CM10 231010T

Announcements

1. Messages from Instructor
    - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
    - Simplify and Visualize:
        - essence, generalize, consider extremes,
        - Venn diagram, necessity and sufficienciency, existence and uniqueness/plurality
        - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical interpretations
    - Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

        Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

        출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>
    - Stop and Think
    - Self-Fulfillment and Contribution to Collective

2. 지난 강의 수정/보완 사항
    - x[n] vs. (x[n])_n, etc.
    - a differentiator/integrator is causal LTI?
        - mathematical
        - physical (w/ causality)
        - initial condition

a_n

$(a_n)_{n=1}^{\infty}$

$(a_n)_{n\in\mathbb{N}}$

$(a_n)_n$

(a_n)

$\{a_n\}_{n=1}^{\infty}$

$\{a_n\}_{n\in\mathbb{N}}$

$\{a_n\}_n$

$\{a_n\}$

(X)

3. HW, Quiz, and Exam
    - HW06 will be assigned by this Friday. Due will be next Tuesday.
    - Midterm Exam is approaching.

3. HW, Quiz, and Exam
  - HW06 will be assigned by this Friday. Due will be next Tuesday.
  - Midterm Exam is approaching.
    - Tentatively, it will be taken on Thursday 10/26 from 19:00-23:00 at LG 102.
    - Those who are not available at this time slot, please let me know by tonight.

학습 學習

4. 강의 자료
  - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
  - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석
  - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform
   7. Sampling
   8. Communication Systems

$x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$, $\forall n$ $\rightarrow \boxed{\text{DT LTI}} \rightarrow y[n]$

$= \mathcal{L}\left(\sum_k x[k]\delta[n-k]\right)$

7. Review
  - Convolution sum y[n] = x[n]*h[n]
    - The output is a weighted summation of the delayed impulse responses.
    - The output at a time instant is a correlation between the input and time-reversed delayed impulse response.
  - More Implications
    - Properties of the convolution sum
      - Lemma 1. The convolution sum is commutative
        - Implication.
          - weighted sum of delayed inputa

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

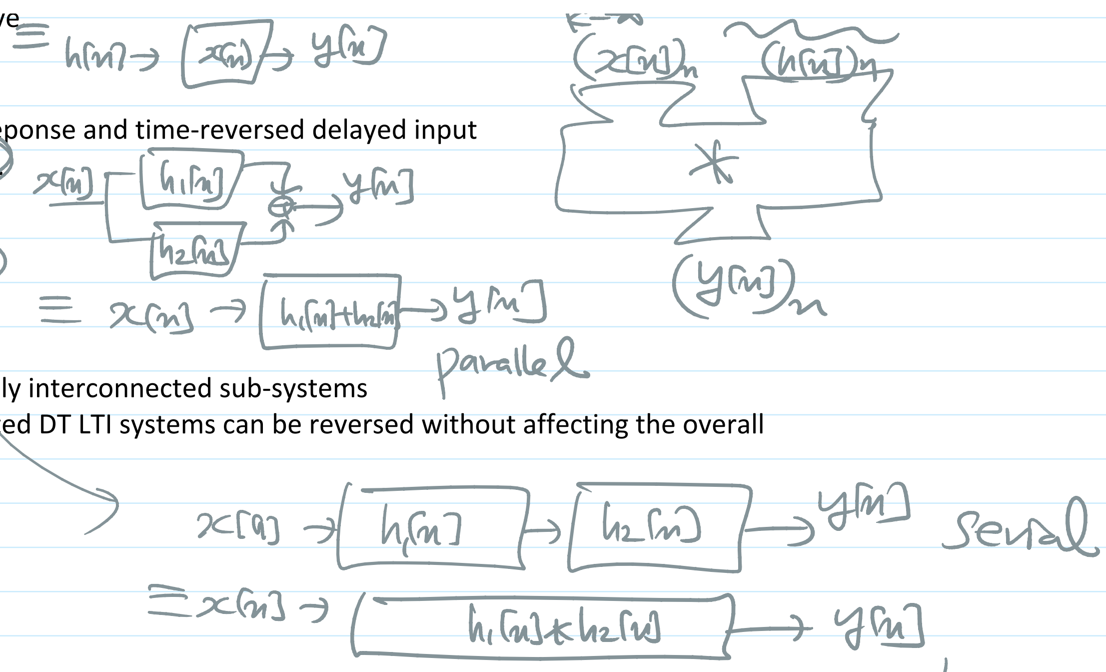
$\equiv h[n] \rightarrow \boxed{x[n]} \rightarrow y[n]$

$= \sum_{k=-\infty}^{\infty} x[k]\, h[n-k]$

$(x[n])_n$ $(h[n])_n$

- Lemma 1. The convolution sum is commutative
  - Implication.
    - weighted sum of delayed inputa
    - sliding correlation b/w the impulse reponse and time-reversed delayed input
- Lemma 2. The convolution sum is distributive.
  - Implication
    - parallel interconnection
- Lemma 3. The convolution sum is associative.
  - Implication
    - the overall impulse response
    - decomposition of a system into serially interconnected sub-systems

$(ab)c = a(bc)$



- Theorem 1. The order of two serially interconnected DT LTI systems can be reversed without affecting the overall impulse reponse of the system.
  - Caution
    - Only for an LTI system!!!!!!!!!!!!!!!!!!!!!

8. Preview

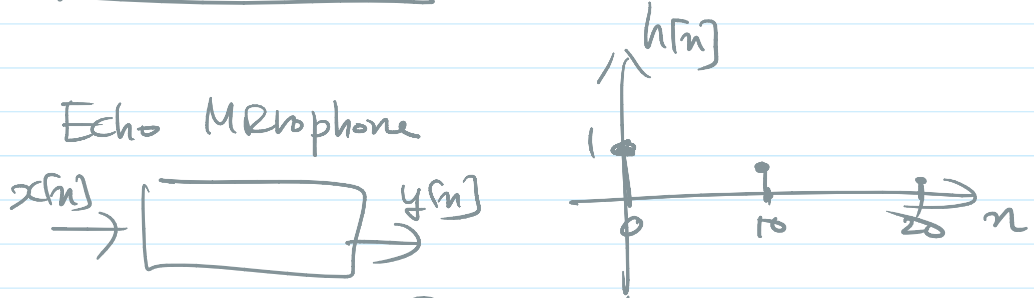
Causality

- Proof of if S then N
  - S is sufficient for N; N is necessary for S.
  - Def. The converse is if N then S.

BIBO stability

- Proof of B iff A
  - if; the direct part (<=); sufficiency, A is sufficient for B.
  - only if; the converse (=>); necessity, A is necessary for B.
- Theorem 2. A DT LTI system is causal iff
- Def. A DT system is BIBO stable if
- Theorem 3. A DT LTI system is BIBO stable iff

Echo Microphone



$h[n]=\delta[n]+\frac{1}{2}\delta[n-10]+\frac{1}{4}\delta[n-20]$

$y[n]=x[n]*h[n]$

$= x[n]*(\quad + \quad + \quad)$

$= x[n]+\frac{1}{2}x[n-10]+\frac{1}{4}x[n-20]$

- Repeat the above for CT LTI Systems
  - Def. unit impulse and the sifting property
  - Lemma. The (unit) impulse response of a CT LTI system ....
  - Thm. The output of a CT LTI system is given by
  - Def. the convolution integral
    - Def. The (unit) step response of a CT LTI system is ...
    - Lemma. 3 properties of the convolution integral
      - Commutativity: echo microphone
      - Distributivity: parallel
      - Associativity: serial
    - Thm. (combination of C and A) Serially interconnected LTIs have order-invariance.

Properties of Convolution

Combining the Commutative property,

$$y[n] = x[n] * h[n] \quad = \quad h[n] * x[n]$$

Distributive property,

$$x[n] * \{h_1[n] + h_2[n]\} = x[n] * h_1[n] + x[n] * h_2[n]$$

and Associative property,

$$x[n] * (h_1[n] * h_2[n]) = (x[n] * h_1[n]) * h_2[n]$$

symbolically, we can treat "$*$" as a "$\times$". Easy, piece of cake! The hard part is the actual calculation of the convolution.

Flip → Slide → Multiply → Sum.

Soon we will develop a clever way (*transformation*) to perform "$\times$" instead of "$*$" operation.

Flip-> Slide-> ..-> Sum은 convolution sum의 sliding correlation interpretation이다.

Def. 결론을 부정하여 이것이 가정과 모순됨을 보임으로써 조건명제가 참임을 증명하는 indirect proof방법을 "proof by contradiction" (귀류법) 이라고 한다.

Def. 참임을 보이고자하는 조건명제의 대우명제가 참임을 보이는 indirect proof방법을 "proof by contraposition/contrapositive"이라고 함.

direct proof와 이들 indirect proofs는 논리적으로 동일한 방법하다.

로 동일한 방법하다.

T/F of { a Conditional statement A, B
$\equiv$ set inclusion A, B

DT LTI

Some Useful Properties of LTI Systems

Thm. 1) Causality $\Leftrightarrow$ $h[n]=0$ for all $n<0$

Q. Prove this result.

$B \Leftrightarrow A$

Thm. 2) Stability $\Leftrightarrow$ $\sum_{k=-\infty}^{+\infty} |h[k]| < \infty$

A DT LTI system is BIBO stable iff ...

BIBO — Bounded Input $\Rightarrow$ Bounded Output

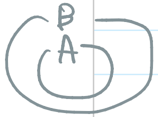
$\exists\, 0<M<\infty$, $|h[n]| \le M, \forall n$

Sufficient condition: For $|x[n]| \le x_{\max} < \infty$.

(i) (if) $B \Leftarrow A$, $A \Rightarrow B$

$$|y[n]| = \left|\sum_{k=-\infty}^{\infty} x[k]h[n-k]\right| \le x_{\max}\boxed{\left|\sum_{k=-\infty}^{\infty} h[n-k]\right| < \infty}.$$

A



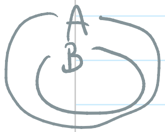
A is sufficient for B.

Necessary condition: If $\boxed{\sum_{k=-\infty}^{\infty} |h[k]| = \infty}$ ~A

(ii) (only if) $B \Rightarrow A$

Let $x[n] = h^*[-n]/|h[-n]|$, then $|x[n]| \equiv 1$ bounded ← 반례

But $y[0] = \sum_{k=-\infty}^{\infty} x[k]h[-k] = \sum_{k=-\infty}^{\infty} h^*[-k]h[-k]/|h[-k]| = \sum_{k=-\infty}^{\infty} |h[-k]| = \infty$



A is necessary for B.

$\sim A \Rightarrow \sim B$

$\sim A \wedge B$ is false.

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Def. An LTI system이 모든 bounded input에 대해 bounded output을 내면, "BIBO stable하다"고 한다.

Q. An LTI system이 BIBO stable하기위한 necessary and sufficient condition은 ?

Q. What is the difference between
\sum_{n=1}^{\infty}a_n
and
\sum_{n\in N} a_n?

Q. BIBO stable할 sufficient condition은? 즉, If an LTI system satisfies (....condition), then it is BIBO stable 에서 (...condition)부분 즉 hypothesis 부분을 찾아라.

Left to Right: proof of necessity

Right to Left: proof of sufficiency

A. 따라서, "the impulse response is absolutely summable"은 sufficient condition이 된다.

Note: the impulse response is absolutely summable하면, bounded input에 대해서 BIBO stable DT system의 y[n]이 각 n에서 absolutely summable한 infinte sum을 갖는다. 따라서 any

에서 (...condition)부분 즉 hypothesis 부분을 찾아라.

summable하면, bounded input에 대해서 BIBO stable DT system의 y[n]이 각 n에서 absolutely summable한 infinte sum을 갖는다. 따라서 any rearrangement에서도 같은 값으로 수렴하므로 physically no problem이다.

Note:
output이 bounded되어 있다는 말은, output y[n]의 각 n 값에서의 infinite sum이 수렴한다는 말이다.
physically meaningful하려면 any rearrangement에 대해서 같은 값으로 수렴해야 하는데

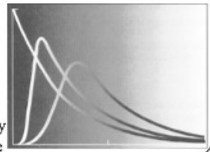
Lecture Note #3에서는 DT LTI system에 대해 배웠고, 여기서는 CT LTI system에 대해 배운다.

거의 대부분의 내용이 parallel하므로 특히 차이점에 대해 관심을 갖도록 하자.

EECE233 Signals and Systems

Lecture Note #4
Prof. Joon Ho Cho
(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

- Covers O & W pp. 90-102, 127-136
- Representation of CT Signals in terms of shifted unit impulses
- Introduction of the unit impulse $\delta(t)$
- Convolution integral representation of CT LTI systems
- Properties and Examples

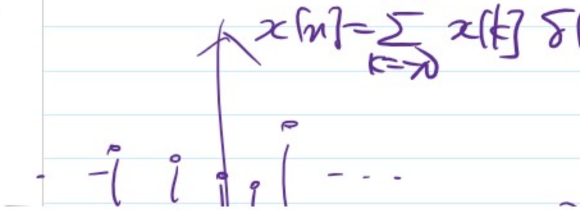


* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

The main ideas in the previous note are summarized as follows.
1. Find an alternate representation of the input signal by using weighted and shifted delta functions.
2. Use linearity and time invariance to derive the notions of impulse response and convolution sum.
3. Examine the properties of the convolution sum and find the physical meanings.
Do the same for CT systems!!!

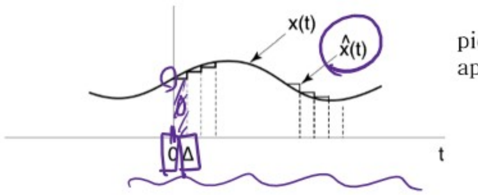
Representation of CT Signals

$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$



Representation of CT Signals

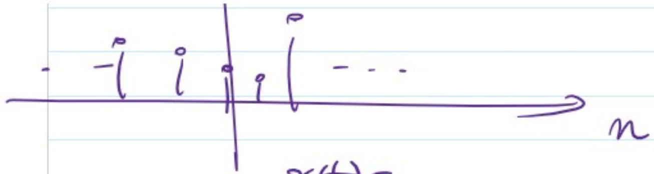
- Approximate any input $x(t)$ as a sum of shifted, scaled square pulses (in fact, that is how we do integration)



piecewise constant approximation of x(t)

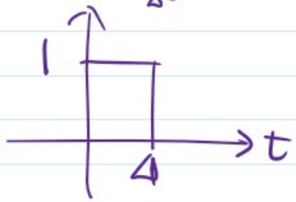
$$\hat{x}(t) = x(k\Delta), \quad k\Delta < t < (k+1)\Delta$$

2



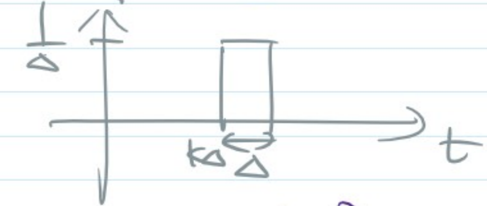
$x(t) =$

$\mathrm{rect}_\Delta(t)$



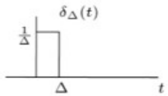
$$\hat{x}(t) \triangleq \sum_{k=-\infty}^{\infty} x(k\Delta)\,\mathrm{rect}_\Delta(t-k\Delta)$$

$$y(t) = \mathcal{L}\big(x(t)\big) \simeq \mathcal{L}\big(\hat{x}(t)\big) = \mathcal{L}\Big(\sum_{k=-\infty}^{\infty} x(k\Delta)\,\mathrm{rect}_\Delta(t-k\Delta)\Big) = \sum_{k=-\infty}^{\infty} x(k\Delta)\,\Delta\,\mathcal{L}\Big(\frac{1}{\Delta}\mathrm{rect}_\Delta(t-k\Delta)\Big)$$



Representation of CT Signals (cont.)

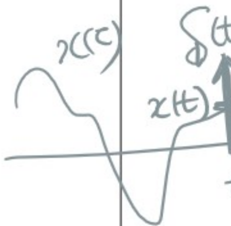
the Dirac Delta ft.

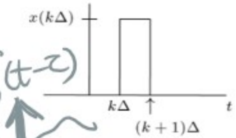


$\delta_\Delta(t)$ has a unit area

$\delta(t)$

the unit impulse ft.





$= x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$

⇓

$$\hat{x}(t) = \sum_{k=-\infty}^{+\infty} x(k\Delta)\delta_\Delta(t-k\Delta)\Delta$$

Def. The sifting property of the unit impulse is given by

↓ limit as $\Delta \to 0$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$$

3

the sifting property of x(t)

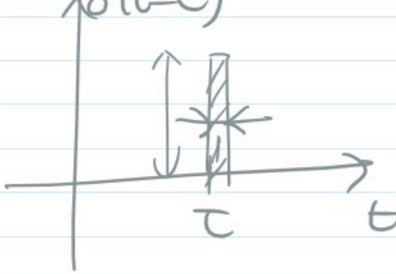
높이 1, 폭 Delta, 면적 Delta

고등학교 구분구적법에서 했던 것과 같은 방식을 쓴다. 다만 적분구간이 이제는 무한대이고 delta_Delta(t)의 극한을 delta(t)로 정의하여 사용한다.

x(t)를 delta(t)의 weighted shifted 'sum'으로 나타낸 것으로 이해할 수 있다.

앞서 배웠던 DT delta function의 sifting property를 CT delta function으로 확장한다.

$\delta(t-\tau)$



Area 1.

$\Delta \to 0 \quad \frac{1}{\Delta}\mathrm{rect}_\Delta(t)$

$\rightarrow \boxed{\text{CT LTI}} \rightarrow h(t)$

$\sim \int^{\infty}$

the sifting property of the D. Delta ft

x(t)를 delta(t)의 weighted shifted 'sum'으로 나타낸 것으로 이해할 수 있다.

Q. What was the parallel result for DT signals?

앞서 배웠던 DT delta function의 sifting property를 CT delta function으로 확장 한다.

$$\simeq \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau \triangleq x(t) * h(t)$$

The conv. integral.

LTI

The Unit-impulse function $\delta(t)$

확률 및 랜덤 프로세스 과목에서 discrete random variable 의 pdf를 나타내기 위해 도입한다.

Paul Dirac first introduced this function in 1920's to describe the charge density of an electron (diameter < 10^{-13} cm), extremely useful, but made many mathematicians uncomfortable. Definition of $\delta(t)$:

Def. delta(t) is defined as the limit of delta_Delta(t) as Delta tends to zero.

그후 수학자들이 distribution또는 generalized function이라는 이름으로 이를 체계화하였다.

오른쪽 정의는 수학적이지 못한 (엉터리같지만 유용한) 정의이다.

$$\left.\begin{aligned} \delta(t) &= 0 \quad for\ t \neq 0 \\ \delta(t) &= \infty \quad for\ t = 0 \end{aligned}\right\} \qquad (1)$$

$$\int_{0^-}^{0^+} \delta(t)\,dt = 1 \qquad (2)$$

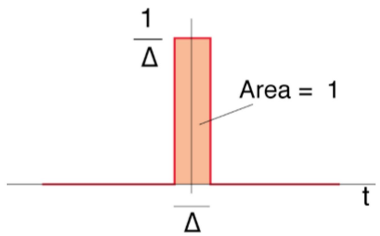
While lie.

0 t

— an infinitesimally sharp pulse with an unity area.

Construction of the Unit-impulse function $\delta(t)$

One of the simplest way — rectangular pulse, taking the limit $\Delta \to 0$.



1사분면에만 있는 rectangular pulse의 극한으로 봐도 되고

2사분면에만 있는 rectangular pulse의 극한으로 봐도 된다.

But this is by no means the only way. One can construct a $\delta(t)$ function out of many other functions, *Eg.* Gaussian pulses, triangular pulses, sinc functions, *etc.*, as long as the pulses are

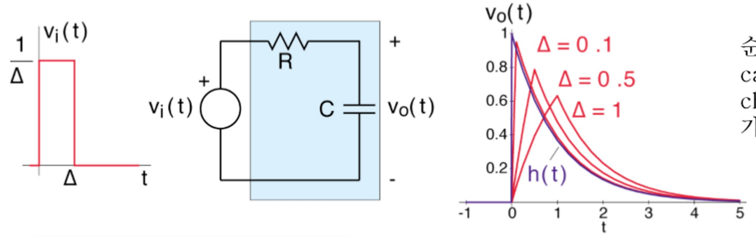
But this is by no means the only way. One can construct a $\delta(t)$ function out of many other functions, *Eg.* Gaussian pulses, triangular pulses, sinc functions, *etc.*, as long as the pulses are short enough — much shorter than the characteristic time scale of the system.

5

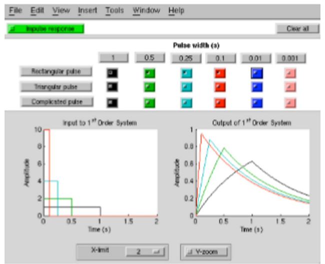
Def. sinc(x) = sin (\pi x) / (\pi x)

혹자는 sinc(x) = sin (x) /x로 정의하기도 하니, sinc를 사용할때는 어느 정의를 따르는지 확실히 할 것.

Demo: Response of an RC circuit to short pulses



순간적으로 capacitor가 charge되었다가 방전됨.



Response from initial rest to pulses of different shapes and durations, but with unit area. As the duration decreases, the response becomes similar for different pulse shapes.

Q. What was the definition of initial rest?

6

Response of a CT LTI System

$x(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t) = \mathcal{L}(x(t)) = \mathcal{L}\left(\boxed{\int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau}\right)$

$x(t) =$

$= \int_{-\infty}^{\infty} x(\tau)\,\mathcal{L}(\delta(t-\tau))\,d\tau$

Def. Given an **LTI** system, the *unit impulse response* $h(t)$ is defined as

$$\delta(t) \longrightarrow h(t)$$

Lemma.

$h(t)$ is defined as

$$\delta(t) \longrightarrow h(t)$$

Lemma.

$$\Downarrow$$

From Time-Invariance:

$$\delta(t-\tau) \longrightarrow h(t-\tau)$$

Theorem.

From Linearity and Time-Invariance:

$$x(t) = \int_{-\infty}^{+\infty} x(\tau)\delta(t-\tau)d\tau \longrightarrow y(t) = \underbrace{\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau}_{\textit{Convolution Integration}} = x(t) * h(t)$$

$$\int_{-\infty} x(\tau)\, \underbrace{\mathcal{L}[\delta(t-\tau)]}_{\triangleq h(t-\tau)}\, d\tau$$

$$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau) d\tau$$

$$\triangleq x(t) * h(t)$$

Interpretation.
(i) The output is a weighted integral of delayed impulse responses.
(ii) The output is the sliding correlation between the input and the time−reversed impulse response.

Operation of CT Convolution

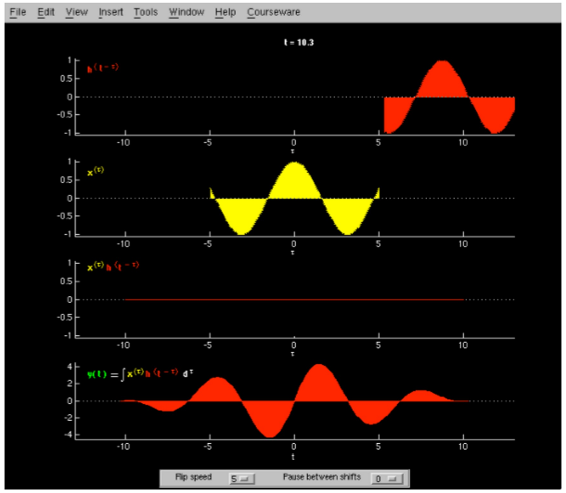
$$y(t) = x(t) * h(t) \equiv \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

$$h(\tau) \xrightarrow{\textit{Flip}} h(-\tau) \xrightarrow{\textit{Slide}} h(t-\tau) \xrightarrow{\textit{Multiply}}$$

$$x(\tau)h(t-\tau) \xrightarrow{\textit{Integrate}} \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau$$

Demo: CT convolution

Demo: CT convolution



9

Properties and Examples

LTI의 output은 weighted integral of delayed input 으로 보는 interpretation 을 얻는다.

1) ~~Commutativity:~~ $x(t) * h(t) = h(t) * x(t)$

2) $x(t) * \delta(t - t_o) = x(t - t_o)$ (Sifting property: $x(t) * \delta(t) = x(t)$)

3) An integrator: $y(t) = \int_{-\infty}^{t} x(\tau) d\tau$

$\Downarrow$

$h(t) = \int_{-\infty}^{t} \delta(\tau) d\tau = u(t)$

That is

$$\boxed{x(t) * u(t) = \int_{-\infty}^{t} x(\tau) d\tau}$$

The impulse repose of the identity system is the impulse itself.
delta(t) in
delta(t) out

Q1. The integrator는 LTI system인가?

Q2. LTI system이라면 impulse response는 어떻게 되는가?

4) Step response

Def. A CT LTI system에 unit step input을 가했을때의 출력을 its unit step response 라 한다.

$$s(t) = u(t) * h(t) = h(t) * u(t) = \int_{-\infty}^{t} h(\tau) d\tau$$

Q1. Find s(t) from h(t).
Q2. Find h(t) from s(t).

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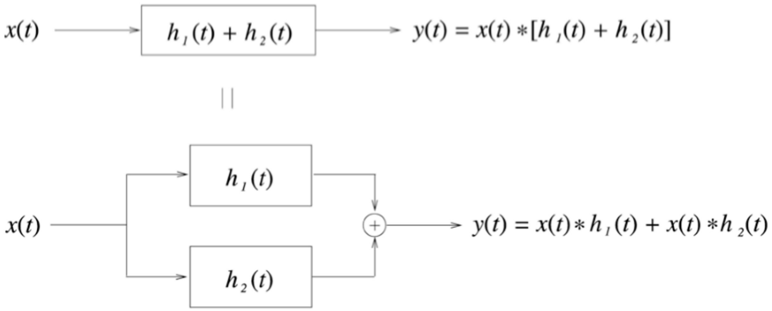
$x(t-t_0) * \delta(t-t_1)$

$= x(t-t_2)$

$t_2 = t_0 + t_1$

Distributivity



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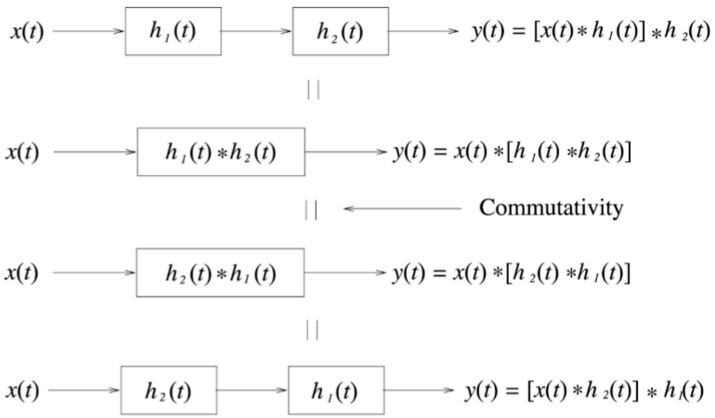
Associativity



$$x(t) \longrightarrow \boxed{h_2(t)} \longrightarrow \boxed{h_1(t)} \longrightarrow y(t) = [x(t) * h_2(t)] * h_1(t)$$

More properties

Theorem. A CT LTI system is causal $\leftrightarrow$ $h(t) = 0$, for all $t < 0$.

As a result:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{t} x(\tau)h(t-\tau)d\tau$$

$y(t)$ only depends on $x(\tau < t)$.

Def. A CT signal is absolutely integrable if ...

Thm. A CT LTI system is BIBO stable $\leftrightarrow \int_{-\infty}^{+\infty} |h(\tau)|\, d\tau < \infty$

Sufficient condition: For $|x(t)| \le x_{\max} < \infty$.

$$|y(t)| = \left| \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)d\tau \right| \le x_{\max} \left| \int_{-\infty}^{+\infty} h(t-\tau)d\tau \right| < \infty.$$

Necessary condition: Suppose $\int_{-\infty}^{+\infty} |h(\tau)| d\tau = \infty$

Let $x(t) = h^*(-t)/|h^*(-t)|$, then $|x(t)| \equiv 1$ bounded

But $y(0) = \int_{-\infty}^{+\infty} x(\tau)h(-\tau)d\tau = \int_{-\infty}^{+\infty} \frac{h^*(-\tau)h(-\tau)}{|h(-\tau)|} d\tau = \int_{-\infty}^{+\infty} |h(-\tau)| d\tau = \infty$

More About $\delta(t)$: The Operation Definition of the Unit Impulse

$\delta(t)$ — idealization of a unit-area pulse that is so short that, for any physical systems of interest to us, the system responds only to the area of the pulse and is insensitive to its duration

Operationally: The unit impulse is the signal which when applied to any LTI system results in an output equal to the

Operationally: The unit impulse is the signal which when applied to any LTI system results in an output equal to the impulse response of the system. That is,

$$\delta(t) * h(t) = h(t) \qquad \text{for all} \quad h(t)$$

보다 정확히 이야기 하면 delta function은 sifting property에 의해 정의되는 mathematical entity이다

— $\delta(t)$ is defined by what it does under convolution.