2023년 9월 14일 목요일 12:49

CM04 230914Th

Announcements

1. 지난 강의 수정 사항

   N/A

2. HW
   - HW02 will be assigned soon. Due will be Tuesday. For details, check the PLMS class homepage.

3. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.html .
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

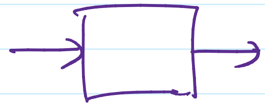
4. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)에 수정입력해 놓았습니다. 정정사항이 있는 학생은 사유를 제출해 주세요.

5. Overview (The numbers are the chapter numbers of the textbook)

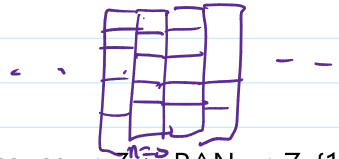
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform
   7. Sampling
   8. Communication Systems

6. Review

6. Review
- EE = twin tower
- a system = a mapping rule with the domain and the range



- a signal
  - scalar, vector, matrix, tensor
  - sequence, waveform (=function of time)
  - field, polynomial, graph, manifold, etc.
  - Any mathematical entity can be viewed as a signal.
- the dimension of a signal
  - When a signal is viewed as a function, the dimension of the function is called the dimension of the signal
    - y = f(x): The argument x is a zero-dimensional signal.
    - **y** = A**x**: The vector **x** is a one-dimensional signal.
    - gray-scale picture: The matrix is a two-dimensional signal.
    - a CT signal x(t), a DT signal x[n]: 1-D signals
    - a length-N vector-valued DT signal **x**[n]: a 1-D signal or a 2-D signal because **x**: Z-> R^N, x: Zx{1,2,...N} -> R



- Limitation of a mathematical model
  - infinite vs. infinitesimal
- a discrete-time signal vs. a digital signal
- Signal and Noise
- Classification
  - Better than no classification
  - limitations, imperfections
  - a Venn diagram (an Euler diagram)
- A classification of CT signals, a classification of DT signals
  - Def. of a periodic signal
  - Def. of a period and the fundamental period
  - Definition, Existence, and Uniqueness

$x: \boxed{\mathbb{R}} \to \mathbb{R}$

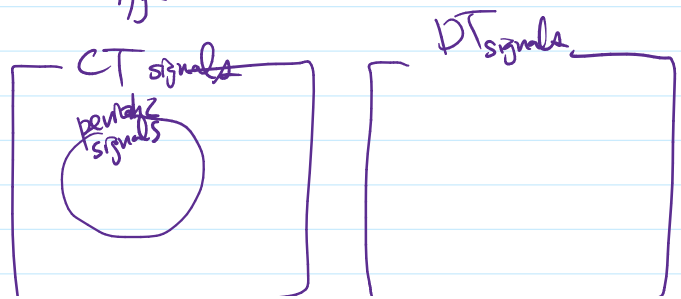
$x: \boxed{\mathbb{R}} \to \mathbb{C}$

Def.

$x(t)$ is periodic $\boxed{\text{if}}$ $\exists T>0$ s.t. $x(t)=x(t+T), \forall t (\in \mathbb{R})$.

A CT signal

iff



7. Preview
  - Def. of a symmetric signal
    - even symmetry
    - odd symmetry
    - intersection

- even symmetry
- odd symmetry
- intersection

○ Def.
- conjugate symmetry
- conjugate anti-(skew-)symmetry

○ Lemma
- even-odd symmetric function decomposition
- conjugate anti-conjugate symmetric function decomposition
- cf) symmetric skew-symmetric matrix decomposition
- cf) Hermitian skew-Hermitian matrix decomposition

○ Def.
- an indicator function
- right-continuity
- the CT unit-step function
- the DT unit-step function
- a right-sided signal, a left-sided signal
- a causal signal, an anti-causal signal

○ Def.
- a bounded signal
- an unbounded siganl

○ 이분법 dichotomy

○ the Euler's identity
- Feynman: the most beautiful equation

Def.
A DT signal $x[n]$ is periodic if $\exists N>0$ s.t. $x[n]=x[n+N], \forall n \in \mathbb{Z}$

$x: \mathbb{Z} \to \mathbb{R}$

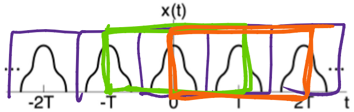
$x: \mathbb{Z} \to \mathbb{C}$

Signals with symmetry

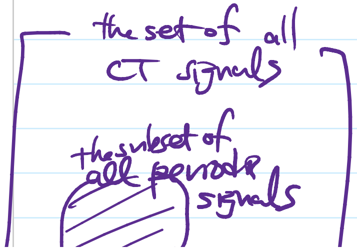
- Periodic signals

CT $x(t) = x(t+T)$ for all t.

definition, existence, uniqueness; a, an, the
정의, 존재성, 유일성



- **Ex.** 60-Hz power line, computer clock, etc.

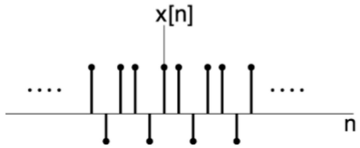


/ex/

$$x(t) = \begin{cases} 0, & \text{for } x \in \mathbb{Q} \\ 1, & \text{for } x \in \end{cases}$$

- **Ex.** 60-Hz power line, computer clock, etc.

DT $x[n] = x[n+N]$ for all n.



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Def. A CT signal x(t) is periodic, if there exists T>0 s. t. x(t) = x(t+T), for all t.

a real positive # T.

Lemma. If periodic, then there exists T>0 s.t x(t) = x(t+nT), for all t and all integer n.

$t \in [0, T)$

Def. For a periodic signal, the smallest period is called the fundamental period, if exists.

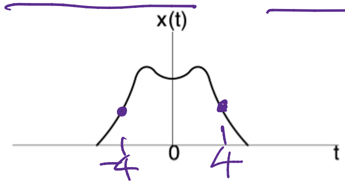
Lemma. For a periodic signal with fundamental period T_0, any period is an integer multiple of T_0.

Signals with symmetry (continued)

- Even and odd signals

Even $x(t) = x(-t), \forall t$ or $x[n] = x[-n], \forall n$

짝우,
우함수



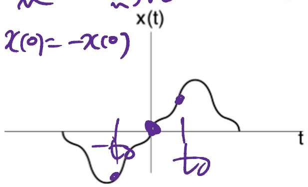
기함수

Odd $x(t) = -x(-t), \forall t$ or $x[n] = -x[-n], \forall n$

$x(0) = -x(0)$

$x(0) = 0$

or $x[0] = 0$



The zero function is even and odd at the same time.

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Signals with symmetry (continued)

Lemma. Any signals can be expressed as a sum of *Even* and

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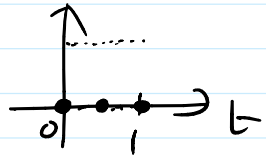
the set of all periodic signals

the set of all DT signals

dichotomy

1, for $x \in \mathbb{Q}^c$



(기준화하자)

Lemma.

$x(t) = x_e(t) + x_o(t), \forall t$

Proof.

$x_e(t) \triangleq \frac{x(t)+x(-t)}{2}$

$x_o(t) \triangleq \frac{x(t)-x(-t)}{2}$

cf) $A \in \mathbb{R}^{n \times n}$

$A = \frac{A+A^T}{2} + \frac{A-A^T}{2}$

CT signals

even

$x(t)=0, \forall t$

odd

cf) $B \in \mathbb{C}^{n \times n}$

$B = \frac{B+B^H}{2} + \frac{B-B^H}{2}$

((G)) Lemma. $x : \mathbb{R} \to \mathbb{C}$

$x(t) = \underbrace{\frac{x(t)+x^*(-t)}{2}}_{\triangleq y(t)} + \underbrace{\frac{x(t)-x^*(-t)}{2}}_{\triangleq z(t)}$

$$\underbrace{\qquad\qquad}_{\triangleq y(t)} \qquad \underbrace{\qquad\qquad}_{\triangleq z(t)}$$

$y^*(t) = y(-t)$

$y(-t) = y^*(t)$.

$z^*(t) = -z(-t)$

Signals with symmetry (continued)

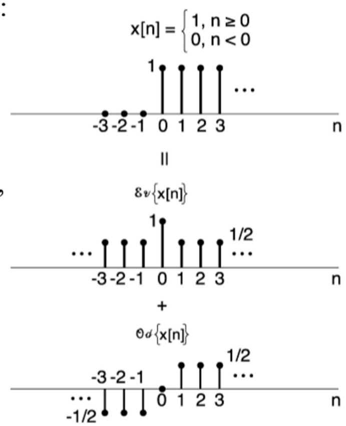
Lemma. Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

$$x(t) = x_{even}(t) + x_{odd}(t) \text{ ,}$$

where

$$x_{even}(t) = [x(t) + x(-t)]/2 \text{ ,}$$
$$x_{odd}(t) = [x(t) - x(-t)]/2 \text{ .}$$

Ex. DT unit-step function.



u[n]은 the unit step function이라 불리며
argument n이 음수이면 u[n]=0,
나머지 모든 경우는 u[n]=1이다.

$$x(t) = \underbrace{\frac{x(t)+x^*(-t)}{2}}_{\text{conjugate symmetric}} + \underbrace{\frac{x(t)-x^*(-t)}{2}}_{\text{conjugate anti-symmetric}}$$

when A is square.

$$A = \frac{A+A^T}{2} + \frac{A-A^T}{2}$$

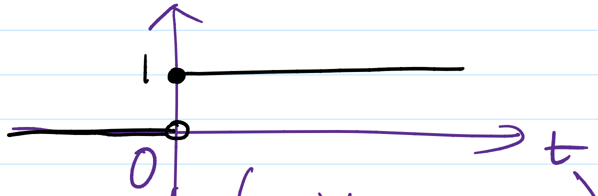
$$A = \frac{A+A^H}{2} + \frac{A-A^H}{2}$$

Def. $u : \mathbb{Z} \to \{0, 1\}$

$$u[n] \triangleq \begin{cases} 1, & n \geq 0 \quad \text{non-negative} \\ 0, & \text{elsewhere } (n<0) \end{cases}$$

Def. $u : \mathbb{R} \to \{0, 1\}$

$$u(t) \triangleq \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



the (right-continuous) unit-step function

Right- and Left-Sided Signals

Def.

A right-sided signal is zero for $t < T$ and a left-sided signal is zero for $t > T$, where T can be positive or negative.

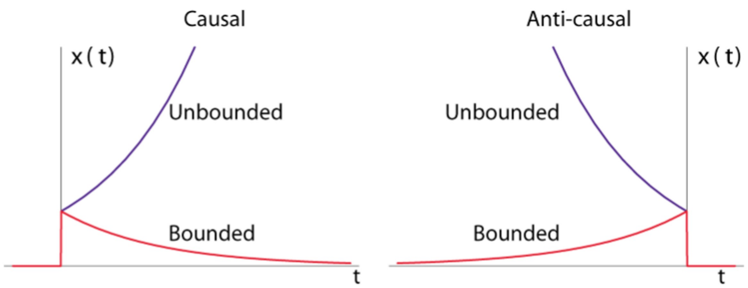
Def. Def.

$\exists T \in \mathbb{R}$

s.t. $x(t) = x(t)u(t-T), \forall t$.

[commonality, difference] → 압축 compress

Bounded and Unbounded Signals



Whether the output signal of a *system* is bounded or unbounded determines the stability of the system.
Eg. **Demo**, the inverted pendulum, unstable but can be stabilized using a feedback control system.

Def. $x(t)$ is bounded if $\exists M \in \mathbb{R}_+$ s.t. $|x(t)| \le M, \forall t$,

Def. $x(t)$ is causal if $x(t)=0, \forall t<0$. $\lfloor x(t) = O(1))$

Def. $x(t)$ is bounded

if $\cancel{\exists M \in \mathbb{R}_+}$

$\cancel{\mathbb{R}_{++}}$

$0 < M < \infty$

s.t. $|x(t)| \le M, \forall t$.

Real and Complex Signals

A very important class of signals is:

- CT signals of the form $x(t) = e^{st}$
- DT signals of the form $x[n] = z^n$

Q Find x such that $\sin x = 2$.

where z and s are complex numbers. For both *exponential* CT and DT signals, x is a complex quantity and has:

- a real and imaginary part, or equivalently
- a magnitude and a phase angle.

We will use whichever form that is convenient.

$j \triangleq \sqrt{-1}$

$i \triangleq \sqrt{-1}$

$z = a + jb = re^{j\theta}$

$a \in \mathbb{R}$ ← real part

$b \in \mathbb{R}$ ← imaginary part

We will use whichever form that is convenient.

$b \in \mathbb{R}$ ← imaginary part

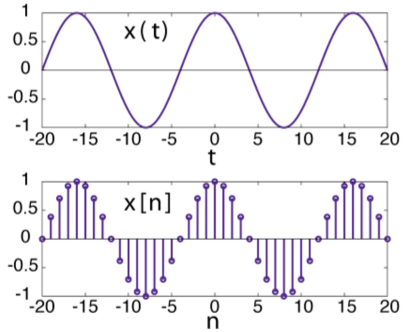
$j \triangleq \sqrt{-1}$

$a = \frac{z + z^*}{2}$ $\quad jb = \frac{z - z^*}{2}$

For example, suppose $s = j\pi/8$ and $z = e^{j\pi/8}$, then the real parts are

$$\mathcal{R}\{x(t) = e^{st}\} = \mathcal{R}\{e^{j\pi t/8}\} = cos(\pi t/8),$$

$$\mathcal{R}\{x[n] = z^n\} = \mathcal{R}\{e^{j\pi n/8}\} = cos[\pi n/8].$$



Singal → Signal
Chanel → Channel

Causal
casual

Summary

- We are awash in a sea of signal — sounds, visual, electrical, thermal, mechanical, etc.
- Signals can be time-varying or spatially varying, can be a function of multiple variables. However, in EECE233, we will mostly deal with 1D signals and use a generic time t to represent the variable,whether

be a function of multiple variables. However, in EECE233, we will mostly deal with 1D signals and use a generic time *t* to represent the variable, whether it is time, space, or something else.

- DT signals and systems become more and more important for signal processing, and will be a major part in EECE233.

Next lecture covers:

O & W pp. 39-56

$e^{j\theta} = \cos\theta + j\sin\theta$

$z = a + jb = re^{j\theta}$, $a, b, r, \theta \in \mathbb{R}$, $r \geq 0$

$|z| = r$ modulus

$z^* = a - jb = re^{-j\theta}$

$z^{-1} = \frac{1}{r}e^{-j\theta}$

$\bar{z}^* = (z^*)^{-1} = \frac{1}{r}e^{j\theta}$

정보통신 시대를 연 천재적인 공학자 클로드 섀넌의 "문제해결의 6단계"

단순화에서 시작하여 다시 복잡화를 통해 본래에 문제를 해결할 뿐만 아니라 새로운 문제를 찾아내고 해결함.

Step 1: Simplify. Simplify. Simplify.

Step 2: Fill your 'mental matrix' with solutions to

Step 1: Simplify. Simplify. Simplify.
Step 2: Fill your 'mental matrix' with solutions to similar problems.
Step 3: Approach the problem from many different angles.
Step 4: Break a big problem down into small pieces.
Step 5: Solve the problem 'backwards.'
Step 6: If you've solved the problem, extend that solution out as far as it will go.