2023년 9월 21일 목요일 13:51

CM07 230921Th

Announcements

1. class meeting #6에서 review를 너무 오래하여 진도를 못 나가는 바람에 연구실에 오자마자 나머지 부분을 녹화를 했는데, 녹화하고 보니 1시간 12분짜리가 되었네요. 따라서 이 녹화 부분을 class meeting #7로 하겠습니다.
2. 그러면 다음주 화요일은 class meeting #8이 됩니다. 학기말에 출석 정산(?)시에 이번 녹화 때문에 1회 더 강의한 것으로 계산하고 모두 출석한 것으로 처리하겠습니다.

□ Causal ← signal, system

Def. A CT signal $x(t)$ is causal if $x(t)=x(t)u(t)$, $\forall t$

Def. A CT system is causal if $x_1(t)\to y_1(t)$ & $x_2(t)\to y_2(t)$ & $x_1(t)=x_2(t)$, $\forall t\le t_0$ then $y_1(t)=y_2(t)$, $\forall t\le t_0$.

□ Def. A DT system is linear if $x_1[n]\to y_1[n]$ & $x_2[n]\to y_2[n]$ then $a x_1[n]+b x_2[n]\to a y_1[n]+b y_2[n]$.

a_1 a_2 a_1 a_2

Proposition. A DT system is linear iff $x_k[n]\to y_k[n]$ for $k\in\mathbb{K}$ $\Rightarrow \sum_{k\in\mathbb{K}} a_k x_k[n] \to \sum_{k\in\mathbb{K}} a_k y_k[n]$

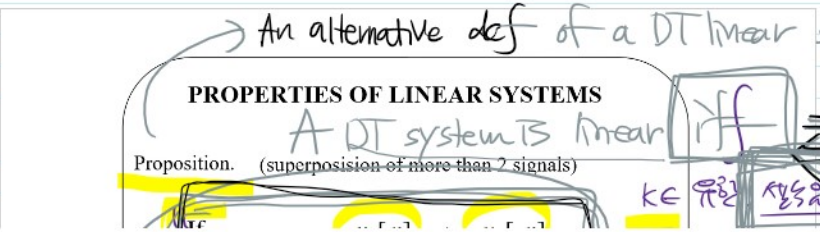
$\mathbb{K}$: a countable set. finite / infinite

$\sum_{k=1}^{2}$ $\mathbb{K}=\{1,2\}$

$\mathbb{K}=\mathbb{N}$ $\sum_{k=1}^{\infty}$

$k\in\mathbb{K}$: the index set of Σ

An alternative def of a DT linear system.



PROPERTIES OF LINEAR SYSTEMS

Proposition. (superposision of more than 2 signals)

If

A DT system is linear iff $\Rightarrow$ $y_k[n]=\mathcal{L}(x_k[n])$, $\forall k$.

$(y_k[n])_{n=-\infty}^{\infty}$

$k\in$ 유한 셀수있고 무한

cf) 함수의 연속 $x=x_0$; $f(x_0)=\lim f(x)$

연속하다

TI DT system is linear

Proposition. (superposision of more than 2 signals)

If $x_k[n] \rightarrow y_k[n]$

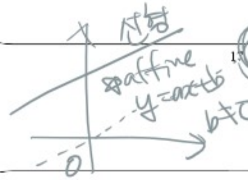
then $\Rightarrow \sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$.

k∈ 유한 셀수있고 무한 ℤ

의미

$a_1 = a$
$a_2 = b$
$a_k = 0$
$k \neq 1,2$ equivalent

This property seems to be almost trivial now, but it is one of the most important ones used in EECE233.



Q How to handle nonlinearity?
A affine ≃ linear. (O) y=ax+b
- linearize. "선형화"
- Volterra series Taylor series.
- Busgang's theorem, etc

$(y_k[n])_{n=-\infty}^{\infty}$

$(y_k[n])_{n\in\mathbb{Z}}$

교환

$$\mathcal{L}\left(\sum_k a_k x_k[n]\right) = \sum_k \mathcal{L}\left(a_k x_k[n]\right) = \sum_k a_k \mathcal{L}\left(x_k[n]\right) = \sum_k a_k y_k[n]$$

4) 극한의 연속 ~~[illegible]~~

$$f\left(\lim_{x\to x_0} x\right) = \lim_{x\to x_0} f(x)$$

$f(x_0) = \lim_{x\to x_0} f(x)$

$\boxed{\lim_{x\to x_0} x = x_0}$

☐ Commonality ⇕ difference   Similarity ⇕ dissimality

Properties of Linear Systems (Continued)

Def. A CT system has the initial rest property if ZI until t_0 implies ZO until t_0, i.e.,

$$x(t) = 0 \ \ for\ t \le t_o \ \rightarrow \ y(t) = 0 \ \ for\ t \le t_o \qquad (*).$$

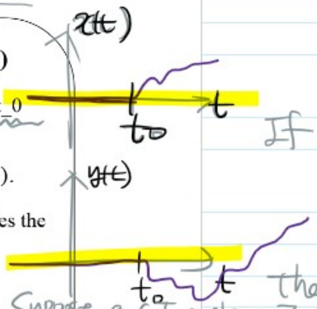
⇐ ⇒ then

Theorem. A linear system is causal if and only if it satisfies the conditions of initial rest: (both necessary and sufficient)

"Proof"

a) Suppose a CT L system is causal. Show that (*) holds.

b) Suppose a CT L system has (*) hold. Show that the system is causal.



If $x_1[n] \rightarrow y_1[n]$ then $a_1x_1[n] + a_2x_2[n] \rightarrow a_1y_1[n] + a_2y_2[n]$,
$x_2[n] \rightarrow y_2[n]$
$x_3[n] \rightarrow y_3[n]$

then $\left(a_1x_1[n] + a_2x_2[n]\right) + a_3x_3[n] \rightarrow \left(a_1y_1[n] + a_2y_2[n]\right) + a_3y_3[n]$.

Suppose a CT system is linear.
If causal
$(x_1(t) = x_2(t), \forall t \le t_0 \Rightarrow y_1(t) = y_2(t), \forall t \le t_0.$
then $x(t) = x_1(t) - x_2(t) = 0, \forall t \le t_0 \Rightarrow y(t) = y_1(t) - y_2(t) \boxed{= 0}, \forall t \le t_0.$

Initial rest.

Causal
⇕ Linear
time-invariance

[Proof by contraposition. (대우증명법)
  " " contradiction (귀류법)

TIME-INVARIANCE (TI) 시불변

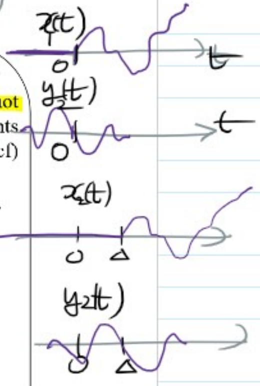
Informally, a system is time-invariant (TI) if its behavior does not depend on the choice of $t = 0$. Then two identical experiments will yield the same results, regardless the starting time. cf) stationary RP.

Def. Mathematically (in DT): A system $x[n] \rightarrow y[n]$ is TI if for *any* input $x[n]$ and *any* time shift n_0,

If $x[n] \rightarrow y[n]$,
then $x[n - n_0] \rightarrow y[n - n_0]$ for any integer n_0.

Def. Similarly for CT time-invariant system,

If $x(t) \rightarrow y(t)$,
then $x(t - t_0) \rightarrow y(t - t_0)$ for any real t_0.



For all pairs of
$x_1(t) \rightarrow y_1(t)$
$x_2(t) = x_1(t-\Delta) \rightarrow y_2(t) = y_1(t-\Delta)$.

Def. Similarly for CT time-invariant system,

If $x(t) \to y(t)$,
then $x(t - t_o) \to y(t - t_o)$ for any real t_0.

Stop & Think.

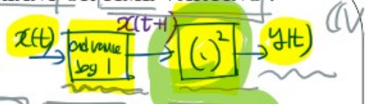
time < invariant
variant (= varying)

TIME-INVARIANT OR TIME-VARYING ?

CT (a) $y(t) = x^2(t+1)$

TI

$= \{x(t+1)\}^2$



((V))

[Justification
[Proof

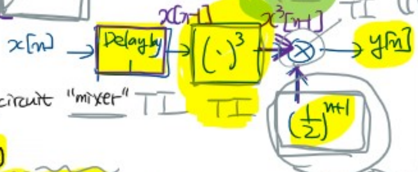
$x(t) \to y(t)$
$\Rightarrow x(t-t_0) \to y(t-t_0), \forall t_0$

TI TI

DT (b) $y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$

TV

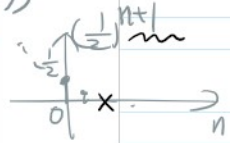
TI ((V))



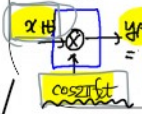
RF circuit "mixer" TI TI

TV.

$(\frac{1}{2})^{n+1}$



CT (c)



$= x(t)\cos 2\pi f_c t$

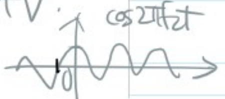
AM

carrier

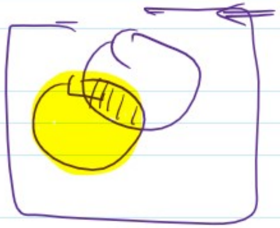
TV

$f_c \neq 0$

$\cos 2\pi f_c t$



Given linear,
causal iff initial rest



$x_2(t) = x_1(t-t_0)$

$y_2(t) = x_1(t-t_0)\cos 2\pi f_c(t-t_0) = y_1(t-t_0)$

$x_2(t) = x_1(t-t_0)$

$y_2(t) = x_1(t-t_0)\cos 2\pi f_c t \neq y_1(t-t_0))$.

$\forall t_0$

NOW WE CAN DEDUCE SOMETHING!

Theorem. If the input to a TI system is periodic, w/ period T then the output is also periodic with the same period.

[w/
[w/o

If $x_1(t) \to y_1(t)$
$\hat{=}\ \exists T > 0$, $x_1(t) = x_1(t-T), \forall t$
then $x_2(t) \hat{=} x_1(t-T) = x_1(t)$
$y_2(t) = y_1(t-T)$
$= y_1(t)$.
$y_1(t)$ is periodic w/ period T.

"Proof": Suppose $x(t + T) = x(t)$
and $x(t) \to y(t)$

Then by TI
$x(t + T) \to y(t + T)$

↑ But these are the same input!

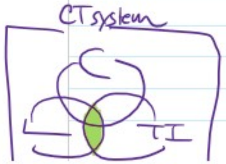
↑ So these must be the same output, *i.e.*, $y(t) = y(t+T)$

분류를 통해 작은 부분집합을 정의하면
그 부분집합의 원소에 대해 공통되는 성
질을 말할 수 있어
이론을 전개해 나갈 수 있게된다.
=> 분류의 힘!!!!

Lemma. If a CT/DT system is memoryless, then the system is time-invariant.

Proof. If $\exists\, g: \mathbb{C} \to \mathbb{C}$ s.t.
$y_1(t) = g(x_1(t)), \forall t \in \mathbb{R}$
then
$x_2(t) \hat{=} x_1(t-t_0), \forall t$
$y_2(t) = g(x_2(t))$
$= g(x_1(t-t_0))$
$= y_1(t-t_0), \forall t$.

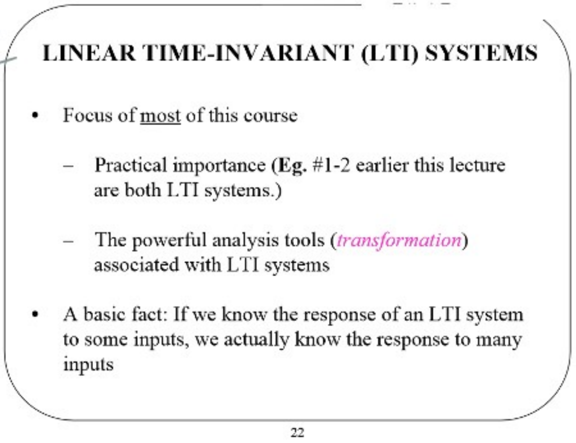
CT system



LTI

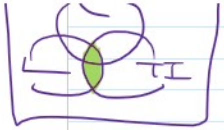
LINEAR TIME-INVARIANT (LTI) SYSTEMS

LTI



LINEAR TIME-INVARIANT (LTI) SYSTEMS

- Focus of most of this course
  - Practical importance (**Eg.** #1-2 earlier this lecture are both LTI systems.)
  - The powerful analysis tools (*transformation*) associated with LTI systems
- A basic fact: If we know the response of an LTI system to some inputs, we actually know the response to many inputs



$= g(x_1(t-t_0))$

$= y_1(t-t_0), \forall t.$

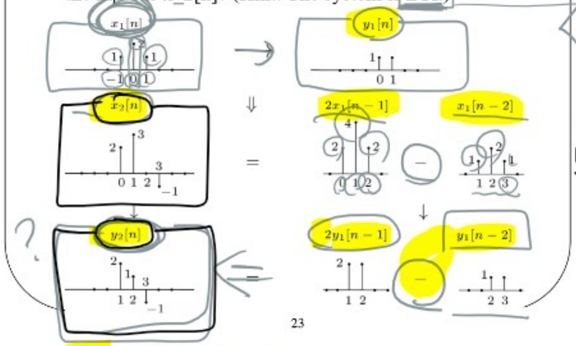
Stop & Think

Example: DT LTI System

Q: Given x_1[n] -> y_1[n] below, what is y_2[n] when the input is x_2[n]? (Hint. The system is LTI.)

linearity

TI



$$x_2[n] = 2x_1[n-1] - x_1[n-2]$$

$$y_2[n] = M(x_2[n])$$ substitution

$$= M(2x_1[n-1] - x_1[n-2])$$ by Lin

$$= M(2x_1[n-1]) - M(x_1[n-2])$$ by lin

$$= 2M(x_1[n-1]) - M(x_1[n-2])$$

$$= \boxed{2\ y_1[n-1]} - y_1[n-2]$$

This problem motivates

(i) Representation of an input signal, and

(ii) Representation of an output signal.

atom.

-> Representing the input signal in terms of elementary signals, finding the outputs to the signals, and constructing the output using the outputs of the elementary signals.

signals

cf) eigen-value and eigen-vector of a square matrix?

이상

1. causality
2. linearity
3. time-invariance

를 배웠는데

각각 이분법이므로 이제 8개의 부분집합을 배운것이다.

그런데 이과목에서는 이렇게 8개로 쪼개서 모두 배우는 것이 아니라

linear time-invariant system 이라는 부분집합에 대해서만 주로 배운다.