2023년 10월 5일 목요일 13:00

CM12 231017T

Announcements

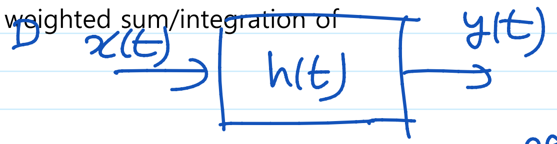
1. Messages from Instructor
   - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
   - ((S))implify and ((V))isualize:
     - essence, generalize, consider extremes,
     - Venn diagram, necessity and sufficienciency, existence and uniqueness/plurality
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications
   - Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

     Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

     출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>

   - Stop and Think
   - Self-Fulfillment and Contribution to Collective
   - Attitude: Exceed expectation.

2. 지난 강의 수정/보완 사항
   - Commutativity of convolution => The output of an LTI system is a weighted sum/integration of delayed input signal.
   - Example:
     - convolution of rectangular pulses with same width



$$y(t)=x(t)*h(t) \triangleq \int_{-\infty}^{\infty} x(\tau)\,h(t-\tau)\,d\tau$$
$$=\int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau$$
$$=h(t)*x(t)$$

ex/ $h(t)=\delta(t)+\frac{1}{2}\delta(t-T)+\frac{1}{4}\delta(t-2T)$

3. HW, Quiz, and Exam
   - HW07 will be assigned by this Friday. Due will be next Tuesday.
     - HW grading policy is changed from HW06.
   - Midterm Exam is approaching.
     - It will be taken on Thursday 10/26 at 19:00-23:00 in LG 102.
     - Open everything
       - Greek letters
     - No communication with human being

- Open everything
    - Greek letters
- No communication with human being
    - You cannot carry any message-exchanging device/stationery when you go to the restroom.

4. 강의 자료
    - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
    - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석
    - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

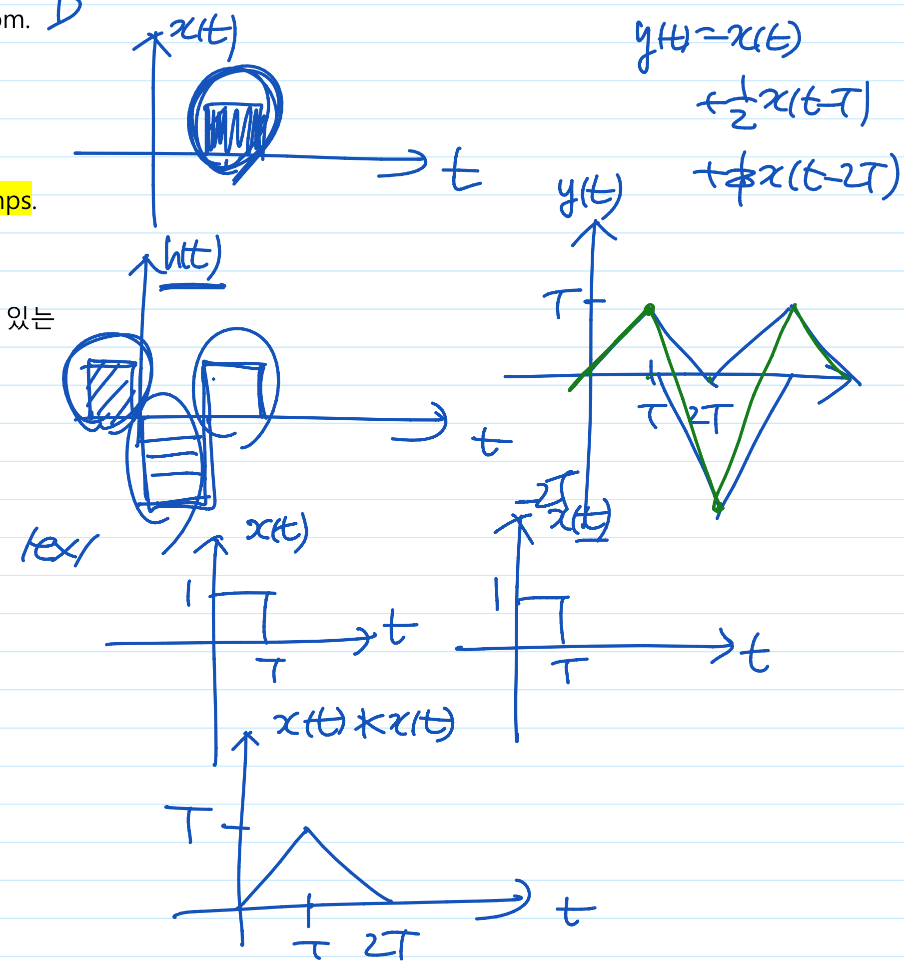
6. Overview (The numbers are the chapter numbers of the textbook)
    1. Signals and Systems
    2. Linear Time-Invariant Systems
    3. Fourier Series Representation of Periodic Signals
    4. Continuous-time Fourier Transform
    5. Discrete-time Fourier Transform
    6. Time and Frequency Characterization of Signals and Systems
    9. Laplace Transform
    10. Z-transform
    7. Sampling
    8. Communication Systems

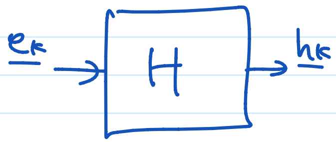
7. Review
    - Def. Generalized functions u_n(t) for n\in Z
        - differentiation of delta(t)
        - integration of delta(t)
        - u_0(t) = delta(t)
    - A convolution of rectangular pulses
        - triangular pulse
        - linear interpolation
    - vector-input vector-output linear system
        - Linear-Algebra version of the sifting property
            - similarity to and differece from the sifting property of the Delta function
        - the I/O relation of a linear system
            - the matrix-vector multiplication
                - same input-output lengths
                - different input-output lengths
            - similarity to and difference from the convolution sum/integral

$= h(t) * x(t)$

$+\frac{1}{4}\delta(t-2T)$

$y(t) = x(t) + \frac{1}{2}x(t-T) + \frac{1}{4}x(t-2T)$





$y = Hx = [h_1\ h_2 \cdots h_M]\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{bmatrix}$

$x = \sum_{k=1}^{M} x_k e_k$ cf) $x[n] = \sum_{k=-\infty}^{\infty} x[k]\delta[n-k]$

$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau$

$y = \sum_{k=1}^{M} x_k h_k$ cf) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

Def. Given $A \in \mathbb{R}^{N \times N}$. If $Av = \lambda v$ then (λ, v)

- the matrix-vector multiplication
  - same input-output lengths
  - different input-output lengths
- similarity to and difference from the convolution sum/integral

8. Preview
   - Linear Algebra for same input-output vector lengths
     - Defs. an eigenvector of a square matrix, and its corresponding eigenvalue
       - non-zero vector?
       - zero eigenvalue?
     - Lemma. The eigenvalues of a square matrix can be found by solving the characteristic equation
       - det(A-lambda I) = 0 and the number of (distinct) eigenvalues
       - algebraic multiplicity of an eigenvalue
     - Corollary. Existence and Uniqueness of (lambda_i, v_i)
       - linearly independent eigenvectors
       - eigen-value
         - geometric multiplicity of an eigenvalue
         - dimension of an eigenspace
   - Q. Why is the eigenstructure of a square matrix important?
     - Q. What about its application to a CT/DT system? Is it useful?
     - A.
     - Q. What about its application to a CT/DT linear system? Is it useful?
     - A.
   - Defs. an eigen-function and the corresponding engen-value of a DT/CT linear system
     - the Fredholm integral equation and the Kernel
   - Defs. an eigen-function and the corresponding engen-value of a DT/CT LTI system
   - Lemmas. complex exponential signal
     - system function
     - Laplace transform, z-transform
- Importance of signals consisting of complex exponential signals
  - ((S)) Signals consisting of complex sinusoidal signals
    - ((S)) Signals consisting of harmonically-related complex sinusoidal signals
      - The set of all CT/DT periodic signals
        - The set of all DT periodic signals
        - DT Fourier series
          - existence
          - uniqueness

Given $A \in \mathbb{R}$: Def. If $A\underline{v} = \lambda \underline{v}$ where $\underline{v} \neq 0$, then $\underline{v}$ is an eigenvector of A & λ is the corresponding (to $\underline{v}$) eigenvalue

an eigenvector

$(\lambda, \underline{v})$

$\det(A - \lambda I) = 0$

λ_1

$A\underline{v} = \lambda_1 \underline{v}$

$\boxed{(A - \lambda_1 I)\underline{v} = 0}$

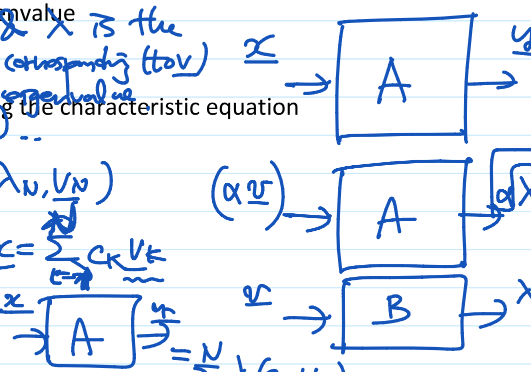
$(\lambda_1 \underline{v}_1)\ (\lambda_2 \underline{v}_2)\ \cdots\ (\lambda_N, \underline{v}_N)$

$\underline{x} = \sum_{k=1}^{N} c_k \underline{v}_k$

$y = \sum_{k=1} x_k h_k$ cf) $y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$

$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$

$A \in \mathbb{R}^{N \times N}$



$= \sum_{k=1}^{N} \lambda_k (c_k v_k)$

(ex) $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

The eigenfunctions $\phi_k(t)$ and their properties

(Focus on CT systems now, but results apply to DT systems as well.)

Def.

linear

$$\phi_k(t) \longrightarrow \boxed{\text{CT System}} \longrightarrow \underbrace{\lambda_k}_{\text{eigenvalue}}\ \underbrace{\phi_k(t)}_{\text{eigenfunction}}$$

$h(t,\tau)$

Eigenfunction in → same function out with a "gain"

Lemma. If a CT system is linear, then ....

From the superposition property of linear systems:

$$x(t) = \sum_k a_k \phi_k(t) \longrightarrow \boxed{\text{CT linear}} \longrightarrow y(t) = \sum_k \lambda_k a_k \phi_k(t)$$

$$\mathcal{L}(x(t)) = \mathcal{L}\Big(\sum_k a_k \phi_k(t)\Big)$$

$$= \sum_k a_k \mathcal{L}(\phi_k(t))$$

$$= \sum_k a_k \lambda_k \phi_k(t)$$

Now the task of finding response of linear systems is to determine λ_k. The solution is simple, general, and insightful.

Fredholm Integral equation.

$$\boxed{\int_{-\infty}^{\infty} h(t,\tau)\phi_k(\tau)\,d\tau = \lambda_k \phi_k(t),\ \forall t.}$$

the Kernel.

$$H\phi_k = \lambda_k \phi_k$$

Theorem. If a CT system is linear, then an eigenfunction and the corresponding eigenvalue can be found by solving the following Fredholm integral equation given by

\int h(t,\tau) x(\tau) d\tau =
\lambda\x(t)

for a non−zero function x(t), where h(t,\tau) is called the Kernel of the integral equation,x(t) is called an eigenfunction, and lamba is called the corresponding eigenvalue.

Remarks.
(i) Existence of an eigenfunction
(ii) Uniqueness of an eigenfunction
(iii) Countably infinite vs. uncoutably infinite

Q1. Given a two−port RLC circuit, how can you compute the output to an arbitrary input?
A1. We can(?) find the output by solving a linear constant coefficient differential equation (LCCDE).

Q2. What if the input is a pure sinusoid? What is the most important property of the output?
A2. The output is a pure sinusoid with the same frequency as the input.

Q3. Do we still need to solve a differential equation when the input is a pure sinusoid?
A3. No. We just need to find the (complex valued) impedance and scale and phase shift the input to obtain the output.

Q4. What is the name of the above mentioned technique to find the output to a pure sinusoid input?
A4. the phasor analysis

Q5. Given an RLC circuit, what is the impedance? How can you find it?
A5. Use KCL and KVL to find the impedance with the mapping of
C −> 1/(j\omega C),
L −> j\omega L,
R − > R

Q6. Why the phasor analysis works?
A6. This question will be answered in this note.

Two Key Questions

1. If they exist, what are the eigenfunctions of a general LTI system?

linear algebra를 기억해보면, matrix마다 eigenvector들의 집합이 다 달랐다. LTI system에서도 일단, 주어진 impulse response마다 해당하는 eigenfunction들의 집합이 다를 것이라 예상할 수 있다. 여기서 질문은, 혹시 모든 LTI system에 대해서 eigenfunction이 되는 그런 signal이 존재할까? 하는 것이다.

2. What kinds of signals can be expressed as superpositions of these eigenfunctions?

Q1. It will turn out that the complex sinusoids (actually not only the sinusoids but also all complex exponential funtions)are all eigenfunctions of ANY LTI system.

Now, the question is: What kind of input signals can be represented by a linear combination of these complex sinusoids?

Q2. True of False: A summation of two periodic signals (e.g. sinusoids) is periodic.

A specific LTI system can have more than one type of eigenfunction

Ex. #1: Identity system

$$x(t) \longrightarrow \boxed{\delta(t)} \longrightarrow x(t) * \delta(t) = x(t)$$

Any function is an eigenfunction for this LTI system.

Linear algebra에서 보았듯, an identity matrix는 any non-zero vector is an eigenvector.

$(\lambda_i: \phi_i(t))$

Ex. #2: A delay

$$x(t) \longrightarrow \boxed{\delta(t-T)} \longrightarrow x(t-T)$$

Any periodic function $x(t) = x(t+T)$ is an eigenfunction.

Linear algebra에서 MNxMN matrix로서 N만큼 down shift하는 matrix 를 생각해보라. 그러면 any downshift invariant vector가 eigenvector가 된다.

Ex. #3: $h(t)$ even

$$\cos\omega t \longrightarrow \boxed{h(t) = h(-t)} \longrightarrow y(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau)\cos[\omega(t-\tau)]d\tau = \int_{-\infty}^{\infty} h(\tau)[\cos\omega t \cdot \cos\omega\tau + \sin\omega t \cdot \sin\omega\tau]d\tau$$

$$= \cos\omega t \underbrace{\int_{-\infty}^{\infty} h(\tau)\cos\omega\tau d\tau}_{H(j\omega)} \quad \text{— } \cos\omega t \text{ is an eigenfunction}$$

Q. 만약 h(t)가 odd이면, sin(wt)가 eigenfunction인가?

많은 경우, complex exponentials라고 하면 s가 pure imaginary일 경우를 가리키나, 그런 경우는 특별히complex sinusoids라고 하기도 한다.

Complex Exponentials are the *only* Eigenfunctions of *any* LTI Systems

$$x(t) = e^{st} \longrightarrow \boxed{h(t)} \longrightarrow \boxed{y(t) = \int_{-\infty}^{+\infty} h(\tau) e^{s(t-\tau)} d\tau}$$

$y(t) = h(t) * x(t)$

$$= \left[\int_{-\infty}^{+\infty} h(\tau) e^{-s\tau} d\tau\right] e^{st}$$

$$= \underbrace{\boxed{H(s)}}_{\text{eigenvalue}}\ \underbrace{e^{st}}_{\text{eigenfunction}}$$

the corresponding eigenvalue an eigenfunction

where s is any complex number that makes the convolution integral well defined.

$s \in \mathbb{C}$ is a constant.

Def. H(s) as a function of s is called the Laplace transform of h(t).

the system function of the LTI system w/ i.r. h(t).

cf) $x(t) = \sum_k a_k e^{s_k t}$

$\Rightarrow y(t) = \sum_k a_k H(s_k) e^{s_k t}$

$$x[n] = z^n \longrightarrow \boxed{h[n]} \longrightarrow y[n] = \sum_{m=-\infty}^{\infty} h[m] z^{n-m}$$

$$= \left[\sum_{m=-\infty}^{\infty} h[m] z^{-m}\right] z^n$$

$$= \underbrace{H(z)}_{\text{eigenvalue}}\ \underbrace{z^n}_{\text{eigenfunction}}$$

where z is any complex number that makes the convolution sum well defined.

Def. H(z) as a function of z is called the z−transform of h(t).

cf) $x[n] = \sum_k a_k (z_k)^n$

$\Rightarrow y[n] = \sum_k a_k H(z_k) (z_k)^n$

$z \in \mathbb{C}$ a constant

Z /zet/ /zi:/

cf) Zak Transform

5

impulse response가 time domain에서 LTI system을 이해하고 해석하고 디자인하는 데 이용된다면, system function은 s또는 z domain에서 LTI system을 이해하고 해석하고 디자인하는 데 이용된다.

System Functions ***H*(*s*) or *H*(*z*)** **– Eigenvalues for now, and more about them later**

((S))

the set of all CT signals

$x(t) = \sum_k a_k e^{s_k t}$

$x(t) = \sum_k a_k e^{j\omega_k t}$

$x(t)$

CT: $x(t) \xrightarrow{e^{st}} \boxed{h(t)} \xrightarrow{H(s)e^{st}} y(t)$

$$H(s) = \int_{-\infty}^{\infty} h(t) e^{-st} dt$$

$$x(t) = \sum a_k e^{s_k t} \longrightarrow y(t) = \sum a_k H(s_k) e^{s_k t}$$

$x(t) = e^{(2-j)t}$

$= e^{2t} e^{-jt}$

DT: $x[n] \xrightarrow{z^n} \boxed{h[n]} \xrightarrow{H(z)z^n} y[n]$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}$$

$$x[n] = \sum a_k z_k^n \longrightarrow y[n] = \sum a_k H(z_k) z_k^n$$

$\sum_k a_k e^{j\omega_k t}$

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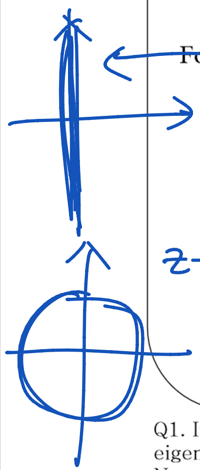
Question 2. What kinds of signals can we represent as "sums" of complex exponentials?



For Now: Focus on restricted sets of complex exponentials

CT: $s = j\omega$ - purely imaginary ,

i.e. signals of the form $e^{j\omega t}$

DT: $z = e^{j\omega}$, i.e. signals of the form $e^{j\omega n}$

z-plane

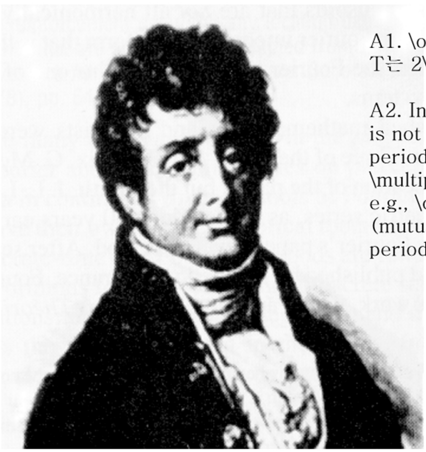
$\Downarrow$

CT & DT Fourier Series and Transforms

Q1. What is the period of e^{j \omega t}?

Q2. What is the period of e^{j \omega n}?

Q1. It turned out that the complex exponential funtions are all eigenfunctions of ANY LTI system.
Now, the question is: What kind of input signals can be represented by a linear combination of these complex exponentials?
Q2. True of False: A summation of two periodic signals (e.g. sinusoids) is periodic.



A1. \omega T = 2\pi에서 주기 T는 2\pi/=omega

A2. In general, e^{j\omega n} is not a periodic function. It is periodic iff \omega is a 2\pi \multiple of a rational number, e.g., \omega = 2\pi M/N (mutually prime). Then, the period is N.

Joseph Fourier (1768-1830)

Q. What happens to H(j\omega) or H(e^{j\omega}) if h(t) or h[n] is real?

A. H(j\omega)=
H(−j\omega)^*.

H(e^{j\omega})=
H(e^{−j\omega})^*=

Q. What is the output of a CT LTI system when x(t) = cos(\omega_0t)?

A. r cos(\omega_0t + theta)

where

r = mag(H(j\omega_0))
theta = phase(H(j\omega_0))

9

Q. Find all DT complex exponential functions with period N.

A. There are only N distinct DT complex exponential functions with period N.

Fourier Series Representation of DT Periodic Signals

- $x[n]$ - periodic with fundamental period N, fundamental frequency

$$x[n+N] = x[n] \text{ for all n} \qquad \textit{and} \qquad \omega_o = \frac{2\pi}{N}$$

- Only $e^{j\omega n}$ which is periodic with period N will appear in the FS

$$\omega N = k2\pi \Leftrightarrow \omega = k\omega_o, \quad k = 0, \pm 1, \pm 2, \cdots$$

- There are only N distinct signals of this form

$$e^{j(k+N)\omega_0 n} = e^{jk\omega_0 n} e^{\overbrace{jN\omega_0 n}^{2\pi n}} = e^{jk\omega_0 n}$$

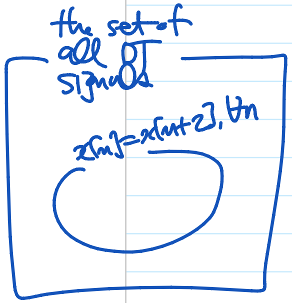
- So we *could* just use $e^{j0\cdot\omega_0 n}, e^{j1\cdot\omega_0 t}, e^{j2\cdot\omega_0 t}, \ldots, e^{j(N-1)\cdot\omega_0 t}$

- However, it is often useful to allow the choice of N consecutive values of k to be *arbitrary* (*E.g.* choose {-(N-1)/2, (N-1)/2} if N is odd and $x[n]$ has definite parity).

⇓

10

((TGTBT))



$x[n] \overset{?}{=} \sum_k a_k (z_k)^n$

$z_k = e^{j\omega k}$???

e^{st} complex exp

$e^{j\omega t}$ " sm —