2023년 9월 14일 목요일 16:36

CM05 230919T

Announcements

1. 지난 강의 수정 사항
   N/A

2. HW
   - HW03 will be assigned soon. Due will be next Tuesday. For details, check the PLMS class homepage.

3. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

4. 출석
   - 출결상태에 대해 전자출결시스템( https://wpsc.postech.ac.kr/eams/login )를 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

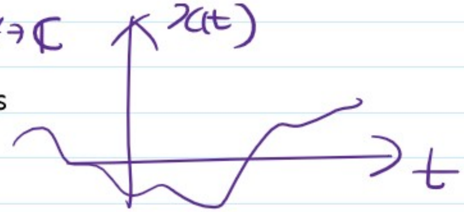
5. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform LCCDE
   10. Z-transform
   7. Sampling
   8. Communication Systems

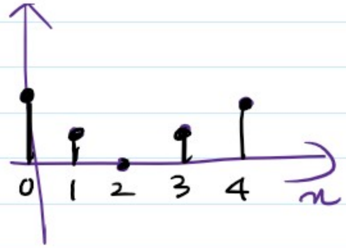
non-periodic

6. Review aperiodic $x(t)$ $x: \mathbb{R} \to \mathbb{C}$ $x: \mathbb{Z} \to \mathbb{C}$ $x(t)$

A classification of CT signals, a classification of DT signals

6. Review aperiodic $x(t)$ $x: \mathbb{R} \to \mathbb{C}$ $x: \mathbb{Z} \to \mathbb{C}$




stem

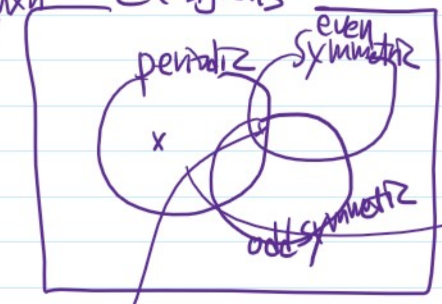
- A classification of CT signals, a classification of DT signals
  - Def. of a periodic signal
  - Def. of a period and the fundamental period
  - Definition, Existence, and Uniqueness
  - Def. of a symmetric signal ↔ asymmetric
    - even symmetry
    - odd symmetry
    - intersection
  - Def.
    - conjugate symmetry
    - conjugate anti-(skew-)symmetry
  - Lemma
    - even-odd symmetric function decomposition
    - conjugate anti-conjugate symmetric function decomposition
    - cf) symmetric skew-symmetric matrix decomposition
    - cf) Hermitian skew-Hermitian matrix decomposition
  - Def.
    - right-continuity
    - the CT unit-step function
    - the DT unit-step function
    - a right-sided signal, a left-sided signal
    - a causal signal, an anti-causal signal
  - Def.
    - a bounded signal
    - an unbounded siganl
  - 이분법 dichotomy
  - cf) an indicator function
  -
- Complex-valued CT and DT signals
  - a complex number
  - j, the square-root of negative 1
  - real part and imaginary part
  - the Euler's identity
    - Feynman: the most beautiful equation

$x(-t) = x^*(t), \forall t$

$A \in \mathbb{R}^{n \times n}$

$\begin{cases} A^H = A \\ A^H = -A \end{cases}$ $\begin{cases} A = A^T \\ A^T = -A \end{cases}$

CT signals




$x(t) = 0, \forall t$

dichotomy

$T > 0$

$x(t) = x(t+T), \forall t$

$= x(t+nT), \forall t \in [0, T)$

$n \in \mathbb{Z}$

$T_0 \leftarrow$ fundamental period

$1_A(x) = \begin{cases} 1, & x \in A \\ 0, & x \notin A \end{cases}$

$1_{[0,\infty)}(t)$

$u: \mathbb{R} \to \{0, 1\}$

$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$

$u: \mathbb{Z} \to \{0, 1\}$

$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

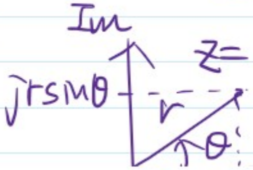
$x(t) = x(t)\, u(t), \forall t$

$|x(t)| < M, \forall t.$

Def. $x(t)$ is right-sided if $\exists T \in \mathbb{R}$ s.t. $x(t) = x(t)\, u(t-T), \forall t$.


$z = a + jb$, $\mathrm{Re}\{z\} \triangleq a = \dfrac{z + z^*}{2}$

$z^* \triangleq a - jb$, $\mathrm{Im}\{z\} \triangleq b = \dfrac{z - z^*}{2j}$




$z = r\cos\theta + jr\sin\theta$

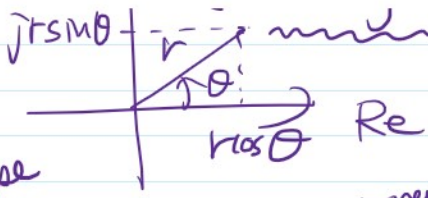
$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$

- real part and imaginary part
- the Euler's identity
  - Feynman: the most beautiful equation

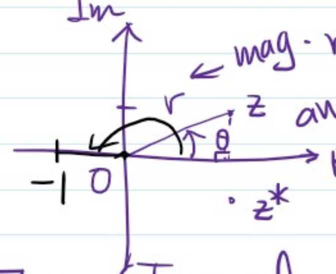
$z = re^{j\theta} = r(\cos\theta + j\sin\theta)$

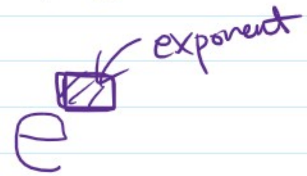
$r=1 \quad \theta=\pi \qquad \boxed{e^{j\pi}+1=0}$

$-1 = e^{j\pi}$

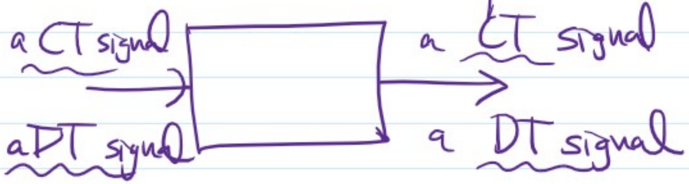



7. Preview
   - magnitude and phase
   - modulus
   - the Euler's identity $e^{j\theta} = \cos\theta + j\sin\theta$
     - Feynman: the most beautiful equation

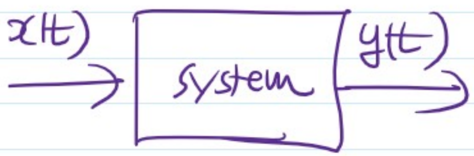



- A classification of systems
  - a CT system
  - a DT system
  - a CT/DT converter
  - a DT/CT converter
  - etc.

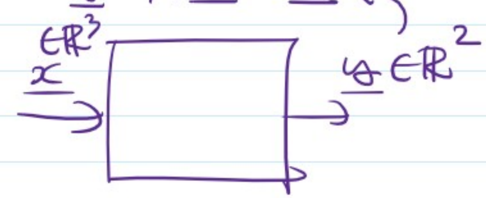



- Description of the mapping rule of a system = $\boxed{\text{Input/Output (I/O) relation}}$
- How to roughly visualize an I/O relation?
  - a block diagam vs. a graph

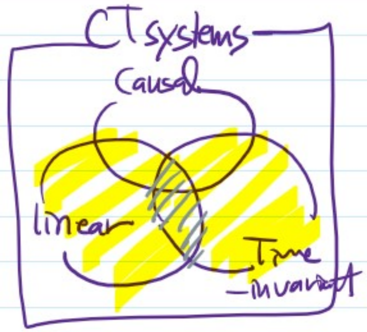


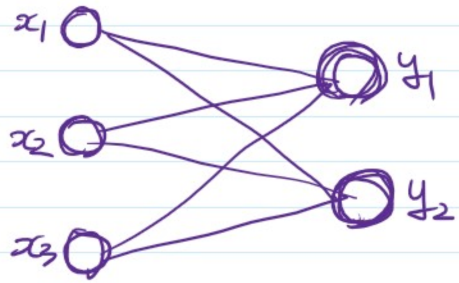

I/O

$y = Ax + b$




- How to precisely describe an I/O relation?
  - examples
    - differential equation for CT system 미분방정식
    - difference equation for DT system 차분방정식
    - etc.
- A classification of CT systems
  - Def. A CT system is causal if ...
    - differential/difference equations with initial conditions, causality...
  - Def. A CT system is memoryless if ...
    - Lemma. A CT memoryless system is causal. The converse is not necessarilly true.
  - Def. A CT system is linear if ...
    - Def. A CT system satisfies the ZIZO condition if ...
    - Lemma. If a CT system is linear, then it satisfies the ZIZO condition. The converse is not necessarily true. Its contrapositive is very useful.
    - Proposition. more than 2 signals
    - Def. A CT system satisfies the initial rest condition if ...
    - Theorem. Given a CT linear system, it is causal iff it satisfies the initial rest condition.
  - Def. A CT system is time-invariant if ...
    - Lemma. If a CT system is memoryless, then it is TI.



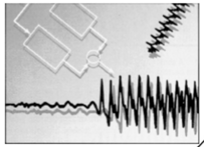
- Theorem. If the input to a CT TI system is periodic, then its output is also periodic with the same period.

# EECE233 Signals and Systems

Lecture Note #2

앞서 lecture note 1에서는 본 과목에서 다루는 signal에 대해 여러가지 방법으로 분류해 보았고
여기 lecture note 2 에서는 본 과목에서 다루는 system에 대해 여러가지로 분류해 본다

- Covers O & W pp. 39-56
- Several examples of systems
- System properties and examples
    a) Linearity
    b) Time Invariance
    c) Causality



* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

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## SYSTEM EXAMPLES

In EECE233, *systems* are described from input/output perspective,that is, input to the system $x$ causes the output $y$

$x(t) \rightarrow$ CT System $\rightarrow y(t)$ $x[n] \rightarrow$ DT System $\rightarrow y[n]$

① $x(t) \rightarrow$ CT system $\rightarrow y(t)$ ③ $x(t) \rightarrow$ CT/DT $\rightarrow x[n]$

② $x[n] \rightarrow$ DT system $\rightarrow y[n]$ ④ $x[n] \rightarrow$ DT/CT $\rightarrow x(t)$

– A classification of systems in this course:

- A classification of systems in this course:
  - ○ a CT system
  - ○ a DT system
  - ○ a CT/DT converter
  - ○ a DT/CT converter
  - ○ etc.

- Description of the mapping rule of a system = Input/Output (I/O) relation
  - How to roughly visualize an I/O relation?
    - ○ a block diagam vs. a graph
  - How to precisely describe an I/O relation?
    - ○ examples
      - § differential equation for CT system
      - § difference equation for DT system

**Ex. #1 RLC circuit — an electrical system**

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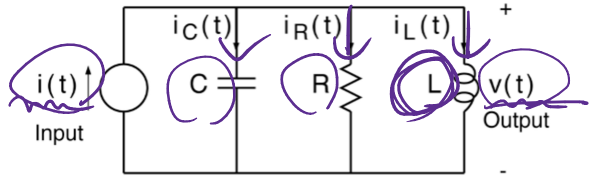
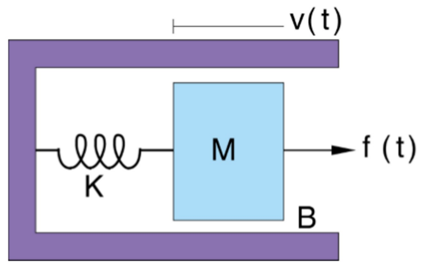



$$i(t) = \underbrace{C\frac{dv(t)}{dt}}_{\text{capacitance}} + \underbrace{\frac{v(t)}{R}}_{\text{resistance}} + \underbrace{\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau}_{\text{inductance}}.$$



**Ex. #2 A shock absorber – a mechanical system**



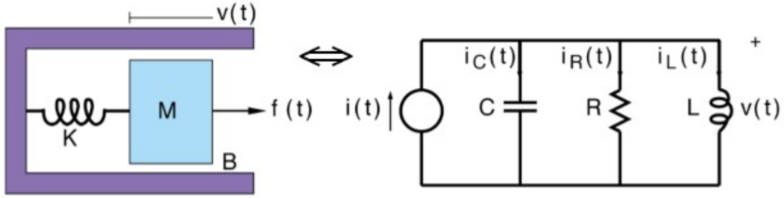

Force Balance: $f$ — input, $v$ — output

$$f(t) = \underbrace{M\frac{dv(t)}{dt}}_{inertial\ force} + \underbrace{Bv(t)}_{friction} + \underbrace{K\int_{-\infty}^{t} v(\tau)d\tau}_{spring\ force}.$$

This equation looks quite familiar, we just saw it earlier.

## Observation: different systems could be described by the same input/output relations — they are the same as far as EECE233 is concerned

((S)) ≡ ((G))



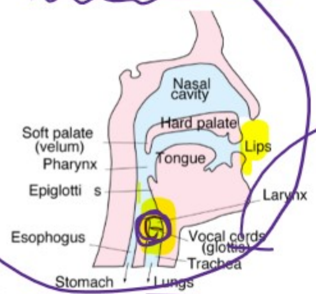

Largely for this reason, EECE233 is the most general among all the EECS subjects, and perhaps the most important.



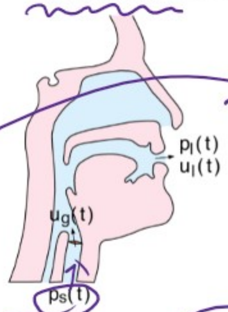
## Example #3 — Human vocal system

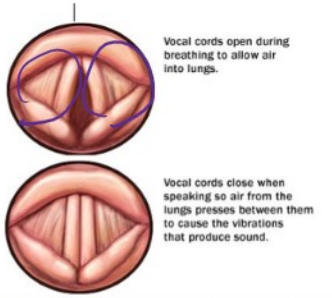
vocal cord: 성대
vocal tract: 구강, 혀, 비강, (치아, 입술) 를 묶어

Med. school description

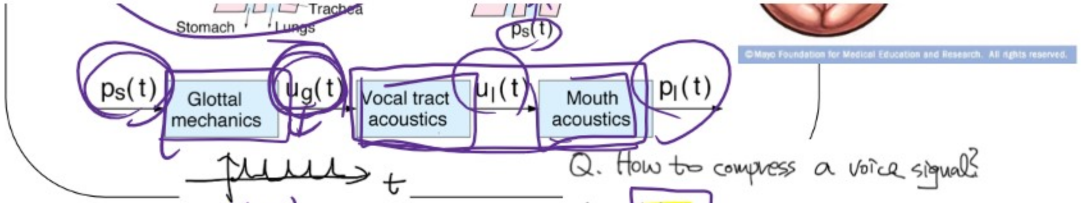



EECE233 version






$U_g(t)$



vocal tract가 이 진동적인 공기의 흐름을 여러 가지로 바꾼다.

Q. How to compress a voice signal?

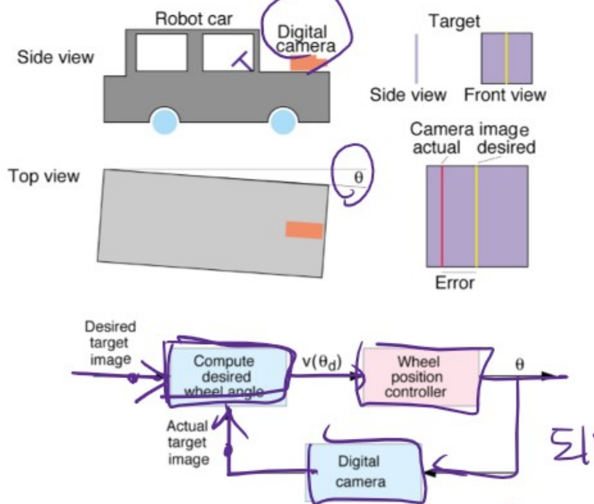
A. LPC linear predictive coding

CELP code-excited linear prediction

Q-CELP cdmaOne IS-95

VOCODER

## Example #4: A robot car – a close-loop system




되먹임

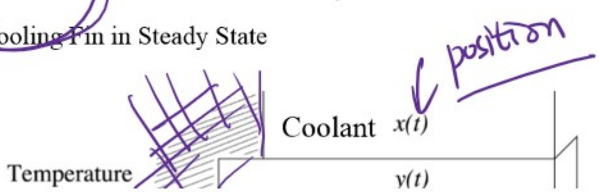
자동제어

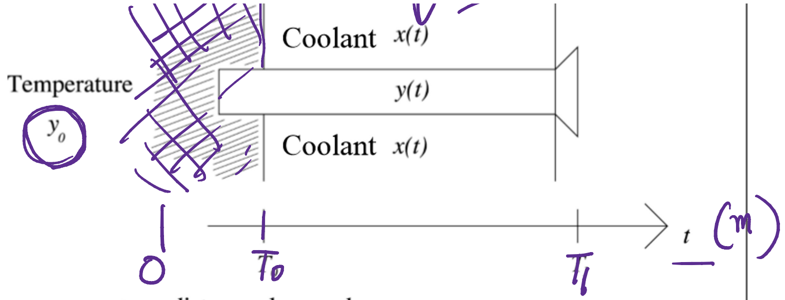


이런 system을 feedback system이라 부름.

## Ex. #5 A thermal system

Cooling Fin in Steady State






$t$ = distance along rod

$y(t)$ = Fin temperature as function of position

$x(t)$ = Surrounding temperature along the fin



이 과목에서는 parameter가 시간인 예가 많지만, 이 과목에서 배운 지식은 반드시 시간이 아니더라도 같은 방식으로 모델링 되는 신호와 시스템을 다루는데 이용/확장 가능하다.

## Ex. #5 (Continued)

$$\begin{aligned}\frac{d^2y(t)}{dt^2} &= k[y(t) - x(t)] \\ y(T_0) &= y_0 \\ \frac{dy}{dt}(T_1) &= 0\end{aligned}$$

Observations

- Independent variable can be something other than time, such as space.
- Such systems may, more naturally, have boundary conditions, rather than "initial" conditions.



CT system

Linear coefficient

## Ex. #6 A DT system

Linear constant coefficient difference eq.

LCCDE

- A rudimentary "edge" detector

input

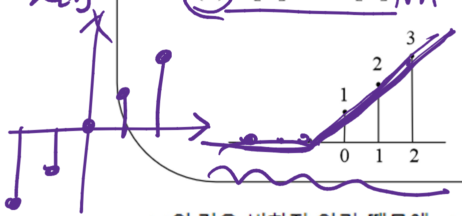
output

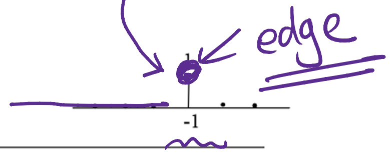
$$y[n] = x[n+1] - 2x[n] + x[n-1]$$
$$= \{x[n+1] - x[n]\} - \{x[n] - x[n-1]\}$$
$$= \text{"Second difference"} \propto \text{"curvature"}$$

- This system detects changes in signal slope.

(a) $x[n] = n$, $\forall n$ $\Rightarrow$ $y[n] = 0$ for all $n$. no idea

(b) $x[n] = nu[n]$, $\forall n$ $\Rightarrow$ $y[n] = ?$

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(a)의 경우 변하지 않기 때문에 edge가 없는 것,이 아니라 변하는데 uniform한 rate로 변하기 때문에 edge가 없는 경우이다.

edge detector는 군사위성이 찍은 사진으로 부터 철도, 도로, 하천 등을 식별하는 데 쓰인다. 또한, 배경화면에서 물체를 분리해내는 일, 그림file을 압축하는 일 등에 쓰인다.

## Observations

1) A very rich class of systems (but by no means all systems of interest to us) are described by differential and difference equations.

2) Such an equation, by itself, does not completely describe the input-output behavior of a system: we need auxiliary conditions (initial conditions, boundary conditions).

3) In some cases the system of interest has time as the natural independent variable and is causal. However, that is not always the case.

4) Very different physical systems may have very similar mathematical descriptions.

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Casual
Causal

Channel
Chanel

Signal
Singal

## SYSTEM PROPERTIES
## (Causality, Linearity, Time-invariance, etc.)

Why bother with such general properties?

- Important practical/physical implications. We can make many important predictions of the system behaviors without having to do any mathematical derivations.

- They allow us to develop powerful tools (*transformations*, more on this later) for analysis and design.

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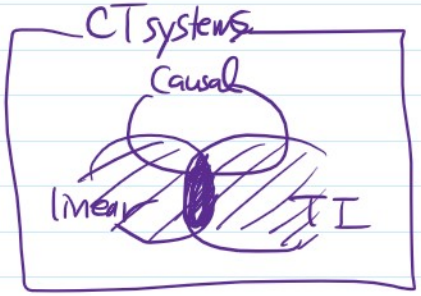
Venn diagram

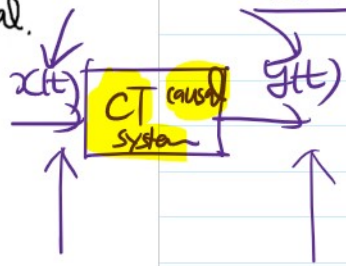



Singal → Signal
Chanel → Channel

Causal
casual

cause & effect
원인 결과
causal.
casual.

END of TIME




## CAUSALITY

- A system is causal if the output does **not** anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time.

- **All** real-time physical systems are **causal**, because time only moves forward. Effect occurs after cause. (Imagine if

- **All** real-time physical systems are **causal**, because time only moves forward. Effect occurs after cause. (Imagine if you own a noncausal system whose output depends on tomorrow's stock price.)
- Causality does **not** apply to spatially varying signals. (We can move both left and right, up and down.)
- Causality does **not** apply to systems processing *recorded* signals, e.g. taped sports games vs. live broadcast.



## CAUSALITY (continued)

표기법 notation

**Def.** Mathematically (in CT): A system $x(t) \rightarrow y(t)$ is causal

if $\boxed{x_1(t)} \rightarrow y_1(t)$, $\boxed{x_2(t)} \rightarrow y_2(t)$

and $x_1(t) = x_2(t)$ for all $t \le t_o$,

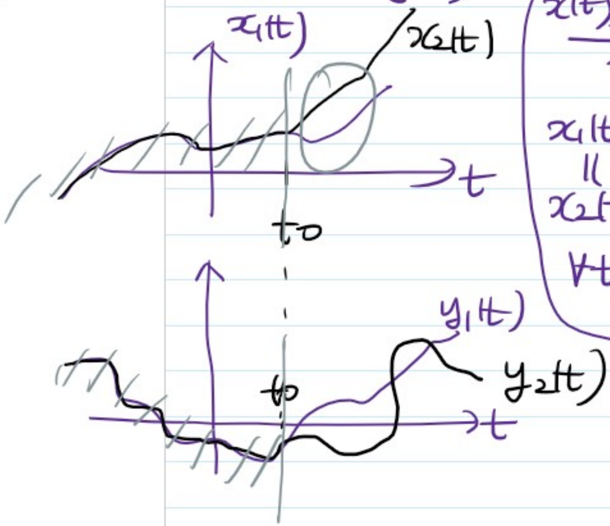
then $y_1(t) = y_2(t)$ for all $t \le t_o$.

That is, for any t_o, the values of $x_1$ and $x_2$ after $t_o$ have **no** influence on $y$ at $t \le t_o$ for a causal system.

- More explicit mathematical definition of causality later.

input ↓ output ↑

$x(t) \rightarrow y(t)$

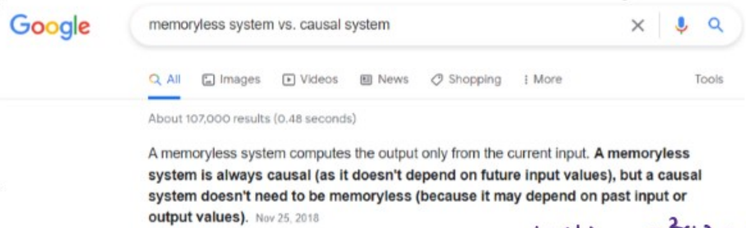



$x(t) \rightarrow \boxed{\text{CT causal}} \rightarrow y(t)$

$x_1(t) = x_2(t) \quad \forall t \le t_0$

$y_1(t) = y_2(t) \quad \forall t \le t_0$

Visualize
Simplify


Google
memoryless system vs. causal system
All Images Videos News Shopping More Tools
About 107,000 results (0.48 seconds)
A memoryless system computes the output only from the current input. **A memoryless system is always causal (as it doesn't depend on future input values), but a causal system doesn't need to be memoryless (because it may depend on past input or output values).** Nov 25, 2018

$y(t) = x^2(t) + 2x(t) + 1$

A memoryless system computes the output only from the current input. **A memoryless system is always causal (as it doesn't depend on future input values), but a causal system doesn't need to be memoryless (because it may depend on past input or output values).** Nov 25, 2018

https://dsp.stackexchange.com › questions › difference-be...

Difference between causality and memorylessness

$x(t) \rightarrow y(t)$

I/O relations

$y(t) = x^2(t) + 2x(t) + 1$

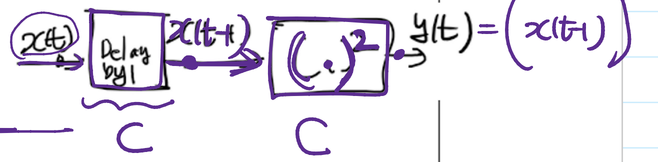
$y = x^2 + 2x + 1$

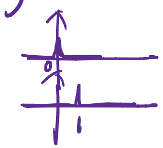
Memoryless

## CAUSAL OR NONCAUSAL

C (a) $y(t) = x^2(t-1)$

$(x(t-1))^2$

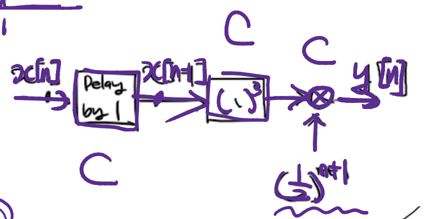


NC (b) $y(t) = x(t+1)$

NC

$x[n] \rightarrow y[n]$ (c) $y[n] = x[-n]$



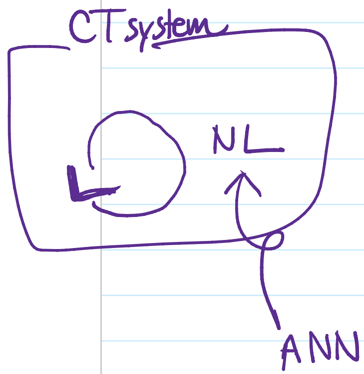

C (d) $y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$

$y[n] = \left(\frac{1}{2}\right)^{n+1} (x[n-1])^3$



## LINEAR AND NONLINEAR SYSTEMS

- Many systems are nonlinear.
  **Ex.** Economic system.
  Input: fiscal and monetary policies, labor, resources, etc.
  → Output: GDP, inflation, etc.
  System behavior is very unpredictable because it is highly nonlinear.
- In EECE233, we will deal with only linear systems, which are good approximations of nonlinear systems in certain ranges.
  **Ex.** Small-signal conductance of a nonlinear diode.
- Linear systems can be analyzed accurately.



**Ex.** Small-signal conductance of a nonlinear diode.

- Linear systems can be analyzed accurately.

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## LINEARITY

Def. A (CT) system is linear if it has the superposition property, (linear) i.e.,

If $x_1(t) \to \boxed{y_1(t)}$ and $x_2(t) \to \boxed{y_2(t)}$,

then $\boxed{ax_1(t) + bx_2(t)} \to \boxed{ay_1(t) + by_2(t)}$.

Lemma. If a system is linear, then zero input $\to$ zero output. (ZIZO)

CT, DT

Question: Is the converse true? Not

"Proof" $x[n]=0 \cdot x_1[n]=0 \;\to\; y[n]= 0 \cdot y_1[n]=0.$

Counter example

$y(t) = (x(t))^2$

$x(t)=0, \forall t$

$x[n]=0, \forall n$

$y(t)=0, \forall t$

$y[n]=0, \forall n$

Question: Is the system $y = A + x$ linear? $A \neq 0$

Recall the contraposition of L => ZIZO

ZIZO $\not\Rightarrow$ L

not ZIZO $\Rightarrow$ nonlinear

$y = ax + b$

linear, $b=0$

affine, $b \neq 0$

$y = Ax$

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