# EECE 695F: Advanced Digital Communications

Midterm Exam # 1 (Spring 2015)

Time allowed: 4.5 hours

Name: ________________________________

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| Problem 1: | _____ | / 50 |
| Problem 2: | _____ | / 40 |
| Problem 3: | _____ | / 10 |
| Problem 4: | _____ | / 20 |
| Problem 5: | _____ | / 28 |
| Problem 6: | _____ | / 19 |
| Problem 7: | _____ | / 25 |
| Problem 8: | _____ | / 15 |
| Problem 9: | _____ | / 10 |
| Total: | _____ | / 217 |

**Problem 1.** (50 points) Answer the following questions. You can answer either in English or in Korean. Use more than 3 sentences to each answer.

(a) (5 points) Explain why a real-valued bandpass signals and random processes are important in modern wireless communications.

(b) (5 points) Explain why a complex-valued baseband signals and random processes are important in modern wireless communications.

(c) (5 points) Explain why IF up-conversion transmitters have been more popular than direct up-conversion transmitters in modern wireless communications.

(d) (5 points) Explain why an bandpass filter is needed immediately after an up-conversion mixer.

(e) (5 points) Explain why heterodyne receivers have been more popular than homodyne receivers in modern wireless communications.

(f) (5 points) Explain what is the function of a pre-select filter in heterodyne receivers?

(g) (5 points) Explain what is the function of a channel select filter in heterodyne receivers?

(h) (5 points) Explain why do we use a PA and an LNA in the Tx and the Rx, respectively.

(i) (5 points) Explain why Gaussian random variables, vectors, and processes are important in modern wireless communications.

(j) (5 points) Explain what is the difference between a PA and an LNA? Explain why an LNA can replace a PA in some cases?

**Problem 2.** (40 points) Answer the following questions. You can answer either in English or in Korean.

(a) (5 points) Sketch a block diagram of a direct up-conversion transmitter. Specify in particular what are the in-phase signal, the quadrature (-phase) signal, the power amplifier.

(b) (5 points) Sketch a block diagram of a heterodyne (IF) up-conversion transmitter. Specify in particular what are the intermediate-frequency signal, the up-conversion mixer, and the image reject filter.

(c) (5 points) In (b), sketch and explain the difference between the upper-side tuning and the lower-side tuning.

(d) (5 points) Sketch a digital IF transmitter. What is the difference between the digital IF transmitter and the heterodyne transmitter?

(e) (5 points) Sketch a detailed block diagram of a direct down-conversion receiver. Specify in particular what are the pre-select filter, the low-noise amplifier, and the bandpass filter.

(f) (5 points) Sketch a detailed block diagram of a heterodyne receiver. Specify in particular what is the down-conversion mixer.

(g) (5 points) In (f), sketch and explain the difference between the upper-side tuning and the lower-side tuning.

(h) (5 points) Sketch a digital IF receiver. What is the difference between the digital IF receiver and the digital baseband receiver?

**Problem 3.** (10 points) Answer the following questions.

(a) (5 points) When a real-valued narrowband bandpass signal is given by

$$x(t) = \cos(2\pi f_c t + \phi(t))$$

find the in-phase component and the quadrature component.

(b) (1 point) When the input to an up-conversion mixer is given by

$$x(t) = x_c(t)\cos(2\pi f_{IF} t) - x_s(t)\sin(2\pi f_{IF} t)$$

and

$$y(t) = \cos(2\pi(f_{RF} - f_{IF})t),$$

what is the output $z(t)$?

(c) (1 point) When the input to an up-conversion mixer is given by

$$x(t) = x_c(t)\cos(2\pi f_{IF} t) - x_s(t)\sin(2\pi f_{IF} t)$$

and

$$y(t) = \cos(2\pi(f_{RF} + f_{IF})t),$$

what is the output $z(t)$?

(d) (3 points) Compare the upper-side tuned output signal of (b) and the lower-side tuned output signal of (c). Are they the same? If not, how do they differ? (Hint. What are the complex envelopes?)

**Problem 4.** (20 points) Suppose that a real-valued narrowband bandpass random process

$$X(t) = X_c(t)\cos(2\pi f_c t) - X_s(t)\sin(2\pi f_c t)$$

is wide-sense stationary. Answer the following questions.

(a) (3 points) Find $\mathbb{E}\{X_c(t)\}$, $\mathbb{E}\{X_s(t)\}$, and $\mathbb{E}\{X(t)\}$, as functions of $t$.

(b) (3 points) When $\mathbb{E}\{X_c^2(0)\} = P$, find $\mathbb{E}\{X_c^2(t)\}$, $\mathbb{E}\{X_s^2(t)\}$, and $\mathbb{E}\{X^2(t)\}$, as functions of $t$.

(c) (5 points) When $\mathbb{E}\{X_c(0)X_c(1)\} = Q$, find $\mathbb{E}\{X_c(1)X_c(2)\}$, $\mathbb{E}\{X_c(3)X_c(2)\}$, $\mathbb{E}\{X_s(0)X_s(-1)\}$, $\mathbb{E}\{X_s(-1)X_s(-2)\}$, and $\mathbb{E}\{X_c(-3)X_c(-2)\}$.

(d) (5 points) When $\mathbb{E}\{X_c(0)X_s(1)\} = R$, find $\mathbb{E}\{X_c(0)X_s(0)\}$, $\mathbb{E}\{X_c(1)X_s(1)\}$, $\mathbb{E}\{X_c(-2)X_s(-2)\}$, $\mathbb{E}\{X_c(1)X_s(2)\}$, and $\mathbb{E}\{X_c(3)X_s(2)\}$.

(e) (4 points) Find $\mathbb{E}\{X_c(t)X_s(t)\}$ as a function of $t$.

**Problem 5.** (28 points) Given i.i.d. Gaussian random variables $X_1, X_2, ...$ with mean $\mu_i$ and variance $\sigma^2$, answer the following questions.

(a) (5 points) Define the Q-function $Q(x)$ in terms of $X_1, \mu_1$, and $\sigma$.

(b) (5 points) Define Marcum's Q-function $Q\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$ in terms of $X_i$'s.

(c) (5 points) Define generalized Marcum's Q-function $Q_m\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$.

(d) (1 point) Find the relation betwen $Q\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$ and $Q_m\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$.

(e) (1 points) Find the cdf of the Rayleigh random variable with unit mean in terms of $Q_m\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$.

(f) (5 points) Define the central Chi-square distribution with $n$ degrees of freedom in terms of $X_i, \mu_i$, and $\sigma$.

(g) (1 points) What is the relation between the generalized Marcum's Q-function $Q_m\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$ and the cdf of the central Chi-square distribution with $n$ degrees of freedom?

(h) (5 points) Why $Q(x)$, $Q\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$, and $Q_m\left(\frac{\chi}{\sigma}, \frac{x}{\sigma}\right)$ are tabulated?

**Problem 6.** (19 points) Suppose that two real-valued random vector $\underline{X}_c$ and $\underline{X}_s$ are jointly Gaussian with

$$\mathbb{E}\{\underline{X}_c\} = \underline{\mu}_c, \mathbb{E}\{\underline{X}_s\} = \underline{\mu}_s, \mathrm{Cov}\{\underline{X}_c\} = C_c, \mathrm{Cov}\{\underline{X}_s\} = C_s, \mathrm{Cov}\{\underline{X}_c, \underline{X}_s\} = C_{cs}.$$

Let a complex-valued random vector be $\underline{X} = \underline{X}_c + \mathrm{j}\underline{X}_s$ and answer the following questions.

(a) (3 points) What are the definitions of $\mathrm{Cov}\{\underline{X}_c\}, \mathrm{Cov}\{\underline{X}_s\}$, and $\mathrm{Cov}\{\underline{X}_c, \underline{X}_s\}$?

(b) (6 points) Define the mean vector, the covariance matrix, and the pseudo-covariance matrix of $\underline{X}$, and find them in terms of $\underline{\mu}_c, \underline{\mu}_s, C_c, C_s$, and $C_{cs}$.

(c) (10 points) Find the condition that $\underline{X}$ is proper.

**Problem 7.** (25 points) Suppose that $\underline{X} = \underline{X}_c + \mathrm{j}\underline{X}_s$ is a proper-complex Gaussian random vector with zero mean and unit covariance matrix. Answer the following questions. You can use the Q-function, Marcum's Q-function, and generalized Marcum's Q-function in your answer.

(a) (5 points) Using the pdf of a real-valued Gaussian random variable, derive the pdf of $\underline{X}$.

(b) (2 points) When $\underline{Y} = A\underline{X} + \underline{b}$, show that $\underline{Y}$ is proper.

(c) (3 points) In (b), what is the pdf of $\underline{Y}$? You don't need to derive.

(d) (5 points) Find $\Pr(|\underline{a}^H \underline{X}| \geq c)$ when $c > 0$.

(e) (5 points) Find $\Pr(\|\underline{X}\| > r)$ when $r > 0$.

(f) (5 points) Find $\Pr(\|\underline{X} + \underline{c}\| > r)$ when $r > 0$.

**Problem 8.** (15 points) Suppose that a real-valued bandpass signal

$$s_1(t) = \mathrm{Re}\{s_{1,l}(t)e^{\mathrm{j}2\pi f_1 t}\}$$

passes through a specular channel

$$h(t) = \delta(t) + \frac{1}{2}\delta(t-\tau).$$

Answer the following questions.

(a) (5 points) Define $r_1(t) \triangleq s_1(t) * h(t)$ and $r_{1,l}(t)$ such that $r_1(t) = \mathrm{Re}\{r_{1,l}(t)e^{\mathrm{j}2\pi f_1 t}\}$. Find $g_1(t)$ such that

$$r_{1,l}(t) = s_{1,l}(t) * g_1(t)$$

(b) (5 points) Repeat (a) when $s_w(t) = \mathrm{Re}\{s_{2,l}(t)e^{\mathrm{j}2\pi f_2 t}\}$, i.e., find $g_2(t)$ such that

$$r_{2,l}(t) = s_{2,l}(t) * g_2(t)$$

where $r_{2,l}(t)$ satisfies

$$\begin{aligned} r_2(t) &\triangleq s_2(t) * h(t) \\ &= \mathrm{Re}\{r_{2,l}(t)e^{\mathrm{j}2\pi f_2 t}\}. \end{aligned}$$

Use Dirac delta functions for $g_2(t)$.

(c) (5 points) Discuss why the answers in (a) and (b) are different.

**Problem 9.** (10 points) Answer the following questions

(a) (5 points) Design a system that has a real bandpass signal $x(t) = \mathrm{Re}\{x_l(t)e^{j2\pi f_c t}\}$ as the input and the envelope $|x_l(t)|$ as the output. All your components must have real inputs & real outputs.

(b) (5 points) Suppose that $f(t)$, $g(t)$, and $h(t)$ are all real-valued baseband signal with bandwidth $< f_c$. Find the output $v(t)$ of the following system.