Consider a noisy observation model

$$Y = a + X \tag{1}$$

where $a$ is a deterministic signal and $X \sim \mathcal{N}(0, \sigma^2)$. Here we can calculate the SNR of $Y$ as following.

$$\text{SNR}(Y) = \frac{a^2}{E[X^2]} = \frac{a^2}{\sigma^2} \tag{2}$$

Now, also consider another observation $Z$ which satisfies $\begin{bmatrix} X \\ Z \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \sigma^2 \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right)$. Can we increase the SNR by using $Z$? The answer is YES.

Consider the SNR of $Y + bZ$,

$$\text{SNR}(Y + bZ) = \frac{a^2}{\sigma^2(b^2 + 2b\rho + 1)}. \tag{3}$$

This value is maximized when $b = -\rho$ and the maximum value is $\frac{a^2}{\sigma^2(1-\rho^2)}$. This gives the inequality

$$\frac{a^2}{\sigma^2} \le \frac{a^2}{\sigma^2(1 - \rho^2)} \tag{4}$$

because $-1 \le \rho \le 1$. Thus, $\text{SNR}(Y) \le \text{SNR}(Y - \rho Z)$. This implies that we can always increase the SNR by observing other correlated noises.