Lec. 02

2014년 3월 3일 월요일
오후 3:06

Lec. #2

- Review

uncertainty in electrical systems

Q. What is a system?
A. A system is a mapping rule

$$\text{input signal} \longrightarrow \boxed{\text{system}} \longrightarrow \text{output signal}$$

Q. How do we model signals in electrical systems?
A. scalar
vector
matrix
sequence ← scalars, vectors, matrices
waveform ← scalar, vector, matrix
polynomial

Page 01

2014년 3월 3일 월요일

오후 3:06

- signals w/ uncertainty
  - random variable
  - " vector
  - " matrix
  - " sequence
  - " process

scalar-valued, vector ", matrix-val.

- point-to-point uni-directional comm.

source → Tx → channel → Rx → sink

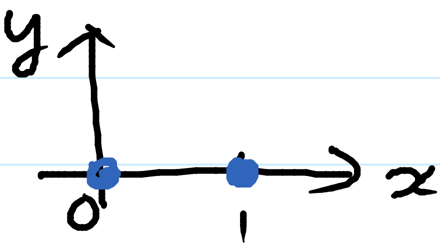
usually deterministic

usually w/ uncertainty

$x \rightarrow \boxed{} \rightarrow y$

$\mathcal{X} = \{0,1\}$ $\quad$ $\mathcal{Y} = \{0,1\}$

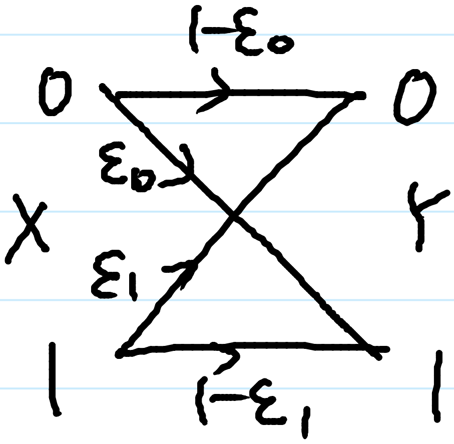
$(0,0), (1,0) \equiv$

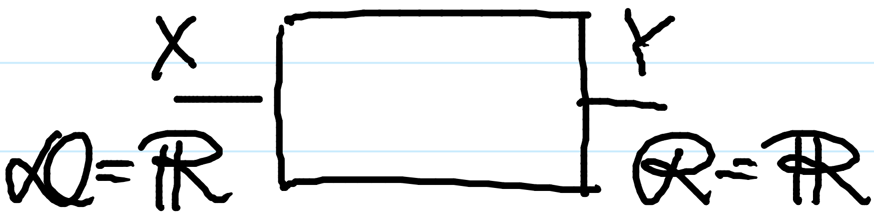


Page 01-1

2014년 3월 3일 월요일

오후 3:06





$$Y = X + N \quad \text{where } N \sim \mathcal{N}(0,1)$$

$\vdots$

Page 02

2014년 3월 3일 월요일

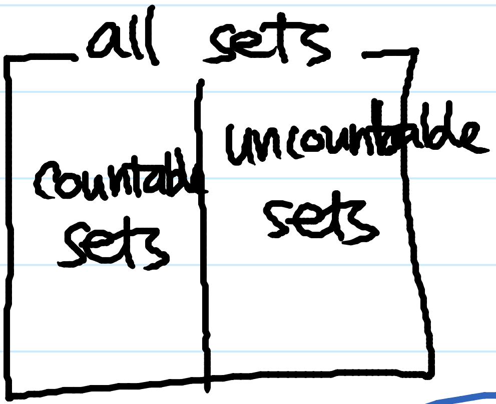
오후 3:23

Set Theory

$\mathbb{R}$: the set of all real numbers

$\mathbb{Q}$: " " " " rational "

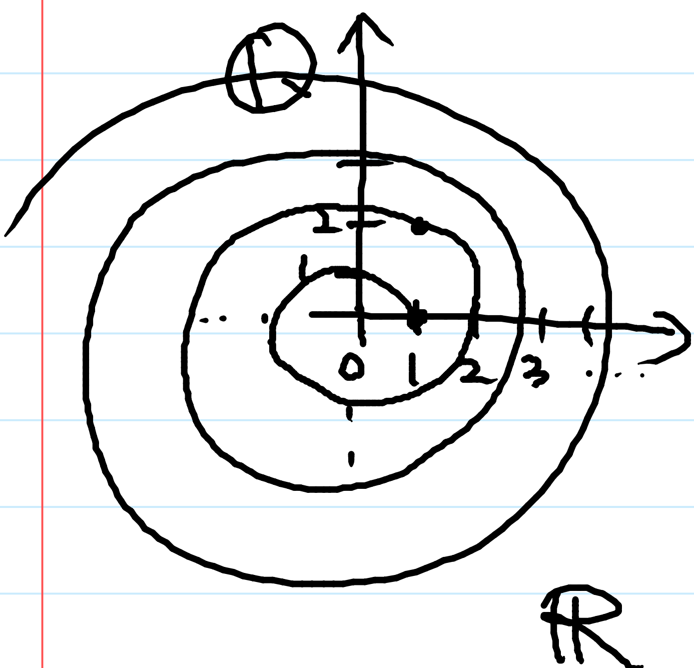
ratio $= \frac{n}{m}$



$\{1, 2, \cdots, 6\}$

$\mathbb{N} = \{1, 2, 3, \cdots\}$

- set
  - countable
    - finite : $\{1, 2, \cdots, 6\}$
    - infinite : $\mathbb{N}, \mathbb{Q}$
  - uncountable – infinite : $\mathbb{R}, \mathbb{C}$



$x \in \mathbb{Q}$

$x = \frac{n}{m}$

1	(1,0)
2	(2,1)
3	⋮

- empty set vs. null set

  $\phi \triangleq \{ \quad \}$

  ϕ : phi

  $\varnothing$: phi

- A & B are disjoint.

  {1, 2} & {3, 4} are "disjoint"

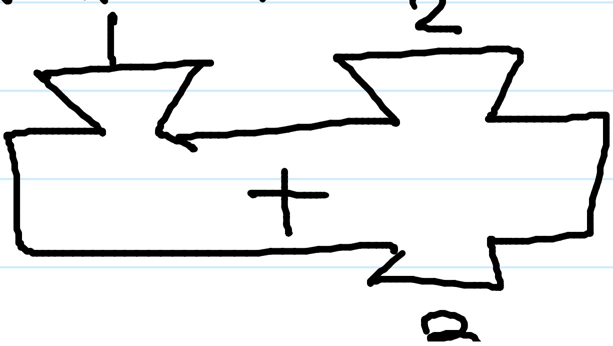
  "mutually exclusive"

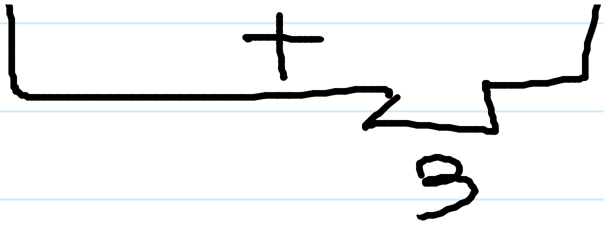
- Set

  element, member

  - countable
    - finite
    - infinite
  - uncountable
    - infinite

- Operations.





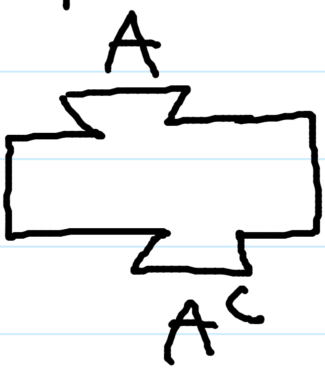
Page 04

2014년 3월 3일 월요일

오후 3:23

- Set operations

(i) complementation



Given a universal set S & a subset A

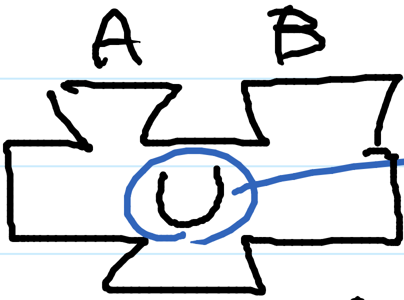
$S \rightarrow \{1, 2, \cdots, 6\}$

$A \rightarrow \{2, 4, 6\}$

A^c is defined as ...

$\rightarrow \{1, 3, 5\}$

(ii) Union



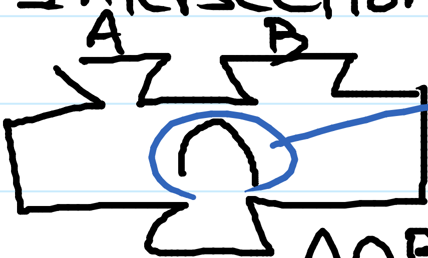
cup

$$A \cup B \triangleq \{x \in S : x \in A \text{ or } x \in B\}$$

colon

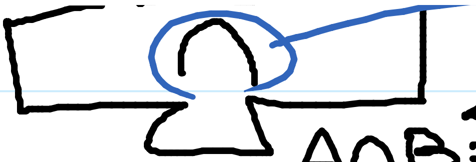
"such that"

(iii) Intersection



cap

$$A \cap B \triangleq \{x \in S : x \in A \text{ and } x \in B\}$$

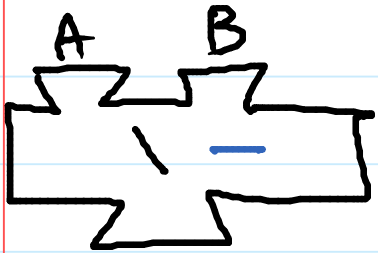


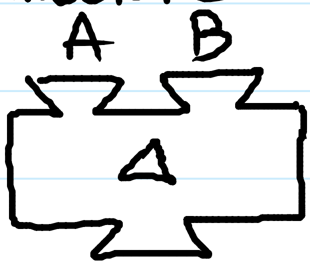
$$A \cap B \triangleq \{x \in S : x \in A \text{ and } x \in B\}.$$

2014년 3월 3일 월요일

오후 3:23

- difference vs. symmetric difference



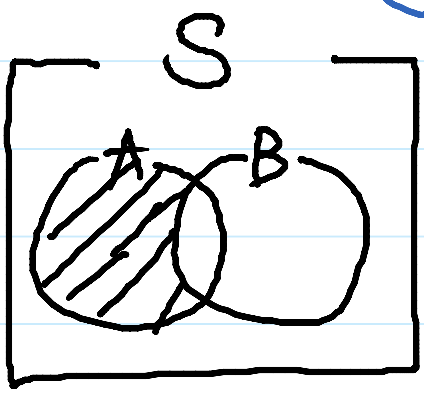


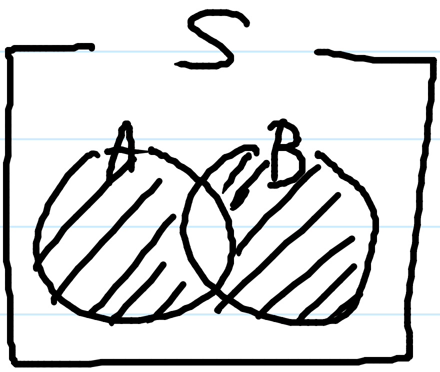
$A \setminus B \triangleq \{x \in S : x \in A$ and $x \notin B\}$

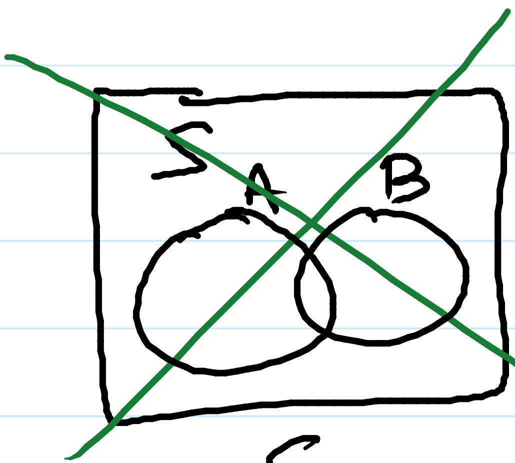
A−B

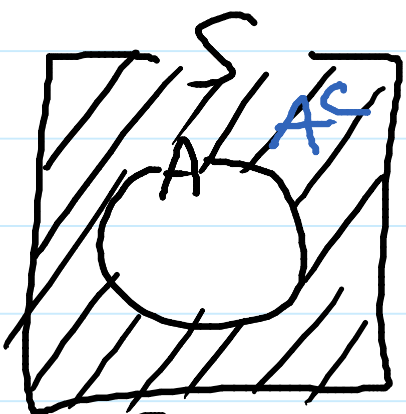
$A \triangle B \triangleq \{x \in S :$ $x \in A \setminus B$ or $x \in B \setminus A\}$
$= (A \setminus B) \cup (B \setminus A)$

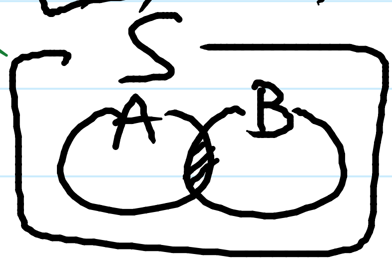
- Venn Diagram

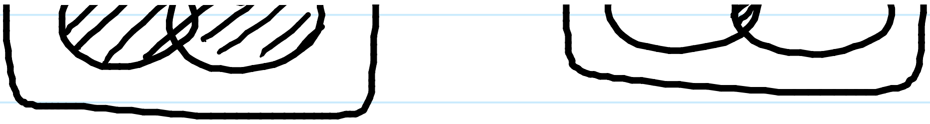












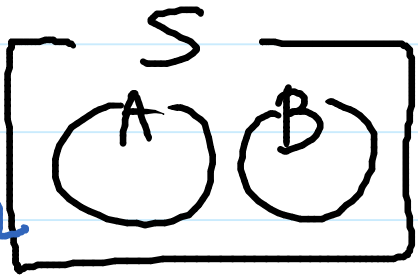
- Equality of two sets

"$A = B$" "A is equal to B"

if $x \in A \Rightarrow x \in B$ and
$x \in B \Rightarrow x \in A$.

- Q. $A \cap B = \emptyset$

visualize!



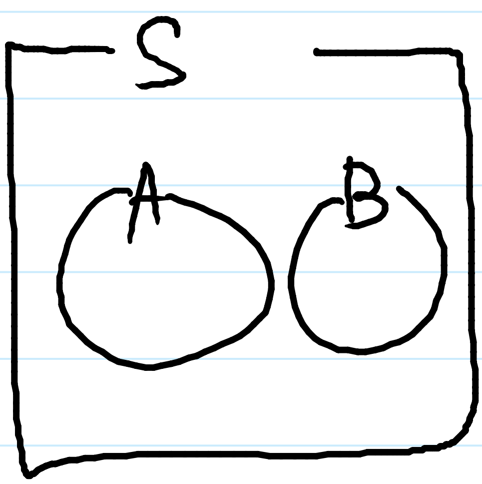
generalize

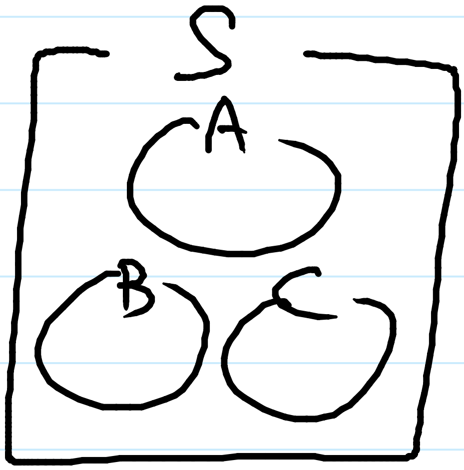
A & B are mutually exclusive
disjoint.

Q. $A_1, A_2, \cdots$
$A_m \cap A_n = \emptyset, \forall m \neq n$

for every
for all

"$A_1, A_2, \cdots$ are pairwise disjoint.





- Q. $A \triangle B \triangleq (A \setminus B) \cup (B \setminus A)$

  $\overset{?}{=} (A \cup B) \setminus (A \cap B)$

- Given a set I, define a subset A_α of S, where $\alpha \in I$.

  a countable set

  /ex/ $I = \mathbb{N}$

  $A_1, A_2, A_3, \cdots$

Q. $\bigcup_{i=1}^{\infty} A_i$

(infinite union)
countable union.

$\bigcap_{i=1}^{\infty} A_i$

countable intersection

$A_1 \cup A_2$

$A_1 \cup A_2 \cup A_3$

$\bigcup_{i=1}^{N} A_i$

finite union

2014년 3월 5일 수요일

오후 1:49

Q. Given an uncountable set I,

$$\bigcup_{\alpha \in I} A_\alpha \triangleq \{ x \in S : \qquad \}$$

$$\bigcap_{\alpha \in I} A_\alpha \triangleq \{ x \in S : \qquad \}$$

- Reading Assignment

Ch. 2. (Ch. 1).

Peebles Ch. 1, 2, 3.