2023년 10월 19일 목요일 13:12

CM15 231026Th

Announcements

1. Messages from Instructor
   - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
   - ((S))implify and ((V))isualize:
     - essence, generalize, consider extremes,
     - Venn diagram, necessity and sufficiencency, existence and uniqueness/plurality
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications
   - Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

     Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

     출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>
   - Stop and Think; similarity to and difference;
   - Self-Fulfillment and Contribution to Collective
   - Attitude: Exceed expectation.

the limit of $(a_n)_n$

Def. $(a_n)_{n\in\mathbb{N}}$ $a_n \in \mathbb{C}$ $a \in \mathbb{C}$

a_n converges to a if

$\forall \varepsilon > 0, \exists N(\varepsilon) \in \mathbb{N}$ s.t. $n \geq N(\varepsilon) \Rightarrow |a_n - a| < \varepsilon.$

$\boxed{\lim_{n\to\infty} a_n = a}$

2. 지난 강의 수정/보완 사항
   - Convergence and epsilon-N method
3. HW, Quiz, and Exam
   - HW07 will be assigned by this Friday. Due will be next Tuesday.
   - Midterm Exam is approaching.
     - It will be taken tonight at 19:00-23:00 in LG 102.
     - It covers upto CM14.
     - Open everything
       - Greek letters
     - No communication with human being
       - You cannot carry any message-exchanging device/stationery when you go to the restroom.

4. 강의 자료

- Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
- Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석

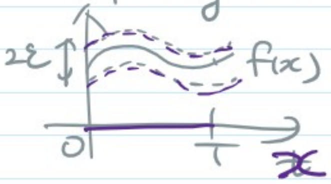
- 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)

1. Signals and Systems
2. Linear Time-Invariant Systems
3. Fourier Series Representation of Periodic Signals
4. Continuous-time Fourier Transform
5. Discrete-time Fourier Transform
6. Time and Frequency Characterization of Signals and Systems
9. Laplace Transform
10. Z-transform
7. Sampling
8. Communication Systems

— converge everywhere
— " almost everywhere

Def. $(f_n(x))_{n\in\mathbb{N}}$, $f(x)$
"uniformly converges" to $f(x)$ if



$N(\varepsilon)\in\mathbb{N}$ s.t. $n\geq N(\varepsilon) \Rightarrow |f_n(x)-f(x)|<\varepsilon$ $\forall x\in[0,T)$

Def. "converges to $f(x)$ in the MS sense" if $\int_0^T |f_n(x)-f(x)|^2 dx \to 0$
squared error

7. Review

- CTFS representation of a CT periodic signal
  - synthesis equation
    - How?
    - Dirichlet conditions
    - Gibbs' phenomenon
  - analysis equation
    - How?
    - Convergence in the MS sense
    - Error energy minimization and Fourier coefficients
      - quadratic form and minimization
      - a Hilbert space = Complete normed linear metric space with inner product
- Application of CTFS to compress a time-limited CT signals using a finitely many coefficients
  - From uncountable infinity to countable infinity under certain constraints

(i) $\int_{\langle T\rangle}|x(t)|dt<\infty$
(ii) BV
(iii)
$\Rightarrow$ (i).

Given $x(t)=x(t+T)$, $\forall t$ $T>0$

Synthesis Eq. $x(t)=\sum_{k=-\infty}^{\infty} a_k \boxed{e^{j\frac{2\pi}{T}kt}}$, ~~$\forall t$~~.

$e^{s_k t}$, $s_k\in\mathbb{C}$

Analysis Eq. $a_k=\frac{1}{T}\int_{\langle T\rangle} x(t)e^{-j\frac{2\pi k t}{T}}dt$

$\frac{2\pi}{T}=\omega_0$

고조파 (harmonics)

Lemma. $\underset{a_k,\,-n,\dots,n}{\text{minimize}}\int_{\langle T\rangle}\left|x(t)-\sum_{k=-n}^{n} a_k e^{j\frac{2\pi k}{T}t}\right|^2 dt$

$=\int_{\langle T\rangle}|x(t)|^2-2\mathrm{Re}\{x(t)\sum_{k=-n}^{n} a_k^* e^{-j\frac{2\pi k}{T}t}\}$

8. Preview

- Generalized Fourier Series for periodic signals involving generalized functions
- Fourier series and Response of LTI system
  - LPF
  - BPF

power series

○ Fourier series and Response of LTI system
- LPF
- BPF
- HPF, etc.

power series

FS

$$= \int_{<T>} |x(t)|^2 - 2\mathrm{Re}\left\{x(t)\sum_{k=-n}^{n} a_k^* e^{j\frac{2\pi k}{T}t}\right\} + \left|\sum_{k=-n}^{n} a_k e^{j\frac{2\pi k t}{T}}\right|^2 dt$$

1st term: $\int_{<T>} |x(t)|^2 dt < \infty$

3rd term: $\int_{<T>} \left(\sum_{k=-n}^{n} a_k e^{j\frac{2\pi k t}{T}} \sum_{k'=-n}^{n} a_{k'}^* e^{-j\frac{2\pi k' t}{T}}\right) dt$

$$= \sum_{k}\sum_{k'} a_k a_{k'}^* T\delta[k-k']$$

$$= \sum_{k=-n}^{n} |a_k|^2 T$$

2nd term: ...

$$f(\underline{a}) = \underline{a}^H T\cdot I\,\underline{a} - 2\mathrm{Re}\left\{\underline{a}^H \begin{bmatrix} \int_{<T>} x(t) e^{-j\frac{2\pi a t}{T}} dt \\ \vdots \end{bmatrix}\right\}$$

+

$$\nabla f|_{a=a^*} = 0 \Rightarrow a_k = \frac{1}{T}\int_{<T>} x(t) e^{-j\frac{2\pi k t}{T}} dt$$

Another (important!) example: Periodic Impulse Train

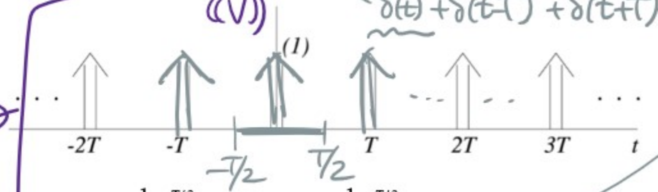
Q. Given

A.

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$ — Sampling function, important for sampling later

(V)

$\delta(t) + \delta(t-T) + \delta(t+T) + \cdots$



Impulse Train

$$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_{-T/2}^{T/2} \delta(t)e^{-jk\omega_0 t}dt$$

$$= \frac{1}{T} \quad \text{for all } k\,!$$

⇓

$$x(t) = \frac{1}{T}\sum_{k=-\infty}^{+\infty} e^{jk\omega_0 t}$$

— All components have:
(1) the same amplitude,
&
(2) the same initial phase.

우변을 한주기 동안 적분(?)하면 1이 된다. 당연하다!

$$a_k = \frac{1}{T}\int_{<T>} x(t) e^{-j\frac{2\pi k t}{T}} dt = \frac{1}{T}, \forall k.$$

여기서 \omega0는 2\pi/T이다. 우변을 한 주기 동안 적분하면 1이 된다. 당연하다?

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Q. DTFS에서 이와 유사한 결과는?

(A few of the) Properties of CT Fourier Series

Lemma

Lemma

Linearity $\quad x(t) \longleftrightarrow a_k,\ y(t) \longleftrightarrow b_k \Rightarrow \alpha x(t) + \beta y(t) \longleftrightarrow \alpha a_k + \beta b_k$

T T

Conjugate Symmetry

$x(t)$ real $\quad\Rightarrow\quad a_{-k} = a_k^*$

T

ex/ $a_1 = 1+j$
$a_{-1} = 1-j$

Proof: $\quad a_{-k} = \frac{1}{T}\int_T x(t)e^{jk\omega_0 t}dt = \left[\frac{1}{T}\int_T x^*(t)e^{-jk\omega_0 t}dt\right]^* = a_k^*$

Corollary

$$\Downarrow \quad a_k = \mathrm{Re}\{a_k\} + j\,\mathrm{Im}\{a_k\}$$

$$= |a_k| e^{j\angle a_k}$$

$\mathrm{Re}\{a_k\}$ is even, $\quad \mathrm{Im}\{a_k\}$ is odd

Corollary

$\Downarrow \; a_k = \mathrm{Re}\{a_k\} + j\,\mathrm{Im}\{a_k\}$

$= |a_k| e^{j\angle a_k}$

$\mathrm{Re}\{a_k\}$ is even , $\mathrm{Im}\{a_k\}$ is odd

or $|a_k|$ is even, $\angle a_k$ is odd

Q. DTFS에서 이와 유사한 property는?

Lemma

- Time shift $x(t) \longleftrightarrow a_k$

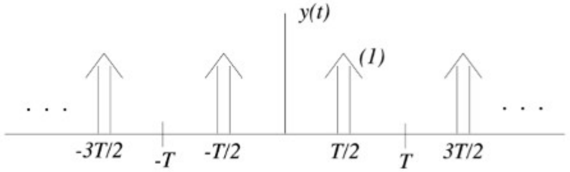
$x(t-t_o) \longleftrightarrow a_k e^{-jk\omega_o t_o} = a_k e^{-jk2\pi t_o/T}$

bk

Introduce a linear phase shift $\propto t_o$

Example: Shift by half period

$$y(t) = x(t - T/2) \xleftrightarrow{e^{-jk\omega_o T/2} = e^{-jk\pi}} a_k e^{-jk\pi} = (-1)^k a_k$$



$$y(t) \longleftrightarrow (-1)^k a_k \qquad \left(a_k = \frac{1}{T} = \text{F.C. of} \sum_{n=-\infty}^{+\infty} \delta(t - nT)\right)$$

$$\| $$

$$\frac{(-1)^k}{T}$$

Def. $\int_{-\infty}^{\infty} |x(t)|^2 dt \triangleq E_{xx}$

$x(t) = x(t+T), \forall t \rightarrow P_{xx} \triangleq \frac{1}{T}\int_{(T)} |x(t)|^2 dt$

- **Thm. Parseval Relation**

$$P_{xx} \triangleq \underbrace{\frac{1}{T}\int_T |x(t)|^2 \, dt}_{\text{Average signal power}} = \sum_{k=-\infty}^{+\infty} \underbrace{|a_k|^2}_{\text{Power in the } k_{th} \text{ harmonic}}$$

Q. DTFS에서 이와 유사한 결과는?

Energy is the same whether measured in the time-domain or the

Q1. Given

Average signal power

Power in the k_{th} harmonic

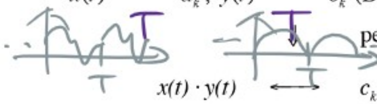
Q1. Given x(t) = \sum_k a_k e^{2pi kt/T} and y(t) = \sum_k b_k e^{2pi kt/T}, find c_k of

z(t) = x(t)y(t) = \sum_k c_k e^{2pi kt/T}.

Energy is the same whether measured in the time-domain or the frequency-domain

- **Thm. Multiplication Property**

$x(t) \longleftrightarrow a_k$, $y(t) \longleftrightarrow b_k$ (Both $x(t)$ and $y(t)$ are periodic with the same period T)



$$x(t)\cdot y(t) \longleftrightarrow c_k = \sum_{l=-\infty}^{+\infty} a_l b_{k-l} = a_k * b_k$$

T

Proof: $$\underbrace{\sum_l a_l e^{jl\omega_o t}}_{x(t)} \cdot \underbrace{\sum_m b_m e^{jm\omega_o t}}_{y(t)} = \sum_{l,m} a_l b_m e^{j(l+m)\omega_o t} \xrightarrow{l+m=k} \sum_k \underbrace{\left[\sum_l a_l b_{k-l}\right]}_{c_k} e^{jk\omega_o t}$$

Q. 같은 주기를 갖는 두 주기함수의 곱은 주기함수인가? 그렇다면 주기는? 주기함수라면 이 주기함수의 CTFS는?

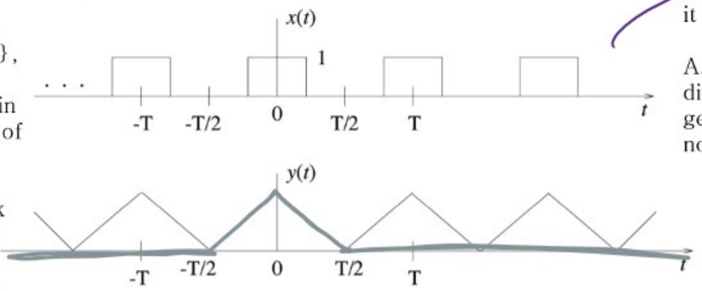
Q. DTFS에서 이와 유사한 결과는?

Q2. Given x(t) = \sum_k a_k e^{2pi kt/T} and y(t) = \sum_k b_k e^{2pi kt/T}, find the time-domain description of

z(t) = \sum_k (a_k b_k) e^{2pi kt/T} in terms of x(t) and y(t).

Thm. Periodic Convolution

Given **$x(t)$, $y(t)$ periodic with period T**



$$x(t) * y(t) = \int_{-\infty}^{+\infty} x(\tau) y(t-\tau) d\tau$$ — not very meaningful

E.g. If both $x(t)$ and $y(t)$ are positive definite, then $x(t) * y(t) = \infty$

Q. What is the convolution of two periodic signals of the same period? Is it well defined?

A. x(t)*y(t) diverges in general. So, it is not well defined.

! $x(t) * y(t) = \infty \quad \forall t$

Def. $x(t) = x(t+T), \forall t \rightarrow a_k$

$y(t) = y(t+T), \forall t \rightarrow b_k$

$z(t) = x(t) \otimes y(t)$ the periodic convolution of two periodic signals w/ comm. T.

Lemma. $z(t) = z(t+T), \forall t \rightarrow c_k$

~~Thm~~ Thm.

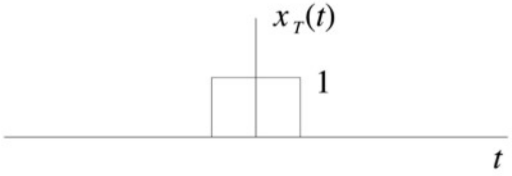
Periodic Convolution (continued)

Periodic Convolution (continued)

Periodic convolution: Integrate over *any* one period
(**e.g.** -T/2 to T/2)

$$z(t) = \int_{-T/2}^{T/2} x(\tau)y(t-\tau)d\tau = \int_{-\infty}^{\infty} x_T(\tau)y(t-\tau)d\tau$$

where $$x_T(t) = \begin{cases} x(t) & -T/2 < t < T/2 \\ 0 & \text{otherwise} \end{cases}$$



Periodic convolution
의 두가지 해석법

1. convolution에서 한 주기 동안만 적분함.

2. 두 함수중 하나를 한 주기만 취한 후, 일반적 convolution을 함.

Periodic Convolution (continued) Facts

1) $z(t)$ is periodic with period T (why?)

From Lecture #2, $x(t)=x(t+T) \to y(t)=y(t+T)$ for LTI systems.
In this example, treat $y(t)$ as the input and $x_T(t)$ as $h(t)$.

엄밀히 말해 time invariant 이기만 하면
주기함수 input leads to 주기함수 output of the same period

2) Doesn't matter what period over which we choose to integrate:

$$z(t) = \int_T x(\tau)y(t-\tau)d\tau = x(t)\boxed{\otimes}y(t)$$

3) $$x(t) \longleftrightarrow a_k,\ y(t) \longleftrightarrow b_k,\ z(t) \longleftrightarrow c_k$$

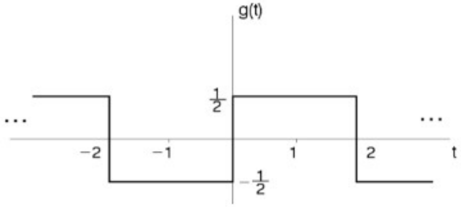
Q. 세 주기함수 of the same period 의 CTFS들의 관계는?

$$c_k = \frac{1}{T}\int_T z(t)e^{-jk\omega_0 t}dt = \frac{1}{T}\int_T\left(\int_T x(\tau)y(t-\tau)d\tau\right)e^{-jk\omega_0 t}dt$$

$$= \int_T \underbrace{\left(\frac{1}{T}\int_T y(t-\tau)e^{-jk\omega_0 \overbrace{(t-\tau)}^{t'}}dt\right)}_{b_k} x(\tau)e^{-jk\omega_0 \tau}d\tau$$

$$= \int_T b_k x(\tau)e^{-jk\omega_0 \tau}d\tau = Ta_k b_k$$

Example: Periodic square wave with zero average



$T = 4$

$\omega_o = \frac{\pi}{2}$

이 신호가
여러 sine 함수들의
linear
combination으로
분해되어 보이는가?

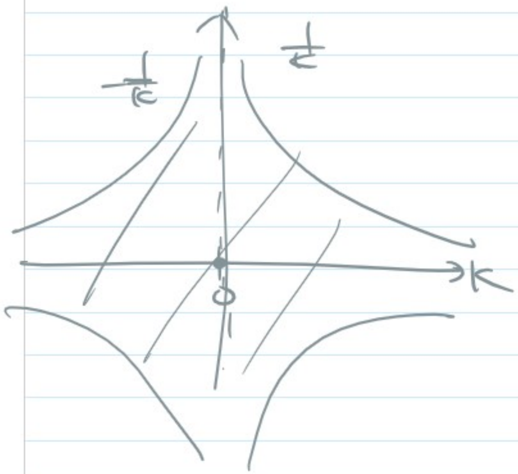
$g(t) \longleftrightarrow d_k \qquad d_k = \int_{-2}^{2} g(t)\, e^{jk\omega_o t} dt$

$$= \begin{cases} \dfrac{\sin(k\pi/2)}{k\pi} e^{-jk\pi/2} & \textit{for } k \neq 0 \;\; \textit{decays as } \dfrac{1}{k} \\ 0 & \textit{for } k = 0 \;\; \textit{no dc component} \end{cases}$$

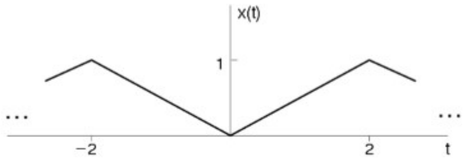
d_k를 MATLAB으
로 그려보라.

A practical example: Pentium 4 processor, clock frequency 3.2 GHz, third harmonic 9.6 GHz, fifth harmonic 16 GHz, ... challenging to avoid cross-talk.

여기서 모든 짝수
FS coefficients는
0이 된다.



Example: Periodic triangle wave



$T = 4$

$\omega_o = \frac{\pi}{2}$

아래 신호가
여러 cosine함수
들의 linear
combination으로
분해되어 보이는
가?

$$\textit{Clearly,} \qquad x(t) = \int_{-\infty}^{t} g(\tau)\, d\tau$$

$$\textit{Then} \qquad x(t) = \sum_k a_k e^{jk\omega_o t} = \int_{-\infty}^{t} \sum_k d_k e^{jk\omega_o \tau} d\tau$$

$$= \sum_k \frac{d_k}{jk\omega_o} e^{jk\omega_o t}$$

Q. square wave 둘
을 periodic
convolution하면?

$a_k = \dfrac{d_k}{jk\omega_o}$ *decays as* $\dfrac{1}{k^2}$, faster than the square wave, because

a_k를 MATLAB으

$\sum_k jk\omega_o$

$a_k = \frac{d_k}{jk\omega_o}$ *decays as* $\frac{1}{k^2}$, faster than the square wave, because the edges are smoother.

a_k를 MATLAB으로 그려보라.