EECE 233 Fall 2014 Lec. #25

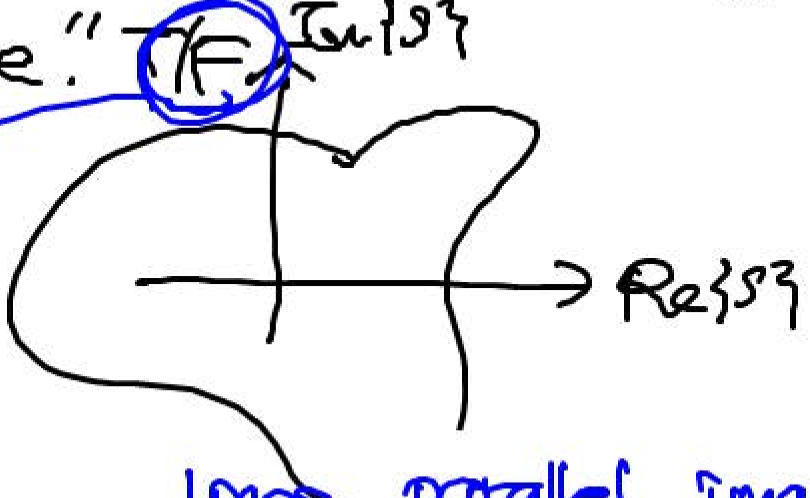
Announcements

1. We will have a supplementary class meeting on this Friday from 09:30 till 10:45.
2. There will be a Quiz & MATLAB session on next Monday from 14:00 till 15:15.
3. The MATLAB HW originally due on this Friday is due on next Monday.

Review

Q1. "$\mathcal{L}\{x(t)\}$ is a function of $s \in \mathbb{C}$,

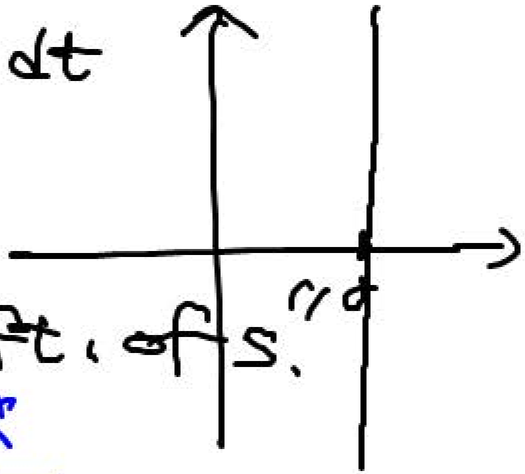
the domain of this function can have an arbitrary shape." T/F.



ROC must be a union of lines parallel imaginary axis.

$$\mathcal{L}\{x(t)\} = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt,$$

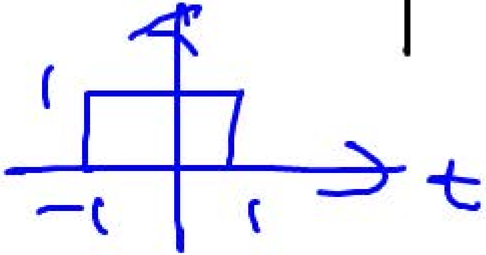
$$\int_{-\infty}^{\infty} |x(t) e^{-st}|\, dt = \int_{-\infty}^{\infty} |x(t) e^{-\sigma t}|\, dt$$



Q2. " $X(s)$ is always a rational ft. of s."

T/F

counter example



$$e^{x} = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$X(s) = \frac{N(s)}{D(s)}$$

Q3. " The inverse Laplace TF must be computed by using contour integral." T/F

Important LT pairs.

(i) $e^{-at}u(t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+a}$, $\{s \in \mathbb{C} : Re(s) > Re(-a)\}$

(ii) $-e^{-at}u(-t) \xleftrightarrow{\mathcal{L}} \frac{1}{s+a}$, $\{s \in \mathbb{C} : Re(s) < Re(-a)\}$

(iii) $\delta(t-T) \xleftrightarrow{\mathcal{L}} e^{-sT}$, entire plane $\mathbb{C}$.

ex/ $\frac{1}{(s+1)(s+2)} \overset{PFE}{=} \frac{A}{s+1} + \frac{B}{s+2}$ $\qquad \int_{-\infty}^{\infty} \delta(t-T)e^{-st}dt =$

$$X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$$

$$\frac{dX(s)}{ds} = \int_{-\infty}^{\infty} x(t) \; (-t) e^{-st} dt$$

$$= \int_{-\infty}^{\infty} (-t x(t)) e^{-st} dt$$

$$t x(t) = -\frac{dX(s)}{ds}$$

$$X(s) = \frac{N(s)}{D(s)}$$

$$= P(s) + \frac{\tilde{N}(s)}{D(s)}$$

$$= P(s) + \frac{\tilde{N}(s)}{\prod_i D_i(s)}$$

ex)

$$\frac{1}{(s+a)^3} = (s+a)^{-3} = \frac{d}{ds}\left(-\frac{1}{2}(s+a)^{-2}\right)$$

ex)

$$\frac{s^2+2s+1}{s+1+s^{-1}} = \frac{s^3+2s^2+s}{s^2+s+1}$$

Recall

$$e^{j\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow H(j\omega_0)e^{j\omega_0 t}$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega \qquad \frac{1}{2\pi}\int_{-\infty}^{\infty} \underbrace{X(j\omega)\, H(j\omega)}\, e^{j\omega t}\, d\omega$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega)\, e^{j\omega t}\, d\omega$$

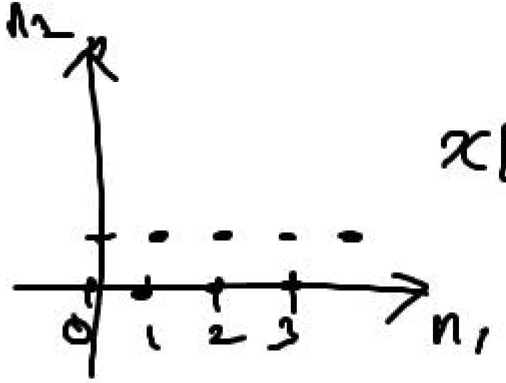
$$e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)e^{st}$$

$$\frac{1}{2\pi j}\oint X(s)\, e^{st}\, ds \qquad \frac{1}{2\pi j}\oint \underbrace{X(s)\, H(s)}_{=Y(s)}\, e^{st}\, ds$$

Q. Define the convolution b/w
$x(t_1, t_2)$ & $h(t_1, t_2)$.

Recall that $x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$

$= x[n] * \delta[n]$



$$x[n_1, n_2] = \sum_{k_1=-\infty}^{\infty} \sum_{k_2=-\infty}^{\infty} x[k_1, k_2]\, \delta^{(2)}[n_1-k_1, n_2-k_2]$$

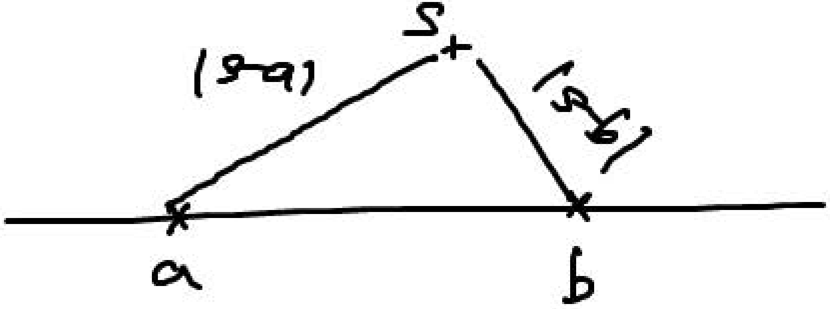
where $\delta^{(2)}[n_1-k_1, n_2-k_2] \triangleq$

$$\begin{cases} 1, & \text{iff } n_1=k_1 \ \& \ n_2=k_2. \\ 0 & \text{elsewhere} \end{cases}$$

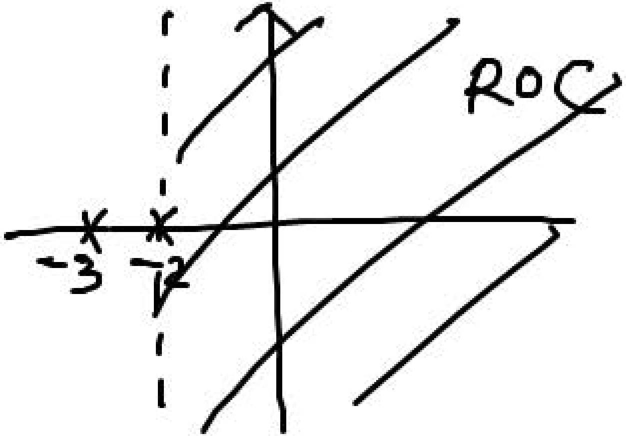
Q. $A \triangleq \{s \in \mathbb{C} : |s-a| = 2|s-b|\}$. $a \neq b$, $s \in \mathbb{C}$.

"conic section"

$\left\{ \begin{array}{l} \text{algebraic} \\ \text{geometric} \end{array} \right\}$ approaches.

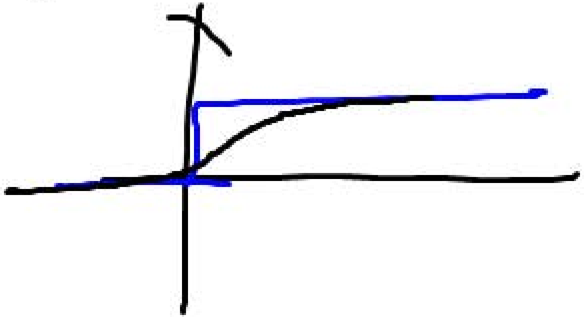


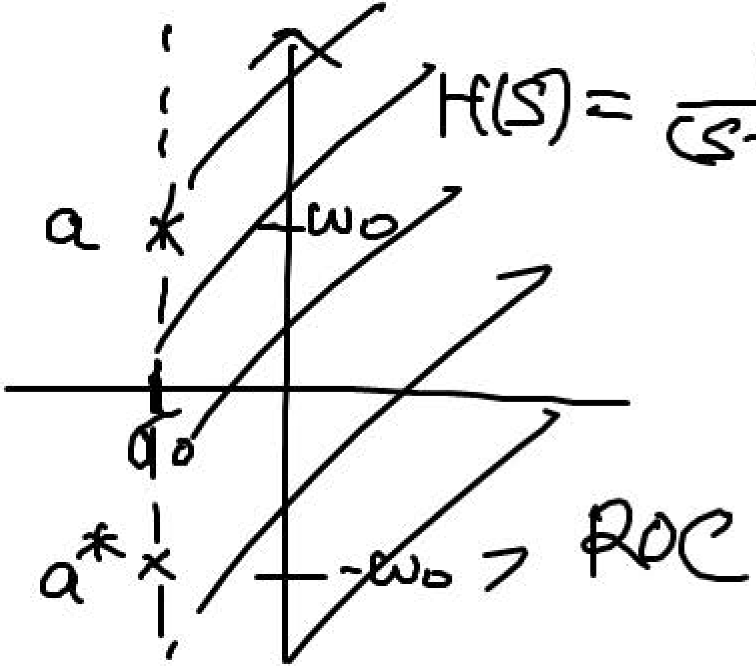
$$H(s) = \frac{1}{(s+2)(s+3)}$$



$$|H(j\omega)| = \frac{1}{|j\omega+2|\,|j\omega+3|}$$

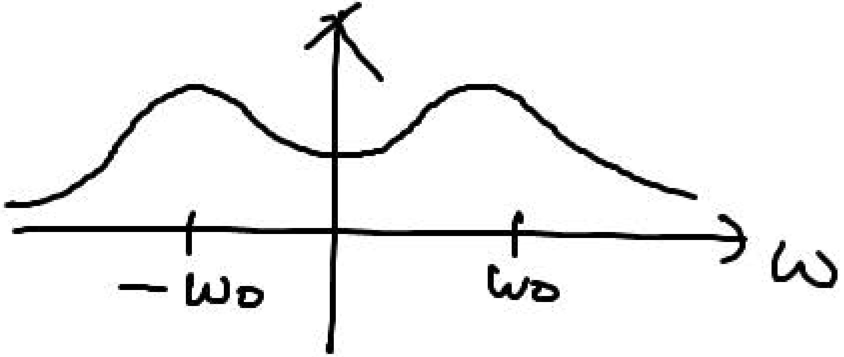
$$= \frac{1}{|j\omega+2|} \cdot \frac{1}{|j\omega+3|}$$





$$H(s) = \frac{1}{(s-a)} \frac{1}{(s-a^*)}$$

$$|H(s)| = \frac{1}{|s-a|} \frac{1}{|s-a^*|}$$



$a = a_0 + j\omega_0$