.

- $$\sum_{m=0}^{M-1} x_{k+m} x^*_{k'+m} = \begin{cases} M & \text{if } k = k' \\ 0 & \text{if } k \neq k' \end{cases}$$

  Better than m-sequence in autocorrelation but harder to generate.

- Not much used for digital communications, more frequently used in radar systems.

  In radar engineering, a pattern that generate a very narrow peak in an **ambiguity function** is preferred. A chirp sequence is one of such sequences.

(iii) Barker Codes

- very short, not repeated > but peakiness under "

  unknown < preceding and succeeding > data surrounding the pattern.

- may be inserted periodically in a transmission stream.

- /ex/ 11-symbol Barker code
  + + + − − − + − − + − (1 1 1 −1 −1 −1 1 −1 −1 1 −1)

  autocorrelation ignoring surrounding data

  1 0 1 0 1 0 11 0 1 0 1 0 1

○ Channel Response : ©

- A communication channel rarely ⟨outputs / generates⟩ a scaled & delayed version of the input.

- Rather, ⟨LTI / LTV⟩ channel we generally encounter.

- Still, a pattern with high correlation peak is preferred because

$$h(t) \simeq \sum_{l} \alpha_l \delta(t-\tau_l).$$

So, the correlator output is approximately

$$\tilde{y}(t) \simeq \sum_{l} \alpha_l \, p(t-\tau_l)$$

where $p(t)$ is the autocorrelation function of the pattern.

○ Noise

If noise is not white, then it distorts the correlator output. One approach is to whiten the noise but it will also affect the desired part in the correlator output.

So, the pattern should be chosen depending on the noise statistic.

◎ Pointers & Add/Delete Methods

- synchronization b/w source message clock and symbol clock.
- However, the two clocks are not perfectly synchronized.
- Cure
  - regularly insert dummy message symbols or
  - bits are not sent

  This method is known as
  < rob/stuff or add/delete > timing & synchronization method.
- The buffer depth is monitored.
- same phenomenon & cure for sink
- " " " " for digital audio signal processing. However, pre-processing is required before add or delete!
- Sometimes, clock reference is broadcast. A pointer points

specific location in time. If a pointer arrives earlier than expected, the local oscillator is too slow.