2023년 9월 26일 화요일 09:55

CM09 231005Th

Announcements

1. Messages from Instructor
    - Imperfect classification rule is better than no classification as long as you recognize the imperfection and update your rules.
    - Simplify and Visualize:
        - essence, generalize, consider extremes,
        - Venn diagram, necessary and sufficient,
        - block diagram, G(V,E); graph of a function, algebraic vs. geometric
    - Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

      Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

      출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>

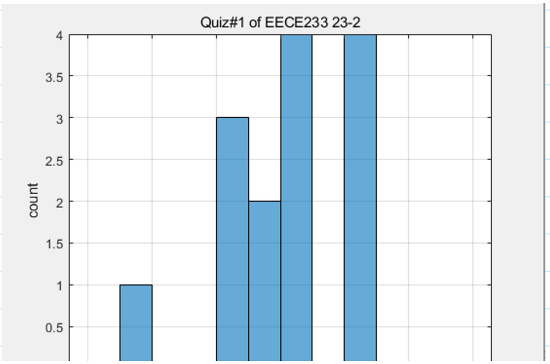
    - Stop and Think
    - Self-Fulfillment and Contribution to Collective

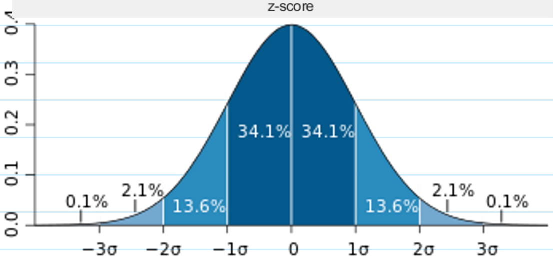
2. 지난 강의 수정/보완 사항

 - linearity x_1->y_1 and x_2->y_2 => ax_1+bx_2 => ay_1+by_2

3. HW, Quiz, and Exam
    - HW05 will be assigned. Due will be next Tuesday.

      Python이나 MATLAB과 같은 tool을 사용하지 않고 손으로 직접 계산하여 Plot한 학생분들이 있습니다. 금주 과제(HW4)까지는 이에 대해 감점을 하지 않습니다만 추후 과제부터는 지시한 사항에 맞춰 풀이해주세요.
    - Quiz 1 has mean 14.86 and standard deviation 2.9. Your z-score = (your score-mean)/standard deviation.



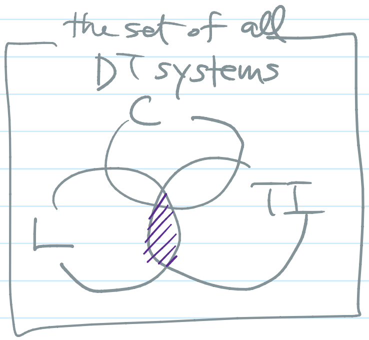


- Midterm Exam is approaching.
  - Tentatively, it will be taken on Tuesday 10/24 from 19:00-23:00 at LG 102.
  - Please let me know by next Tuesday who cannot be available at this time slot.

4. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform
   7. Sampling
   8. Communication Systems



7. Review
   - Def. the Kronecker Delta function, the unit sample function
     - ~~Lemma. A DT signal x[n] can be represented by ...~~
   - Def. Impulse response of a DT linear system — The unit sample response
     - Lemma. Impulse response of a DT LTI system
       - Examples

sum weighted delayed

$\delta[n]$ | DT | $h[n]$ $x[n]$ | DT LTI | $y[n] = \sum^{\infty} x[k]h[n-k]$

- Def. Impulse response of a DT linear system — The unit sample response
  - Lemma. Impulse response of a DT LTI system
    - Examples
      - identity system $\delta[n]$
      - delay system $\delta[n-n_0]$
      - sum $u[n]$
  - Theorem. I/O relation of a DT LTI system in terms of ...
    - Def. the convolution sum(mation)
      - cf) convolutional neural network (CNN)
    - $\boxed{y[n] = x[n]*h[n]}$
    - Implication
      - A DT LTI system is fully characterized by its unit sample response.
      - Ex. Echo microphone's impulse response??
    - Interpretation
      - delayed weighted sum of impulse response
      - ??

$\delta[n] \rightarrow$ DT LTI sys $\rightarrow h[n]$

$x[n] \rightarrow$ DT LTI system w/ $h[n]$ $\rightarrow$ $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$ — sum ... delayed

cf) $\delta[n-k]$ as a ft of n.

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

$$y[n] = \mathcal{L}(x[n]) = \sum_{k=-\infty}^{\infty} x[k]\,\mathcal{L}(\delta[n-k]) = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

$\mathcal{L}(\delta[n-k]) \triangleq h[n-k]$

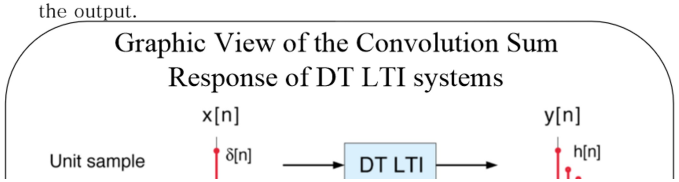
$$h[n] = \delta[n] + \frac{1}{2}\delta[n-n_0] + \frac{1}{4}\delta[n-2n_0]$$

$$\begin{cases} z^* = (z)^* \\ x[n]*h[n] \end{cases}$$

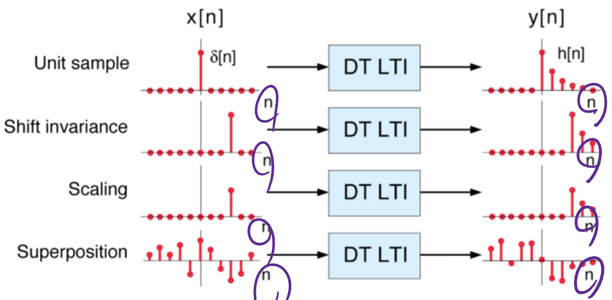
8. Preview

- sliding correlation of time-reversed impulse response
- More Implications?
  - Properties of the convolution sum
    - Lemma 1. The convolution sum is commutative.
      - Implication.
        - delayed weighted sum of input
        - sliding correlation of time-reversed input
    - Lemma 2. The convolution sum is distributive.
      - Implication
        - parallel interconnection
    - Lemma 3. The convolution sum is associative.
      - Implication
        - the overall impulse response
        - decomposition of a system into serially interconnected sub-systems
  - Theorem 1. The order of two serially interconnected DT LTI systems can be reversed without affecting the overall impulse reponse of the system.
    - Caution
      - Only for an LTI system!!!!!!!!!!!!!!!!!!!!
  - Theorem 2. A DT LTI system is causal iff
    - Def. A system is BIBO stable if
  - Theorem 3. A DT LTI system is BIBO stable iff

Understanding what's going on using the graphs of the input and the output.



Response of DT LTI systems



$$x[n] = \sum_{k=-\infty}^{+\infty} x[k]\,\delta[n-k] \xrightarrow{\text{DT LTI}} y[n] = \underbrace{\sum_{k=-\infty}^{+\infty} x[k]\,h[n-k]}_{\text{convolution sum}}$$

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왼쪽 convolutin sum을 (y[n])_n = \sum_k x[k] (h[n−k])_n
로 보고 음미하면, output (y[n])_n은
weighted delayed impulse responses의 합이다. delay k일때의 weight은
x[k]이다.

Visualizing the calculation of $y[n] = x[n]*h[n]$

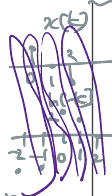
Choose the value of n and consider it fixed

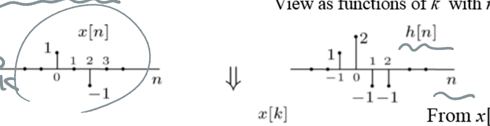
$$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k] \quad , \forall n \in \mathbb{Z}$$

View as functions of k with n fixed

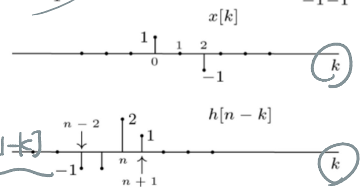
(i) $n=0$

$y[0] = \sum_{k=-\infty}^{\infty} x[k]\,h[-k]$





From $x[n]$ and $h[n]$ to $x[k]$ and $h[n\text{-}k]$



(ii) $n=1$

$y[1] = \sum_{k=-\infty}^{\infty} x[k]\,h[1-k]$

$h[-(k-1)]$

Note, $h[n\text{-}k]$–k is the mirror image of $h[n]$–n with the origin shifted to n

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앞서의 slide에서는 linearity와 time−invariance를 이용하여 convolution sum을 이해하였고

여기서는 alternate way to understand인 sliding correlation interpretation of convolution sum을 이해한다.

두 방법 모두를 숙지해야 하면, 상황에 따라 어느 하나를 선택하면 이해하기가 쉽다.

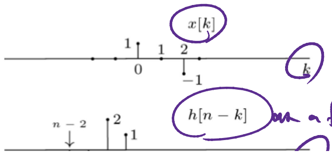
$(1-1)+(1-1)+(1-1)+$

2 interpretation of $x[n] * h[n]$

(i) $\sum_{k=-\infty}^{\infty} x[k]\,h[n-k] = y[n]$

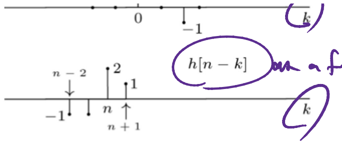
(ii) $y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

Calculating Successive Values: Shift, Multiply, Sum



$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$

각 시각에서의 출력을 계산하는 과정은, sliding correlation interpretation



as a ft of k.

$y[n] = \sum_{k=-\infty}^{+\infty} x[k]h[n-k]$

각 시각에서의 출력을 계산하는 과정을, sliding correlation interpretation 을 통해 convolution sum을 계산해 내는 과정을 설명

$y[n] = 0$ for $n <$
$y[-1] =$
$y[0] =$
$y[1] =$
$y[2] =$
$y[3] =$
$y[4] =$
$y[n] = 0$ for $n >$

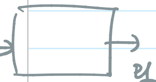
Examples of Convolution and DT LTI Systems

Ex. #1: $h[n] = \delta[n]$ Q. What is the response to x[n] when h[n]=delta[n]?

$$y[n] = x[n] * \delta[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$$

$$= x[n] \quad \text{— An Identity system}$$

analogy



$x \in \mathbb{C}^N$, $y \in \mathbb{C}^N$

$y = Ax$

$\in \mathbb{C}^{N \times N}$

If $y = x$ then $A = I$.

$= \begin{bmatrix} 1 & & \\ & \ddots & \\ & & 1 \end{bmatrix}$

Q. What is the response to x[n] when h[n]=delta[n−n_0]?

Ex. #2: $h[n] = \delta[n - n_o]$ $n_0 \in \mathbb{N}$

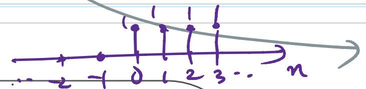
$$y[n] = x[n] * \delta[n - n_o] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - n_o - k] = \sum_{k=-\infty}^{\infty} x[k]\delta[(n-n_0) - k].$$

$$= x[n - n_o] \quad \text{— A Shift}$$

$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n - k]$ $= x[n-n_0]$

Q. MATLAB 과제: Echo mic 의 구현과 관련됨..

$h[n] = \sum_{k=-\infty}^{n} \delta[k], \forall n$



$y = Ax$

Ex. #3 $y[n] = \sum_{k=-\infty}^{n} x[k]$ – An accumulator

$u[n] \triangleq \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

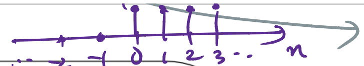
$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

Q. What is the impulse response h[n] of this system?
Unit Sample response

$$h[n] = \sum_{k=-\infty}^{n} \delta[k] = u[n]$$

$\Downarrow$

$h[n] = \sum_{k=-\infty}^{n} \delta[k], \forall n$



$y = Ax$

$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$

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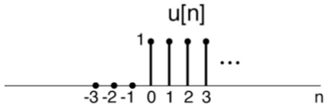


$$\Downarrow$$

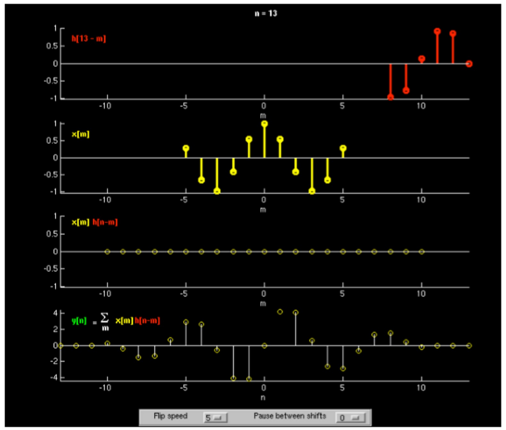
$$y[n] = x[n] * u[n] = \sum_{k=-\infty}^{n} x[k]$$

Convolution operation procedure:

$$h[k] \xrightarrow{\text{Flip}} h[-k] \xrightarrow{\text{Slide}} h[n-k] \xrightarrow{\text{Multiply}} x[k]h[n-k]$$

$$\xrightarrow{\text{Sum}} \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

Demo: DT Convolution



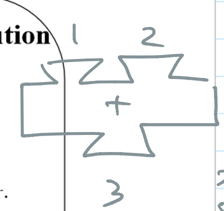
교환법칙이 성립한다

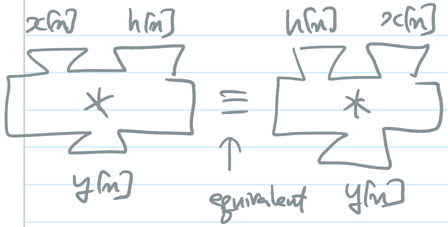
Lemma. The Commutative Property of Convolution

$$y[n] = x[n] * h[n] \quad = \quad h[n] * x[n]$$

$$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n] \quad = \quad h[n] \rightarrow \boxed{x[n]} \rightarrow y[n]$$

impulse response와 system input은 서로 interchageable하다.





Def. The step response $s[n]$ of an LTI system is defined as ...

Lemma. $s[n] = u[n] * h[n] = h[n] * u[n]$

"Input" Unit Sample response of accumulator

$\Downarrow$

$$s[n] = \sum_{k=-\infty}^{n} h[k]$$

cf) $3 \times 5 = 3+3+3+3+3 = 15$
$= 5+5+5 = 15$

$z = f(x,y) = x^2 y$
$\neq f(y,x) = y^2 x$

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The step response can be obtained from the impulse response by the left equality.

Convolution sum의 commutativity를 이용하면, unit step response s[n]은 두가지로 interpret될 수 있다.

1. 말 그대로 system의 unit step function 입력에 대한 response.

2. system의 impulse response를 accumulator에 통과 시켰을 때의 output.

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \qquad l = n-k, \quad k = n-l$$

$$= \sum_{l=\infty}^{+\infty} x[n-l]\, h[l]$$

$$= \sum_{l=-\infty}^{\infty} h[l]\, x[n-l]$$

$$= h[n] * x[n]$$

Q. How can you find the impulse response from the step response?

A. delta[n] = u[n]−u[n−1]. Therefore, h[n] = s[n] − s[n−1].

Q. 덧셈(addition)이란 무엇인가?
A. 덧셈이란, +: R^2 −> R 이다. 즉, two variable function 이다.
쉽게 말해 f(x,y)의 일종이다.

Q. f(x,y)들 중에서 특히 어떤 특성을 갖는 함수인가?
A. f(x,y) = f(y,x),즉 commutative하다.
같은 질문을 곱셈에 대해서도 해 보라. 뺄셈은? 나눗셈은?

commutativity property는 system이 LTI이면 성립한다. What about general linear systems?

앞서의 convolution sum의 두가지 interpretation을 다시 적용하면 결국
우리는 이제
convolution sum에 대한 4가지 다른 interpretation을 갖는다.

1.2. The output is the delayed and weighted sum of the input/IR.
3.4. The output is the sliding correlation between the input/IR and the time−reversed IR/input.

4가지 모두 중요하다!!!!!!
어떤 경우는 이 중 어느 하나가 이해하기 쉽고,
다른 어떤 경우는 이 중 다른 어느 하나가 이해하기 쉽다.

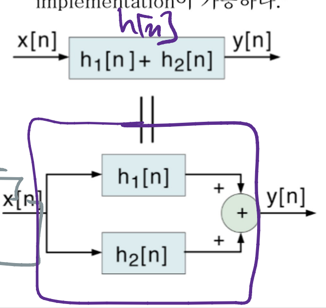
Lemma. The Distributive Property of Convolution

배분법칙

$$y[n] \quad x[n]*\{h_1[n]+h_2[n]\} = x[n]*h_1[n] + x[n]*h_2[n]$$

$3\times(1+4) = 3\times1 + 3\times4$

Interpretation "하나의 DT LTI system을 parallel systems로 implementation이 가능하다."



Q. 곱셈 f(x,y)는 또 어떤 특성을 갖는가? 덧셈과 관련하여 설명하라.
A. f(x, h1+h2) = f(x, h1) + f(x, h2)

"interpretation"
"implication"
이 중요하다고 이미 강조하였다.

수식이나 diagram을 그린 후 반드시 "말" 만으로 표현해 보라.

Q1. 이 property는 LTV system에서도 성립하는가?
Q2. 앞서의 commutativity는 LTV system에서 성립하는가?

결합법칙

$2\times(3\times4) \overset{?}{=} (2\times3)\times4$

Lemma. The Associative Property of Convolution

Lemma. The Associative Property of Convolution

$$\boxed{x[n]} * \boxed{(h_1[n] * h_2[n])} = (x[n] * h_1[n]) * h_2[n]$$

우변에서 먼저 시작하면,
먼저 system with h1[n]에 x[n]이 입력으로 들어가서 (x[n]*h1[n])이 출력으로 나온다. 이를 다시 system with h2[n]의 입력으로 사용하면 최종 출력은 (x[n]*h1[n])*h2[n]이 된다.

이제 associativity를 이용하면,
"cascade된 두 LTI system을 impulse response가 h1[n]*h2[n]인 하나의 LTI system으로 구현하면 equivalent 하다"는 것을 알게 된다.

f(x,y) = xy로 정의하면,
f(x, f(y,z)) = f(f(x,y), z)가 성립한다. 이때 f는 associative하다고 한다.

convolution sum도 associative property를 갖는 mapping rule이다.

그렇다면 system analysis/design에서 이의 implication은?

$x \quad f(y,z)$

$f(\, , \,)$

$f(f(x,y), z)$

Theorem. The associativity combined with commutativity leads to

LHS. $x[n] \to \boxed{h_1[n]} \to \boxed{h_2[n]} \to y[n] \qquad x[n] \to \boxed{h_2[n]} \to \boxed{h_1[n]} \to y[n]$

$$y[n] = (x[n] * h_1[n]) * h_2[n] = (x[n] * h_2[n]) * h_1[n]$$

여기서 commutativity와 associativity를 결합하면
"cascaded (serially connected) 된 두 LTI system의 순서를 바꾸어도 overall response는 바뀌지 않는다"는 결론에 도달한다.

LTI systems의 순서를 바꿔도 같지만, LTV는 일반적으로 그렇지 않다.

Q. modulator y(t) = x(t)cos(2\pi f_ct)는 LTI인가?

Implication (Very special to LTI Systems)

$(x[n] * h_1[n]) * h_2[n]$

$x[n] \to \boxed{h_1[n]} \to \boxed{h_2[n]} \to y[n]$

$x[n] * (h_1[n] * h_2[n])$

$x[n] \to \boxed{h_1[n] * h_2[n]} \to y[n]$

$x[n] * (h_2[n] * h_1[n]) = (x[n] * h_2[n]) * h_1[n]$

$x[n] \to \boxed{h_2[n]} \to \boxed{h_1[n]} \to y[n]$