LN07 FS and Response of LTI Systems

2023년 10월 26일 목요일 14:13

CM16 231031T

Announcements

1. Messages from Instructor
    - Goal of Life: Self-Fulfillment and Contribution to Collective (자아실현, 공동체 기여)
        - Attitude: Exceed the expectations! (기대를 넘어서라!)
    - "아는 것이 힘이다. 배우고야 무슨 일이든 한다." -심훈, 상록수-
    - 학습 (배우고 익히기)
        - Theory Learning (이론 배우기)
        - Problem Solving (문제 풀기)
    - How to Learn
        - Stop and Think; commonality, similarity, and difference
    - How to Practice

      Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

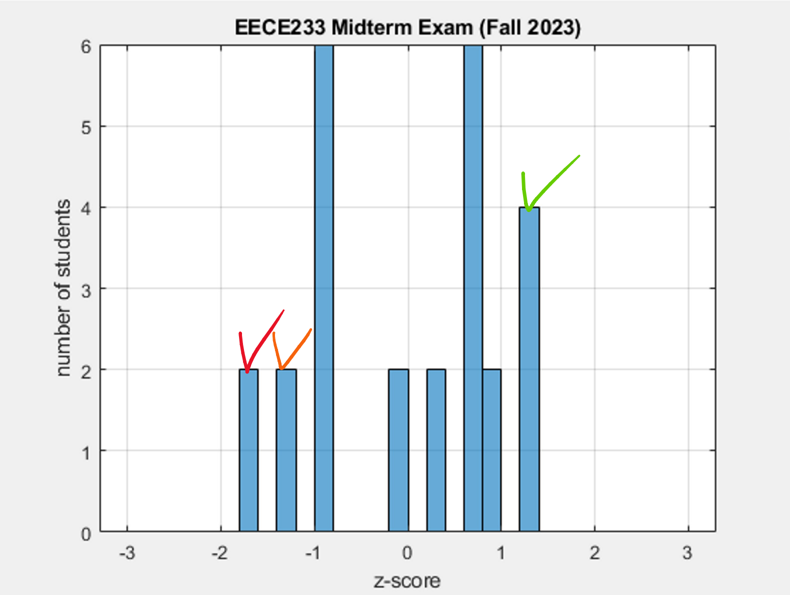
      Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

      출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>

    - ((S))implify and ((V))isualize:
        - essence, generalize, consider extremes,
        - Venn diagram, necessity and sufficiencieny, existence and uniqueness/plurality
            - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
        - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications

2. 지난 강의 수정/보완 사항
    - Derivation of the notion of Periodic Convolution from the multiplication property of Fourier Series
    - Derivation and Interpretation of Parseval's relation: Substitute only one.

3. HW, Quiz, and Exam

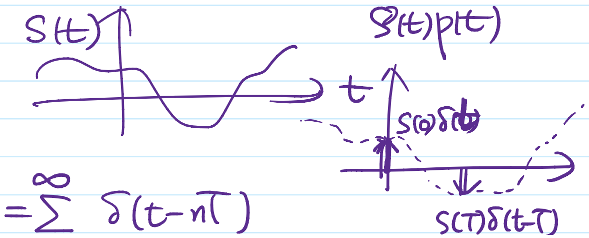
- HW09 will be assigned by this Friday. Due will be next Tuesday.
- Midterm Exam is graded.
  - Total = 100
  - (Tentative) Mean = 61.00, Stdev = 16.21
  - 

  - Graded Exam will be returned on this Thursday.

4. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

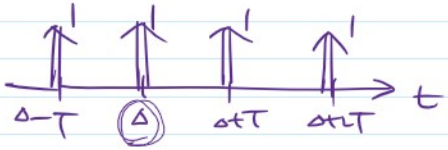
5. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform

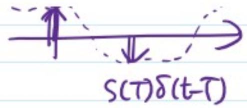


$p(t)=\sum^{\infty} \delta(t-nT)$

6. Time and Frequency Characterization of Signals and Systems
9. Laplace Transform
10. Z-transform
7. Sampling
8. Communication Systems



$$p(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$$
$$\overset{?}{=}\sum_{k=-\infty}^{\infty}a_k e^{j\frac{2\pi}{T}kt} \Rightarrow a_k=\frac{1}{T},\ \forall k\in\mathbb{Z}$$



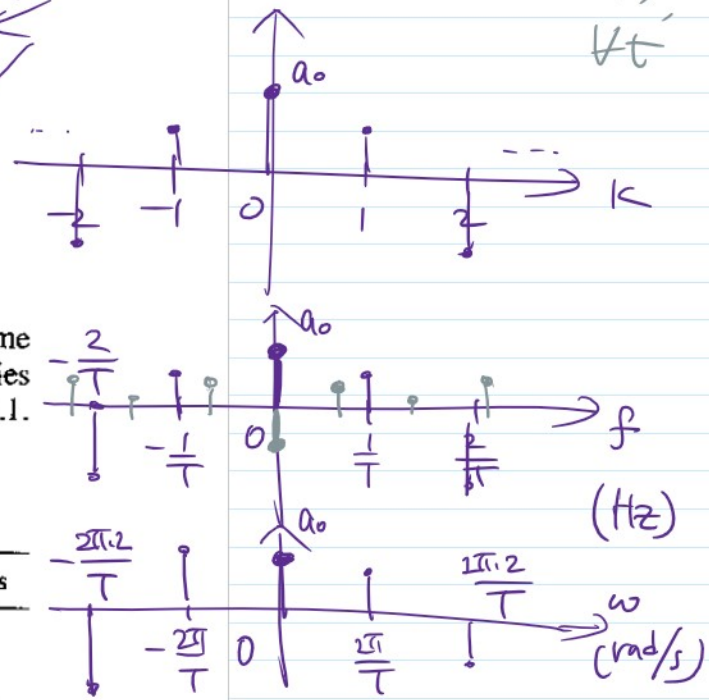
7. Review
   - CTFS
     - A periodic impulse train as a periodic signal has all harmonic components with the same intensity.
     - Shift by half period
   - Properties of DTFS and CTFS
     - The concept of the frequency domain (FD)
     - The FS coefficients and the FD

Q. $\boxed{x(t)=x(t+T),\ \forall t}$ a_k

$y(t)=y(t+T),\ \forall t$ b_k

$$z(t)=\sum_k [a_k b_k e^{j\frac{2\pi k t}{T}}]$$
$$=z(t+T),\ \forall t$$

$w(t)=w(t+T)$ $\forall t$



3.7 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

There are strong similarities between the properties of discrete-time and continuous-time Fourier series. This can be readily seen by comparing the discrete-time Fourier series properties summarized in Table 3.2 with their continuous-time counterparts in Table 3.1.

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$x[n]$, $y[n]$ Periodic with period N and fundamental frequency $\omega_0=2\pi/N$	a_k, b_k Periodic with period N
1 Linearity	$Ax[n]+By[n]$	Aa_k+Bb_k
2 Time Shifting	$x[n-n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
3 Frequency Shifting	$e^{jM(2\pi/N)n}x[n]$	a_{k-M}
4 Conjugation	$x^*[n]$	a^*_{-k}
5 Time Reversal	$x[-n]$	a_{-k}
6 Time Scaling	$x_{(m)}[n]=\begin{cases}x[n/m], & \text{if } n \text{ is a multiple of } m\\ 0, & \text{if } n \text{ is not a multiple of } m\end{cases}$ (periodic with period mN)	$\frac{1}{m}a_k$ (viewed as periodic with period mN)
7 Periodic Convolution	$\sum x[r]y[n-r]$	Na_kb_k

		$\begin{cases} 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	m (with period mN)
7	Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n-r]$	$Na_k b_k$
8	Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
9	First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
10	Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix} \text{finite valued and periodic only} \\ \text{if } a_0 = 0 \end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
11	Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k \rvert = \lvert a_{-k} \rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
12	Real and Even Signals	$x[n]$ real and even	a_k real and even
13	Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
14	Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

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Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$

TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES

	Property	Section	Periodic Signal	Fourier Series Coefficients
			$\left.\begin{array}{l} x(t) \\ y(t) \end{array}\right\}$ Periodic with period T and fundamental frequency $\omega_0 = 2\pi/T$	a_k b_k
1	Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$

$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}$

$y(t) = x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}$

	Property	Section	Periodic Signal	Fourier Series Coefficients
			$x(t)$ } Periodic with period T and	a_k
			$y(t)$ } fundamental frequency $\omega_0 = 2\pi/T$	b_k
1	Linearity	3.5.1	$Ax(t) + By(t)$	$Aa_k + Bb_k$
2	Time Shifting	3.5.2	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
3	Frequency Shifting		$e^{jM\omega_0 t}x(t) = e^{jM(2\pi/T)t}x(t)$	a_{k-M}
4	Conjugation	3.5.6	$x^*(t)$	a^*_{-k}
5	Time Reversal	3.5.3	$x(-t)$	a_{-k}
6	Time Scaling	3.5.4	$x(\alpha t), \alpha > 0$ (periodic with period T/α)	a_k
7	Periodic Convolution		$\int_T x(\tau)y(t-\tau)d\tau$	$Ta_k b_k$
8	Multiplication	3.5.5	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
9'	Differentiation		$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk\frac{2\pi}{T}a_k$
10'	Integration		$\int_{-\infty}^{t} x(t)\,dt$ (finite valued and periodic only if $a_0 = 0$)	$\left(\frac{1}{jk\omega_0}\right)a_k = \left(\frac{1}{jk(2\pi/T)}\right)a_k$
11	Conjugate Symmetry for Real Signals	3.5.6	$x(t)$ real	$a_k = a^*_{-k}$; $\Re e\{a_k\} = \Re e\{a_{-k}\}$; $\Im m\{a_k\} = -\Im m\{a_{-k}\}$; $\lvert a_k\rvert = \lvert a_{-k}\rvert$; $\measuredangle a_k = -\measuredangle a_{-k}$
12	Real and Even Signals	3.5.6	$x(t)$ real and even	a_k real and even
13	Real and Odd Signals	3.5.6	$x(t)$ real and odd	a_k purely imaginary and odd
14	Even-Odd Decomposition of Real Signals		$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real]; $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\Re e\{a_k\}$; $j\Im m\{a_k\}$

$$\cos(2\pi f_c t + \theta) = \mathrm{Re}\{e^{j(2\pi f_c t+\theta)}\}$$

편리함.

phasor analysis

$$y(t) = x(-t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}$$
$$= \sum_{k'=-\infty}^{\infty} a_{-k'} e^{j\frac{2\pi k' t}{T}}$$
$$x^*(t) =$$
$$x^*(-t) =$$

15'

Parseval's Relation for Periodic Signals

$$P_{xx} = \frac{1}{T}\int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

$$\text{LHS} = \frac{1}{T}\int_{<T>} x(t)\,x^*(t)\,dt$$
$$= \frac{1}{T}\int_{<T>} x(t)\left(\sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}\right)^* dt$$
$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} a_k^* \underbrace{\int_{<T>} x(t)\, e^{-j\frac{2\pi k t}{T}} dt}$$
$$= \frac{1}{T}\sum_{k=-\infty}^{\infty} a_k^* T \cdot a_k \triangleq |a_k| = \sum_{k=-\infty}^{\infty} |a_k|^2 = \text{RHS}$$

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi k t}{T}}$$

$$T = \int_{<T>} \left|e^{j\frac{2\pi k t}{T}}\right|^2 dt$$

8. Preview
   - Fourier series and Response of DT/CT LTI system
     - The concept of the frequency domain (FD)
       - radian/sec vs. Hz
       - difference between the FD of CT signals and the FD of DT signal
     - LPF, BPF, HPF, notch filter, etc.
     - ideal LPF, BPF, HPF, etc.

- difference between the FD of CT signals and the FD of DT signal
- LPF, BPF, HPF, notch filter, etc.
- ideal LPF, BPF, HPF, etc.

$$|a_k|^2 = \frac{1}{T}\int_{<T>} \left|a_k e^{j\frac{2\pi k t}{T}}\right|^2 dt$$

$$= \frac{1}{T}\sum_{k=-\infty} a_k^* \, T \, a_k \quad = \sum_{k=-\infty} |a_k|^2 = RHS$$

EECE233 Signals and Systems

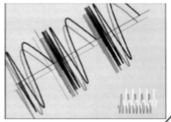
지난 Lecture Note #6에서는 DT periodic signal과 CT periodic signal을 각각 complex sinusoidal functions의 linear combination으로 나타내는 법, 즉 DT/CT Fourier Series Representation을 배웠다.

Lecture Note #7

Prof. Joon Ho Cho
(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

Q. 왜 complex sinusoids의 linear combination으로 나타내려 했었나?

- Covers O & W pp. 226-250
- Fourier Series and LTI Systems
- Frequency Response and Filtering
- Examples and Demos



* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

1

A. complex sinusoids가 모든 DT/CT LTI system의 eigenfunction 후보였기 때문이었다. 즉, output은 input sinusoidal components의 linear combination으로 간단히 나타나기 때문이었다.

Review: The Eigenfunction Property of Complex Exponentials (from Lecture #5)

CT: $e^{st} \longrightarrow \boxed{h(t)} \longrightarrow H(s)e^{st}$

$$H(s) = \int_{-\infty}^{+\infty} h(t)e^{-st}\,dt$$

the system function of a CT LTI system = the Laplace TF of its impulse response

DT: $z^n \longrightarrow \boxed{h[n]} \longrightarrow H(z)z^n$

$$H(z) = \sum_{n=-\infty}^{+\infty} h[n]z^{-n}$$

the system response of a DT LTI system = the z−TF of its impulse response

2

of its impulse response

2

Q. 그렇다면 periodic signal이 입력일 때 LTI system의 출력은? 그 의미는?

Fourier Series and LTI Systems

$$x(t)=\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \longrightarrow \boxed{h(t)} \longrightarrow y(t)=\sum_{k=-\infty}^{\infty} H(jk\omega_0)a_k e^{jk\omega_0 t}$$

$y(t)=\sum_{k=-\infty}^{\infty} a_k H(jk\omega_0) e^{jk\omega_0 t}$

$$a_k \longrightarrow \underbrace{H(jk\omega_0)}_{\text{"gain"}} a_k$$

$H(jk\omega_o)=|H(jk\omega_o)|\,e^{j\angle H(jk\omega_o)}$,

includes both amplitude & phase

$$x[n]=\sum_{k=<N>} a_k e^{jk\omega_0 n} \longrightarrow \boxed{h[n]} \longrightarrow y[n]=\sum_{k=-\infty}^{\infty} H(e^{jk\omega_o})a_k e^{jk\omega_0 n}$$

$$a_k \longrightarrow \underbrace{H(e^{jk\omega_o})}_{\text{"gain"}} a_k$$

$H(e^{jk\omega_o})=|H(e^{jk\omega_o})|\,e^{j\angle H(e^{jk\omega_o})}$,

includes both amplitude & phase

3

For periodic signal inputs, only the system function evaluated at j times the Harmonic frequencies do matter.

Def. Frequency Response of an LTI System

named as such because complex sinusoid input e^{j\omega t} with frequency \omega에 대한 응답은 input에 gain으로 H(j\omega)를 곱한 것이 되기 때문이다.

$(s=j\omega)$

$$e^{j\omega t} \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega)e^{j\omega t}$$

$\omega \in (-\infty, \infty)$

CT Frequency response: $H(j\omega)=\int_{-\infty}^{+\infty} h(t)e^{-j\omega t}dt$

Frequency response of a CT LTI system = System function evaluated on the line s = j\omega for −\infty < \omega < \infty

$$e^{j\omega n} \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow H(e^{j\omega})e^{j\omega n}$$

$\omega \in (-\infty, \infty)$

$\in [0, 2\pi]$

$(z=e^{j\omega})$

$H(e^{j\omega}) = H(e^{j(\omega+2\pi)}),\ \forall\omega$

DT Frequency response: $H(e^{j\omega})=\sum_{n=-\infty}^{+\infty} h[n]e^{-j\omega n}$

Q. Existence for all f?
A. Not for all f.
Q. Find a sufficient condition for the system function to exist?
A1. If h(t) is absolutely integrable then H(j\omega) exists for all \omega.
A2. If h(t) is square integrable then H(j\omega) exists (in some sense which will be discussed later) for all \omega.

지금까지는 complex exponential functions가 CT/DT LTI systems의 eigenfunction이라는 것을 이용하는데 있어, 특히 입력이 주기함수인 경우만을 생각하였다.

지금부터는 다시 조금 더 일반적인 경우를 생각한다. 즉, 입력이 반드시 harmonically related인 것은 아닌 몇개의 complex sinusoids인 경우를 생각한다.

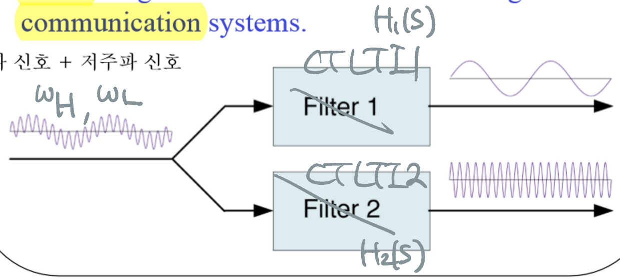
Signal Processing with Filters

Separation of narrowband signals

The input consists of a sum of two narrowband signals, for example, broadcast signals from two different radio stations. Need to separate them for a clear reception.

Filter design — a main task in constructing communication systems.

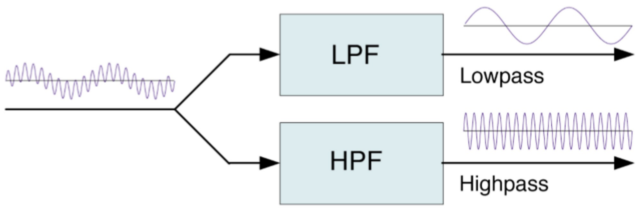
ex1. 고주파 신호 + 저주파 신호



$H_1(j\omega_H) \simeq 0$ (ϵ)

$|H_1(j\omega_L)| \gg |H_1(j\omega_H)|$

Example: Using LPF and HPF to separate signals



Filter design becomes more challenging if the two signals are close in frequency range.

Low Pass Filter:
낮은 주파수만 pass시키고 그 이외는 stop시킨다. 이를 위해서는 낮은 주파수에서의 frequency response의 magnitude가 나머지 주파수에서의 magnitude에 비해 크면된다.

High Pass Filter일때도 유사하게 된다.

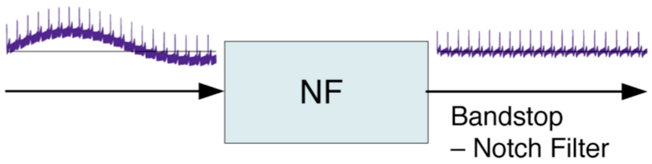
Extraction of narrowband signals from wideband noise – BPF

ex2. 광대역 잡음 + 협대역 신호

예를 들어 사람의 목소리는 (주기함수는 아니지만 후에 보면 알게 되듯.... 이하 성립한다.)80 Hz에서 1200Hz정도인데 가청주파수는 20 Hz 에서 20 KHz 정도이다.
목적이 사람의 목소리만 녹음하는 것이라면 80-1200정도만 pass시키는 filter를 마이크 다음에 달아서 그 output만 녹음하면 불필요한 고주파를 차단하고 녹음할 수 있을 것이다. 주위에 고주파 잡음이 없어도, 마이크 후에 증폭기가 있는데 여기서 백색잡음이 들어간다. 백색잡음 중 고주파 부분이라도 stop시키는 음질이 향상될 것이다.

Filtering out unwanted narrowband signal, for example, 60-Hz power lines — Notch Filter

ex3. 광대역 신호+협대역 잡음

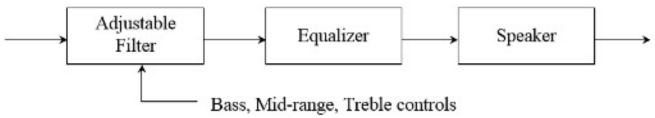


음성 방송 위주의 오디오 시스템이라면 80Hz이상에서만 사람 목소리가 있으므로 60 Hz에서만 notch 시키면 power line에서의 잡음을 줄일 수 있다.

Frequency Shaping and Filtering

- By choice of $H(j\omega)$ (or $H(e^{j\omega})$) as a function of ω, we can *shape* the frequency composition of the output
  - Preferential amplification
  - Selective filtering of some frequencies

Example #1: Audio System



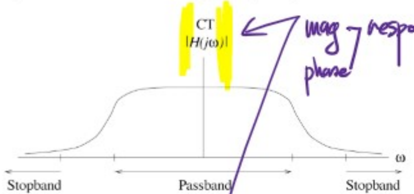
For audio signals, the amplitude is much more important than the phase.

equalizer는 말 그대로는 frequency response가 frequency selective한 경우 이 후에 연결하여 selective 한 response를 전 주파수 범위에서 flat 또는 equal하게 만드는 것, equalizer이다. 그러나, equalizer는 자유롭게 각 주파수 response의 크기를 unequal하게도 할수 있는 기능이 있는 것을 equalizer라 부른다.

Example #2: Frequency Selective Filters

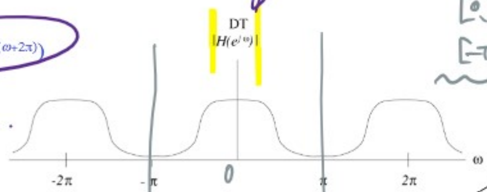
- Filter out signals outside of the frequency range of interest

Lowpass Filters: Only show amplitude here.



mag — response
phase

Note for DT: $H(e^{j\omega}) = H(e^{j(\omega+2\pi)})$

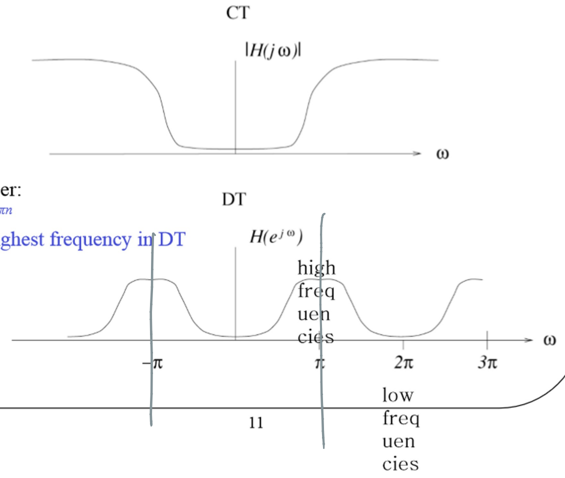


[0, 2π)
[−π, π)

Q. 그런데 CT LTI와 DT LTI의 frequency response는 중대한 차이가 있다. DT LTI의 경우 그것이 (   )하다는 것이다.

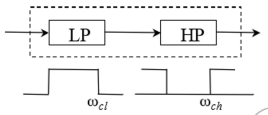
Highpass Filters

Highpass Filters

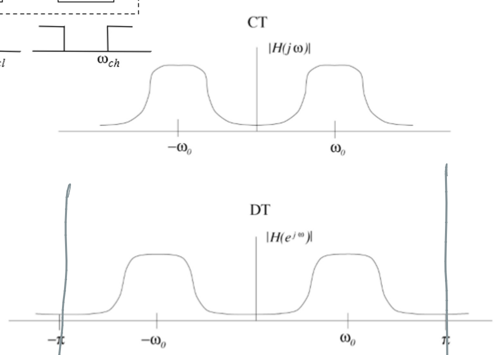


Remember:
$(-1)^n = e^{j\pi n}$
— π = highest frequency in DT

Bandpass Filters



BPF if $\omega_{cl} > \omega_{ch}$



Demo: Filtering effects on audio signals

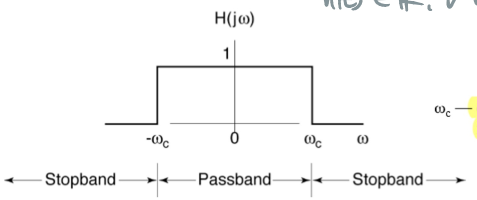
Ideal*ized* Filters

Def. Ideal LPF with cutoff frequency \omega_c

Def. pasband, stopband of an ideal LPF

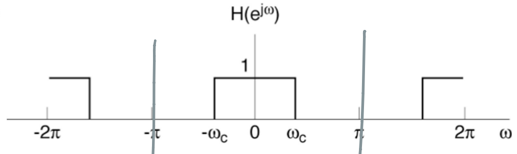
$h(t) \in \mathbb{R},\ \forall t.$

CT



ω_c — cutoff frequency

DT



Note: $|H| = 1$ and $\angle H = 0$ for the ideal filters in the passbands, no need for the phase plot.

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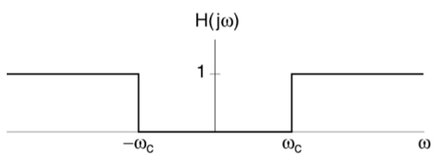
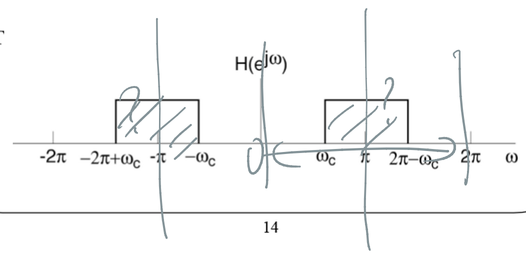
Def. Ideal HPF with cutoff frequency \omega_c

CT



DT



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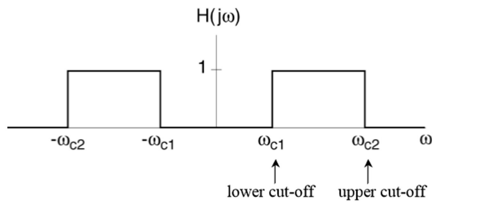
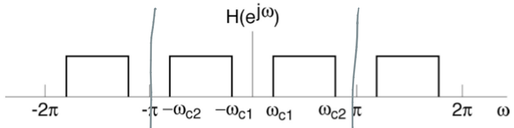
Def. Ideal BPF with lower cutoff and upper cutoff frequencies .... and ..

CT



DT



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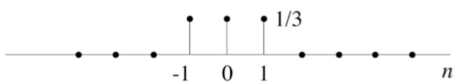
아직 우리는 how to construct a LPF/BPF/HPF에 대해서는 배우지 않았고
다만 주어진 CT/DT LTI system의 frequency response를 계산하여 그
magnitude response의 모양에 따라 LPF/BPF/HPF를 분류하는 것만 배웠다.
일단 여기서도 주어진 필터가 어떤 filter인지만 보고, 어떻게 설계하는 지는 차
차 배우기로 하자.

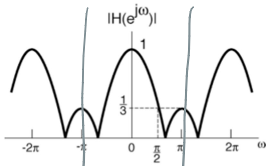
Example #3: DT Averager/Smoother

$$y[n] = \frac{1}{3}\{x[n-1] + x[n] + x[n+1]\}$$

$$h[n] = \frac{1}{3}\{\delta[n-1] + \delta[n] + \delta[n+1]\}$$



$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n} = \frac{1}{3}[e^{-j\omega} + 1 + e^{j\omega}] = \frac{1}{3} + \frac{2}{3}\cos\omega$$



A LPF

완벽한 LPF는 아니지만,
이렇게 부를 수도 있겠다.

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LCCDE 에서 y[n]만 나오고 다른 delayed 된
y[]들이 나오지 않는다는 뜻

Example #4: Nonrecursive DT (*FIR*) filters

FIR: Finite

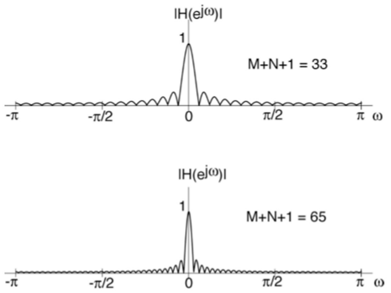
LCCDE 에서 y[n]만 나오고 다른 delayed 된 y[]들이 나오지 않는다는 뜻

Example #4: Nonrecursive DT (*FIR*) filters

FIR: Finite Impulse Response 말 그대로 impulse response가 finite duration 동안만 nonzero를 가지는 경우 이렇게 부른다.

$$y[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M} x[n-k] \longrightarrow h[n] = \frac{1}{N+M+1}\sum_{k=-N}^{M}\delta[n-k]$$

$$H(e^{j\omega}) = \frac{1}{N+M+1}\sum_{k=-N}^{M} e^{-jk\omega} = \frac{1}{N+M+1}e^{j\omega(N-M)/2}\frac{\sin[\omega(M+N+1)/2]}{\sin(\omega/2)}$$



Rolls off at lower ω as M+N+1 increases

roll off = '값이 큰 값에서 작은 값으로 변한다'
로 이해하면 된다.

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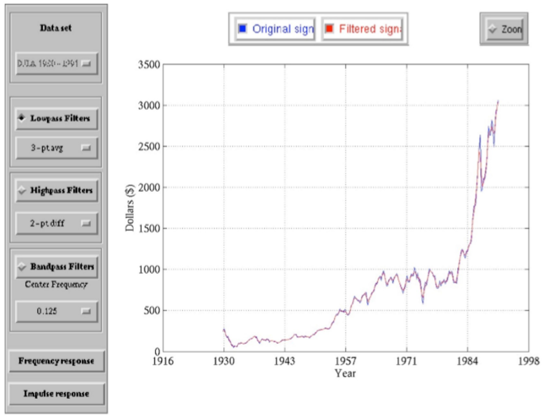
과거의 N개 미래의 M개 그리고 현재값을 평균하는 moving average system은 LPF특성을 갖는다. 어찌보면 당연하다. 이평선!!

증권시장에서 장세 분석시 50일선, 100일선 하는 것들은 moving average를 지난 50일, 지난 100일간 한 것으로, 둘다 LPF이다.

Demo: DT filters, LP, HP, and BP applied to DJ Industrial average



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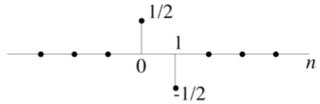
Example #5: Simple DT "Edge" Detector

— DT 2-~~points "differentiator"~~

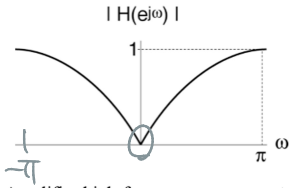
$$y[n] = \frac{1}{2}[x[n] - x[n-1]]$$

$$h[n] = \frac{1}{2}[\delta[n] - \delta[n-1]]$$



$$H(e^{j\omega}) = \frac{1}{2}(1 - e^{-j\omega}) = je^{j\omega/2}\sin(\omega/2)$$

$$|H(e^{j\omega})| = |\sin(\omega/2)|$$

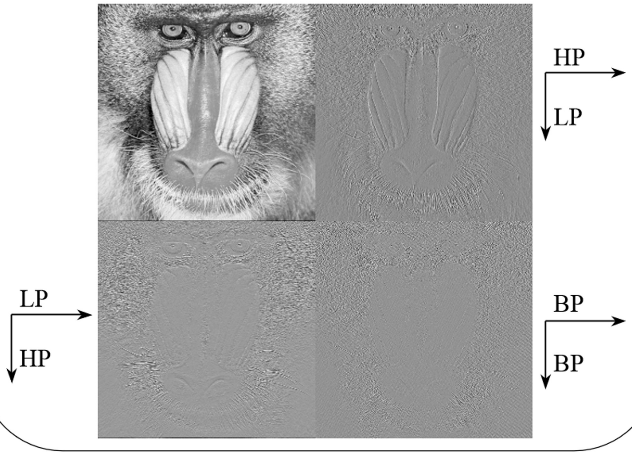


Amplifies high-frequency components

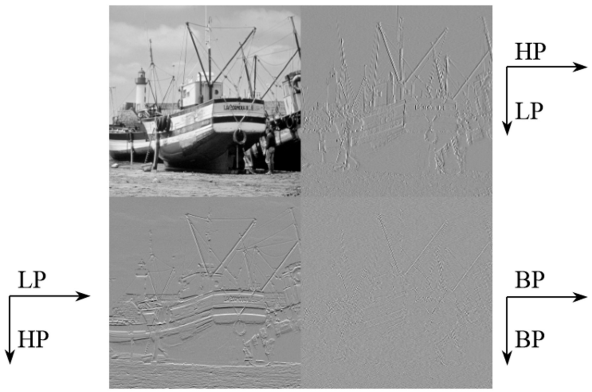
edge는 급격한 변화이므로 고주파 성분이 많을 것이다. 잘 만들어진 edge detector는 따라서 HPF일 것이다.

Example #6: Edge enhancement using DT differentiator

Demo: Apply different filters to two-dimensional image signals.



Demo: Apply different filters to two-dimensional image signals.



Note: To really understand these examples, we need to understand frequency contents of aperiodic signals ⇒ the Fourier Transform