2023년 10월 10일 화요일   12:43

CM11 231012Th

Announcements

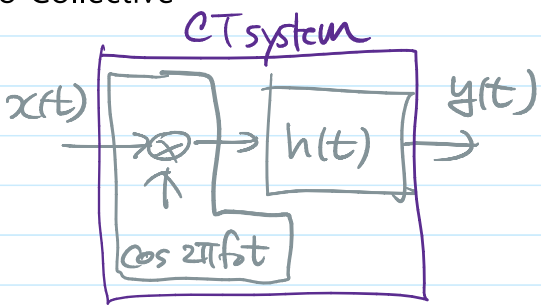
1. Messages from Instructor
   - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
   - Simplify and Visualize:
     - essence, generalize, consider extremes,
     - Venn diagram, necessity and sufficiencency, existence and uniqueness/plurality
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical interpretations
   - Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

     Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

     출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>

   - Stop and Think
   - Self-Fulfillment and Contribution to Collective
   - Greek letters

2. 지난 강의 수정/보완 사항
   - modulator-filter and order
   - RC circuit



3. HW, Quiz, and Exam

- HW06 will be assigned by this Friday. Due will be next Tuesday.
  - HW grading policy will be changed from HW06.
- Midterm Exam is approaching.
  - It will be taken on Thursday 10/26 at 19:00-23:00 in LG 102.
  - Open everything
  - No communication with human being

4. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

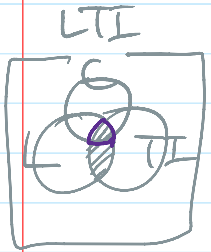
5. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform
   7. Sampling
   8. Communication Systems

7. Review $S \Rightarrow N$
   - Proof of if S then N
     - S is sufficient for N; N is necessary for S.
     - Def. The converse is if N then S

- Proof of if S then N
  - S is sufficient for N; N is necessary for S.
  - Def. The converse is if N then S.

LTI



- Proof of B iff A

iff if only if $A \Rightarrow B$.

  - if; the direct part (<=); sufficiency, A is sufficient for B.
  - only if; the converse (=>); necessity, A is necessary for B.
- Theorem 2. A DT/CT LTI system is causal iff the impulse response is a causal signal.

$h(t)=h(t)u(t),\ \forall t$

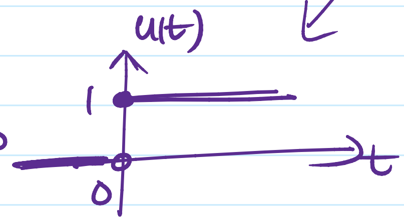
$h[n]=h[n]u[n],\ \forall n$

- Def. A DT/CT system is BIBO stable if ...
- Theorem 3. A DT/CT LTI system is BIBO stable iff the impulse response is absolutely summable/integrable

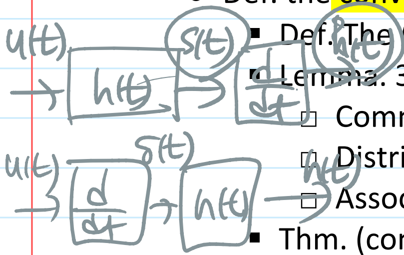
$u[n]=\begin{cases}1, & n\geq 0\\ 0, & \text{elsewhere}\end{cases}$

$\sum_{n=-\infty}^{\infty}|h[n]|<\infty$

$\int_{-\infty}^{\infty}|h(t)|\,dt<\infty$



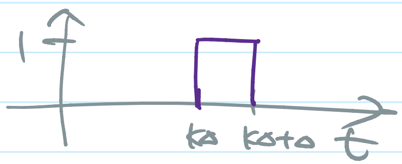
- Repeat the above for CT LTI Systems
  - Def. the unit impulse and the sifting property
  - Lemma. The (unit) impulse response of a CT LTI system ....
  - Thm. The output of a CT LTI system is given by ...
  - Def. the convolution integral
    - Def. The (unit) step response of a CT LTI system is ...
    - Lemma. 3 properties of the convolution integral
      - Commutativity: echo microphone
      - Distributivity: parallel
      - Associativity: serial
    - Thm. (combination of C and A) Serially interconnected LTIs have order-invariance.



8. Preview

   - Def. Generalized functions u_n(t) for n\in Z
     - differentiation of delta(t)
     - integration of delta(t)
     - u_0(t) = delta(t)
   - A convolution of rectangular pulses
     - triangular pulse
     - linear interpolation

$x(t) \simeq \sum_{k=-\infty}^{\infty} x(k\Delta)\ \mathrm{rect}_\Delta(t-k\Delta)$



$= \sum_{k=-\infty}^{\infty} x(k\Delta)\ \frac{1}{\Delta}\mathrm{rect}_\Delta(t-k\Delta)\ \Delta$

$\simeq \int_{-\infty}^{\infty} x(\tau)\,\delta(t-\tau)\,d\tau$

$\delta(t)$ the Dirac Delta function

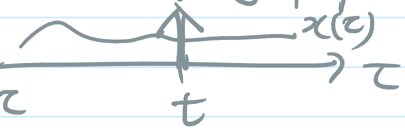
a generalized function

- Defs. an eigen-function and the corresponding engen-value of a DT/CT linear system
  - complex exponential signal
  - system function
    - Laplace transform, z-transform
- The set of all DT periodic signals and DT Fourier series

- existence
- uniqueness

$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\,d\tau$

the sifting property



$= \int_{-\infty}^{\infty} x(t)\delta(t-\tau)\,d\tau$

$= x(t)\int_{-\infty}^{\infty} \delta(t-\tau)\,d\tau$

$= x(t)$

$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\,d\tau$

$h(t)$: the impulse response of the CT LTI system

$y(t) = x(t) * h(t)$

More properties

Theorem. A CT LTI system is causal $\leftrightarrow$ $h(t) = 0$, for all $t < 0$.

As a result:

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau = \int_{-\infty}^{t} x(\tau)h(t-\tau)\,d\tau$$

$y(t)$ only depends on $x(\tau < t)$.

Def. A CT signal is absolutely integrable if ...

Thm. A CT LTI system is BIBO stable $\leftrightarrow \int_{-\infty}^{+\infty} |h(\tau)|\,d\tau < \infty$

Sufficient condition: For $|x(t)| \le x_{\max} < \infty$.

$$|y(t)| = \left|\int_{-\infty}^{+\infty} x(\tau)h(t-\tau)\,d\tau\right| \le x_{\max}\left|\int_{-\infty}^{+\infty} h(t-\tau)\,d\tau\right| < \infty.$$

Necessary condition: Suppose $\int_{-\infty}^{+\infty} |h(\tau)|\,d\tau = \infty$

Let $x(t) = h^*(-t)/|h^*(-t)|$, then $|x(t)| \equiv 1$ bounded

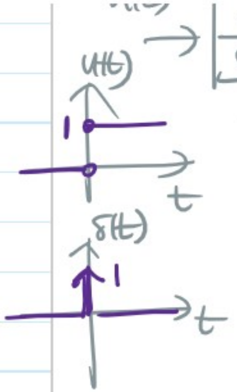
But $y(0) = \int_{-\infty}^{+\infty} x(\tau)h(-\tau)\,d\tau = \int_{-\infty}^{+\infty} \frac{h^*(-\tau)h(-\tau)}{|h(-\tau)|}\,d\tau = \int_{-\infty}^{+\infty} |h(-\tau)|\,d\tau = \infty$

13

$u(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \delta(t)$

More About $\delta(t)$:

The Operational Definition of the Unit Impulse



More About $\delta(t)$: The Operation Definition of the Unit Impulse

$\delta(t)$ — idealization of a unit-area pulse that is so short that, for any physical systems of interest to us, the system responds only to the area of the pulse and is insensitive to its duration

Operationally: The unit impulse is the signal which when applied to any LTI system results in an output equal to the impulse response of the system. That is,

$$\delta(t) * h(t) = h(t) \qquad \text{for all } h(t)$$

— $\delta(t)$ is defined by what it does under convolution.

보다 정확히 이야기 하면 delta function은 sifting property에 의해 정의되는 mathematical entity이다

$\delta(t) \rightarrow \boxed{\int_{-\infty}^{t} \cdot \, d\tau} \rightarrow u(t)$

$e(t) = -L\frac{di}{dt}$

Q1. Is this an LTI system?

Q2. If so, what is the impulse response?

Differentiator and the Unit Doublet

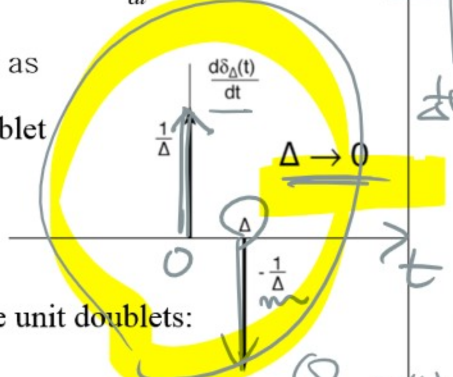
$$x(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow y(t) = \frac{dx(t)}{dt}$$

Def. The unit doublet is defined as

Impulse response = unit doublet

Notation.

$$u_1(t) = \frac{d\delta(t)}{dt}$$



$= \frac{1}{\Delta}(u(t)-u(t-\Delta)) \triangleq \delta_\Delta(t) \rightarrow \delta(t)$

$\frac{1}{\Delta}rect_\Delta(t) \rightarrow \boxed{\frac{d}{dt}} \rightarrow \frac{1}{\Delta}\left(\delta(t)-\delta(t-\Delta)\right)$

$= \frac{1}{\Delta}\delta(t) - \frac{1}{\Delta}\delta(t-\Delta)$

The operational definitions of the unit doublets:

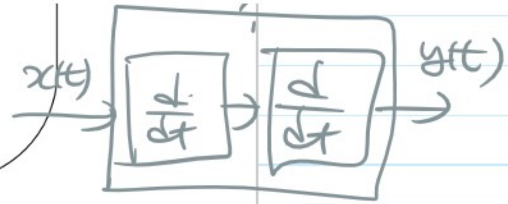
$$x(t) * u_1(t) = \frac{dx(t)}{dt}$$

Q $x(t) \rightarrow \boxed{\boxed{\frac{d}{dt}} \boxed{\frac{d}{dt}}} \rightarrow y(t)$

The operational definitions of the unit doublets:

$$x(t) * u_1(t) = \frac{dx(t)}{dt}$$

Q



1. x(t) -> d/dt -> d/dt -> ... -> d/dt -> y(t)
2. <=> x(t) -> d^n/dt^n ->y(t)
3. LTI?
4. If so, h(t) = ?

Triplets and beyond!

Def.

$n > 0$

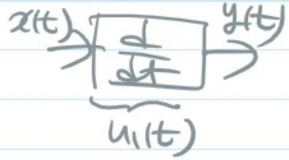
$$u_n(t) = \underbrace{u_1(t) * \cdots * u_1(t)}_{n\ times}$$

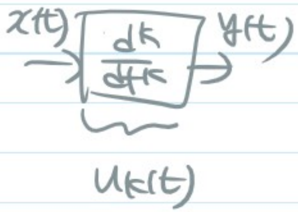
$u_1(t)$ * (n-1) times

$n=1$ $u_1(t) = u_1(t)$

$n=2$ $u_2(t) = u_1(t) * u_1(t)$

$n=k$ $u_k(t) = u_1(t) * \cdots * u_1(t)$

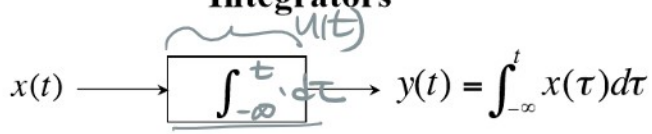




Operational definitions:

$$x(t) * u_n(t) = \frac{d^n x(t)}{dt^n} \qquad (n > 0)$$

Integrators



$u_5(t)$

미분기 5개 거친

$u_{\square}(t)$

$$x(t) \longrightarrow \boxed{\int_{-\infty}^{t} \cdot\, d\tau} \longrightarrow y(t) = \int_{-\infty}^{t} x(\tau)d\tau$$

Q. Is this system LTI? If yes, find the impulse response.

Def. $u_{-1}(t) \equiv u(t)$

(적분기의 impulse response

Operational definition: $x(t) * u_{-1}(t) = \int_{-\infty}^{t} x(\tau)d\tau$

Cascade of n integrators: $(n-1)$ times

serial interconnection

$$u_{-n}(t) = \underbrace{u_{-1}(t) * \cdots * u_{-1}(t)}_{n\ times} \qquad (n > 0)$$

$u_{-1}(t)$

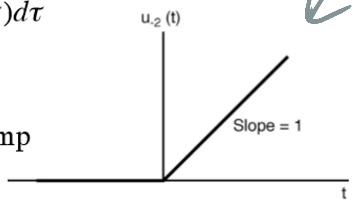
Q. u_{-1}(t)는 BIBO stable한가?
Q. What about u_{-2}(t)?

Integrators (continued)

$u_{-1}(t)$ $u_{-2}(t)$

$$\delta(t) \longrightarrow \boxed{\int} \xrightarrow{u_{-1}(t)} \boxed{\int} \longrightarrow u_{-2}(t)$$

$$u_{-2}(t) = \int_{-\infty}^{t} u_{-1}(\tau)d\tau = \int_{-\infty}^{t} u(\tau)d\tau$$
$$= u(t)\int_{0}^{t} d\tau$$
$$= t \cdot u(t) \quad \text{the unit ramp}$$



More generally, for $n > 0$

$$u_{-n}(t) = \frac{t^{(n-1)}}{(n-1)!}u(t)$$

Mathematical induction의 3 steps:
1. Basis (step)
2. Induction/inductive step

Q. 좌식을 mathematical induction을 이용하여 증명하시오.

$u_5(t)$

$u_{-5}(t)$

적분기 5개 직렬

Q. $u_0(t)$?

$$x(t) \longrightarrow \underbrace{\boxed{\frac{d}{dt}}}_{u_1(t)} \longrightarrow \underbrace{\boxed{\int_{-\infty}^{t} \cdot\, d\tau}}_{u_{-1}(t)} \longrightarrow y(t) = x(t)$$

$= \delta(t)$

$u_0(t) \triangleq \delta(t)$

3. Conclusion
주의! Conclusin은 오직 fininte한 parameter 값에 대해서만 성립함이 증명된 것임.

(Mathematical induction vs. transfinite induction Wikipedia찾아봐요~)

Notation

Def. $u_0(t) = \delta(t)$

Lemma. $u_n(t) * u_m(t) = u_{n+m}(t)$

n and m can be $\pm$

E.g. $u_1(t) * u_{-1}(t) = u_0(t)$

$$\left(\frac{d}{dt}u(t)\right) = \delta(t)$$

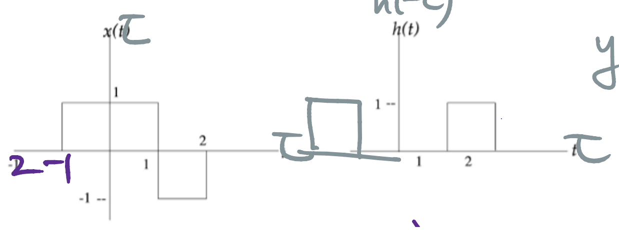
Note: The order of integration and differentiation may not be exchangeable.

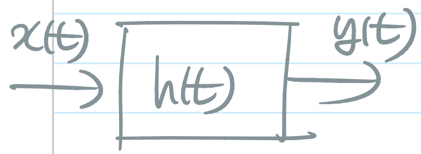
Initial rest 조건...

Example: Calculating $x(t) * h(t) = y(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$

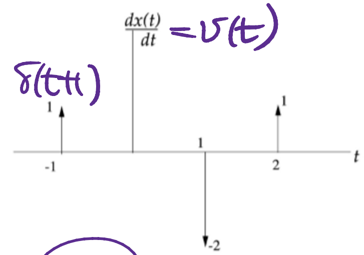
$h(-\tau)$

$y(0) = 0$

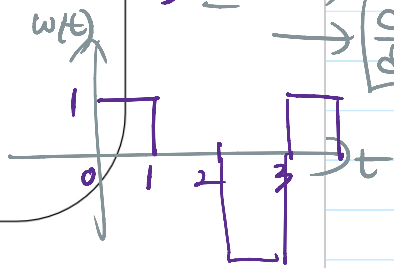


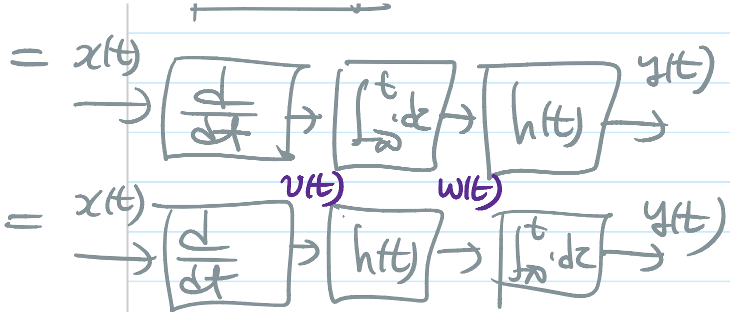


$= x(t)$ ... $y(t)$



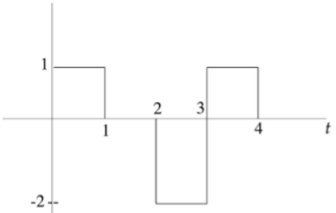
$$\frac{dx(t)}{dt} = \delta(t+1) - 2\delta(t-1) + \delta(t-2)$$





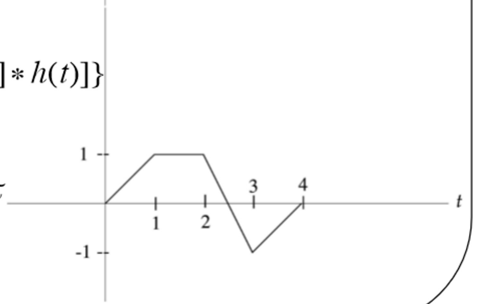
Example (continued)

$$\frac{dx(t)}{dt} * h(t) = h(t+1) - 2h(t-1) + h(t-2)$$

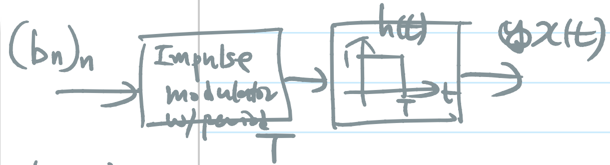


$$x(t) * h(t) = u_{-1}(t) * \{[u_1(t) * x(t)] * h(t)]\}$$

$$= \int_{-\infty}^{t} \left[\frac{dx(\tau)}{d\tau} * h(\tau) \right] d\tau$$



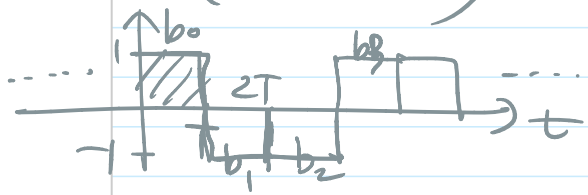
Q. 폭이 같은 두 사각파의 convolution integral은 ?
A. 폭이 2배인 삼각파.
Q. 이를 이용하여 이 example 의 x(t)*h(t)를 구하시오.



$b_n \in \{1, -1\}$

$$\sum_{n=-\infty}^{\infty} b_n \delta(t - nT)$$

$$x(t) = \left(\sum_{n=-\infty}^{\infty} b_n \delta(t-nT) \right) * h(t)$$



A. 폭이 2배인 삼각파.
Q. 이를 이용하여 이 example
의 x(t)*h(t)를 구하시오.

Note: 여기서의 trick은,
rectangular pulse의 미분은 delta들로 나오고 delta입력에 의한 출력은
impulse response로 쉽게 구해지며 구형파의 적분도 쉽다는 것을 이용하여

y(t) = x(t)*h(t)
= x(t)* delta(t) *h(t)
= x(t)* u_0(t) *h(t)
= x(t)*[u_1(t)*u_{-1}(t)]*h(t)
= ([x(t)*u_1(t)]*h(t))*u_{-1}(t)
로 alternate way to compute y(t)를 찾아내는 것이다.

사실 이 문제는 u_1(t)*u_{-1}(t) = u_0(t) = delta (t)을 사용하는 예를
보여주기 위해 위와 같이 풀었으나, 이렇게 복잡하게 할 필요 없이, 폭이 같
은 두 rectangular pulses의 convolution이 삼각파임에 주목하여 inearity
와 time invariance를 이용하면 output y(t)를 쉽게 구할 수 있다.
input이 하나의 pulse의 weighted and uniformly time shifted versions
의 합으로 되어 있으면 이 방법으로 이해하는 것이 아주 유용하다-> 디지
털 통신에서 pulse amplitude modulation을 이해하기 위해 꼭 필요하다.

EECE233 Signals and Systems

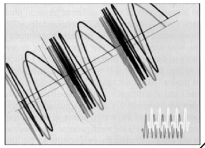
Lecture Note #5

Prof. Joon Ho Cho

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

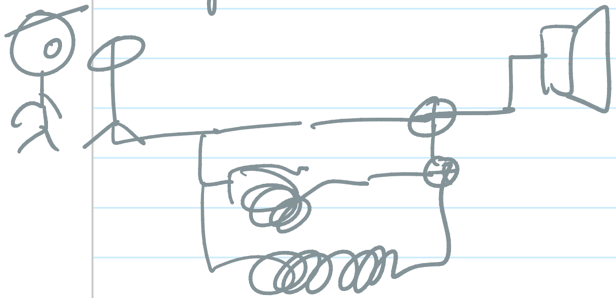
- Covers O & W pp. 177-185,211-225
- Complex Exponentials as Eigenfunctions of LTI Systems
- DT Fourier series
- DT Fourier series examples

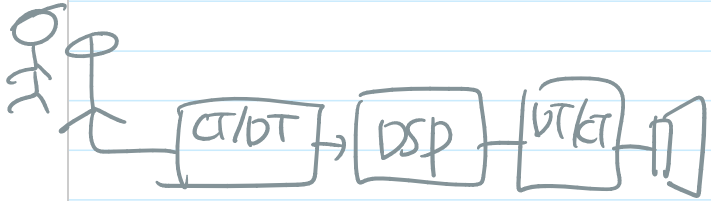


* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

$h(t) = \delta(t) + \frac{1}{2}\delta(t-T) + \frac{1}{4}\delta(t-2T)$

delay line





지금까지 우리는 DT LTI system과 CT LTI system에 대해 time domain에서 그 해석법을 배웠다.

가장 중요한 식은, convolution sum과 integral이었고
이로부터 LTI system은 impulse response로 characterize됨을 알 수 있다.

1. Analysis
LTI system의 출력은 입력의 delayed weighted sum/integral또는 등가적으로 impulse response의 delayed weighted sum임을 보았다.

2. Synthesis
echo mic과제에서 보았듯, 원하는 시스템을 만든다는 것은 원하는 기능을 하도록 impulse response를 만들어 내는 것이다. 또한 commutativity, distributivaty, associativity등을 이용해 synthesize할 수 있음도 보았다.

하지만, 과제에서처럼 주먹 구구로 만들어서는 engineering이라 할 수 없다. 게다가 원하는 impulse response가 무엇인지 알더라도 예를 들어 어떤 미분/차분 방정식으로 어떻게 근사해야 하는가?

Q. 어떤 보다 나은 해석/디자인 방법이 없을까?
A. 힌트는, 우리가 이미 배운 선형대수학에 있다. 행렬의 곱, row space, column space 만 배우면 선형대수가 끝났던가? eigenvalue, eigenvector는 왜 필요한가?

The eigenfunctions $\phi_k(t)$ and their properties

(Focus on CT systems now, but results apply to DT systems as well.)

Def.

$$\phi_k(t) \longrightarrow \boxed{\text{CT System}} \longrightarrow \underbrace{\lambda_k}_{\text{eigenvalue}} \; \underbrace{\phi_k(t)}_{\text{eigenfunction}}$$

Eigenfunction in → same function out with a "gain"

Lemma. If a CT system is linear, then ....

From the superposition property of linear systems:

$$x(t) = \sum_k a_k \phi_k(t) \longrightarrow \boxed{\text{CT linear}} \longrightarrow y(t) = \sum_k \lambda_k a_k \phi_k(t)$$

Now the task of finding response of linear systems is to determine λ_k. The solution is simple, general, and insightful.

$$\underline{x} \in \mathbb{C}^{M\times 1} \longrightarrow \boxed{\text{linear}} \longrightarrow \underline{y} \in \mathbb{C}^{M\times 1}$$

$$\underline{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix} = \sum_{k=1}^{M} x_k \underline{e_k}$$

cf) $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

where $\underline{e_k} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$ ← the k-th position

$$\underline{y} = \mathcal{L}(\underline{x}) = \mathcal{L}\left(\sum_{k=1}^{M} x_k \underline{e_k}\right) = \sum_{k=1}^{M} x_k \underbrace{\mathcal{L}(\underline{e_k})}_{\triangleq\, \underline{h_k}}, \quad \mathcal{L}(\underline{e_k}) \in \mathbb{C}^{M\times 1}$$

$$= \sum_{k=1}^{M} x_k \underline{h_k} = \sum_{k=1}^{M} \underline{h_k} x_k$$

Theorem. If a CT system is linear, then an eigenfunction and the corresponding eigenvalue can be found by solving the following Fredholm integral equation given by

\int h(t,\tau)x(\tau)d\tau =
\lambda\x(t)

for a non-zero function x(t), where h(t,\tau) is called the Kernel of the integral equation,x(t) is called an eigenfunction,
and lamba is called the corresponding eigenvalue.

Remarks.

(i) Existence of an eigenfunction

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(ii) Uniqueness of an eigenfunction
(iii) Countably infinite vs. uncoutably infinite

$$= \sum_{k=1} x_k \underline{h_k} = \sum_{k=1} \underline{h_k} x_k$$

$$= \begin{bmatrix} \underline{h_1} & \underline{h_2} & \cdots & \underline{h_M} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_M \end{bmatrix} = H \underline{x}$$