CM17 231102Th on CTFT and inverse CTFT

2023년 10월 26일 목요일 14:22

CM17 231102Th

Announcements

1. Messages from Instructor
   - Goal of Life: Self-Fulfillment and Contribution to Collective (자아실현, 공동체 기여)
     - Attitude: Exceed the expectations! (기대를 넘어서라!)
   - "아는 것이 힘이다. 배우고야 무슨 일이든 한다." -심훈, 상록수-
   - 학습 (배우고 익히기)
     - Theory Learning (이론 배우기)
     - Problem Solving (문제 풀기)
   - How to Learn
     - Stop and Think; Find commonality, similarity, and difference
   - How to Practice

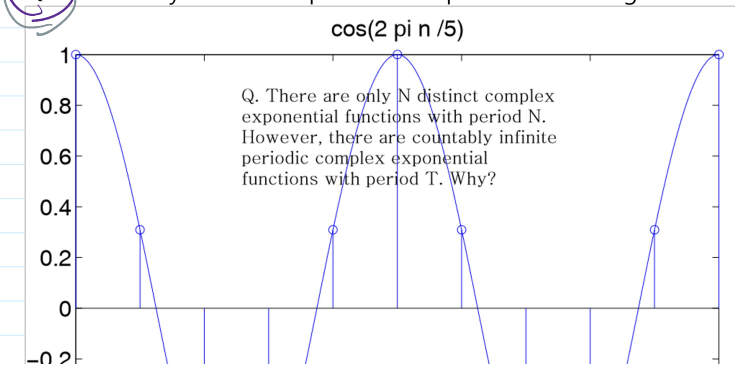
     Purposeful practice with 3F: Time, Focused Action, Experience, Lesson, Immediate Feedback, Fix, Growth

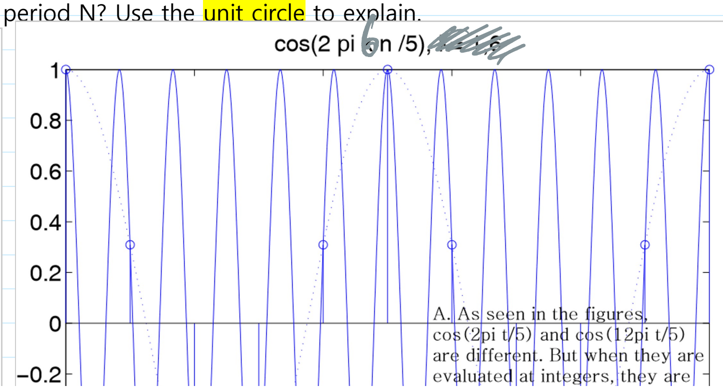
     Purposeful practice is designed to improve performance, and is characterized by staying focused and positive, setting clear and specific goals, allocating time during your week, taking on tasks that go beyond your existing level of ability, and seeking feedback.

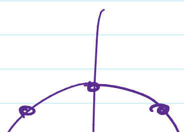
     출처: <https://www.google.com/search?q=purposeful+practice&rlz=1C1CHBD_koKR890KR890&oq=Purposeful+practice&gs_lcrp=EgZjaHJvbWUqBwgAEAAYgAQyBwgAEAAYgAQyBwgBEC4YgAQyBwgCEAAYgAQyBwgDEAAYgAQyBwgEEAAYgAQyCAgFEAAYFhgeMggIBhAAGBYYHjIICAcQABgWGB4yCAgIEAAYFhgeMggICRAAGBYYHtIBCDMzNTVqMGo3qAIAsAIA&sourceid=chrome&ie=UTF-8>

   - How to solve? How to understand? ((S))implify and ((V))isualize:
     - essence, generalize, consider extremes,
     - Venn diagram, necessity and sufficiency, existence and uniqueness/plurality
       - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications

2. 지난 강의 수정/보완 사항
   - Q1. How many distinct DT periodic complex sinusoidal signals with period N? Use the unit circle to explain.

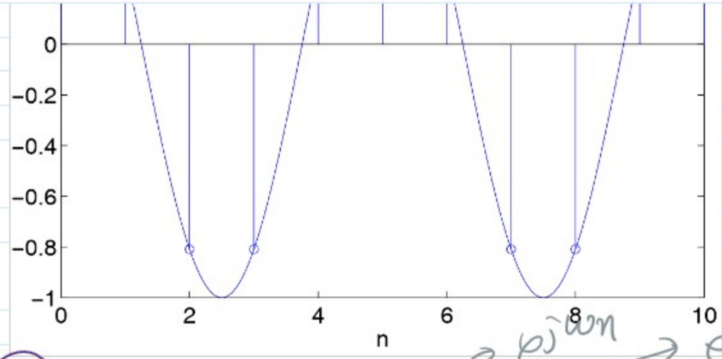


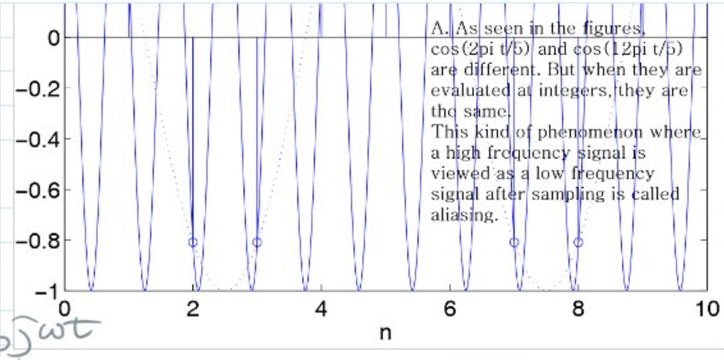


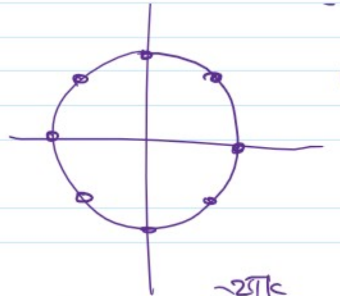


N=8

$e^{-j\frac{2\pi}{N}k}$







$e^{j\frac{2\pi}{N}k}$

$k=0, \cdots, 7$

$\to e^{j\omega n}$ $\to e^{j\omega t}$

- Q2. How many distinct frequencies(?) for DT and CT complex sinusoidal signals? Are they periodic? Use the unit circle to explain.
  - A weighted sum of harmonically/non-harmonically-related complex sinusoidal functions is always periodic?
  - If not, what about a single term-case?
- Q3. Periodicity of frequency responses? DT vs. CT
- Q4. Properties of frequency/magnitude/phase responses of an LTI system with a real-valued impulse response?
  - ~~전거장~~, ~~자기장~~, 안테나 real-valued
- Q5. Design of a bandpass filter using a LPF and a HPF?
  - No frequency component unpresent in the input appears at the output of an LTI system.

$\to \boxed{LPF} \to \boxed{HPF} \to \; = \; \to \boxed{BPF} \to$

$x_k[n] = e^{j\frac{2\pi k}{8}n}, \; k=0$

$x_0[n] = 1, \; \forall k$

$x_1[n] = e^{j\frac{2\pi}{8}n}$

$x_7[n] = e^{j\frac{2\pi}{8}7 \cdot n}$

$= e^{-j\frac{2\pi n}{8}}$

$e^{j(\omega+2\pi)n}$

$= e^{j\omega n} \underbrace{e^{j2\pi n}}_{1}$

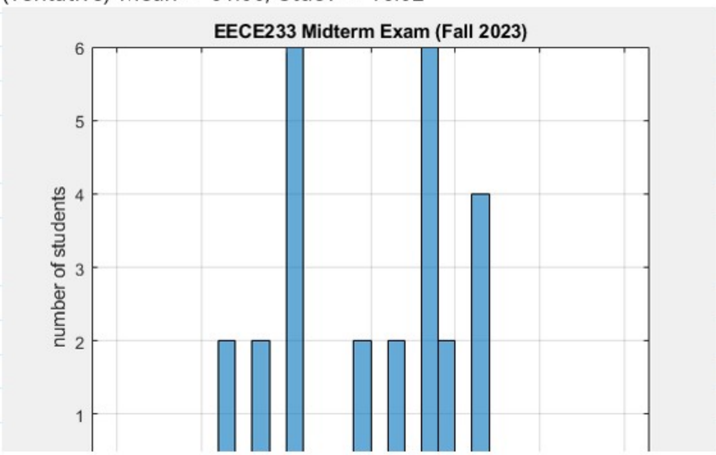
$= e^{j\omega n}$

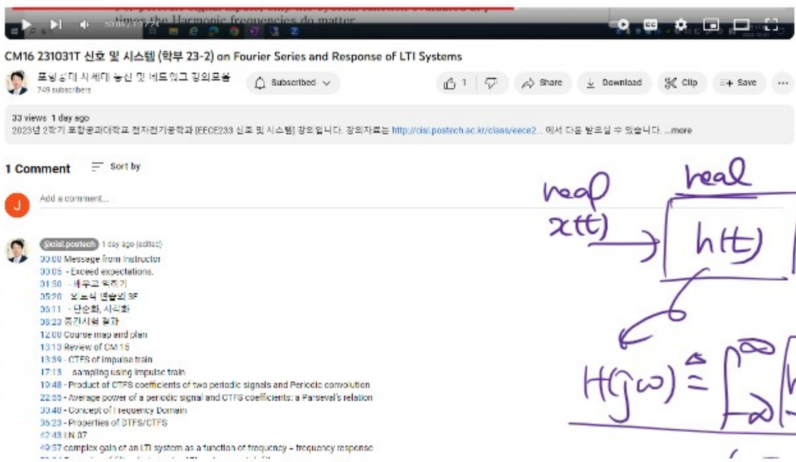
$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} \Big|_{e^{j\omega}}$ $\omega_k = \frac{2\pi}{N}k$ $k=0, N, 2N, \cdots$

$= \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$

$H(e^{j(\omega+2\pi)}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j(\omega+2\pi)n} = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$

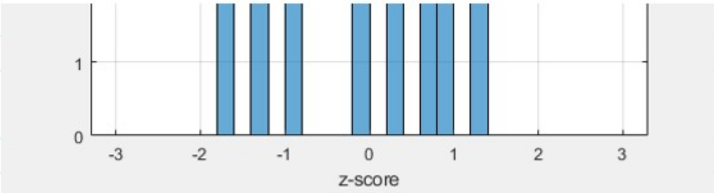
$= H(e^{j\omega})$

3. HW, Quiz, and Exam
   - HW09 with MATLAB problems will be assigned by this Friday. Due ~~will be next Tuesday~~. $\to \infty$
     - Def. of 이평선 and the Dirichlet kernel
     - Solve all the problems in the midterm. (Collaboration encouraged. You may ask help to Tutor.)
   - Midterm Exam is graded.
     - Total = 100
     - (Tentative) Mean = 61.00, Stdev = 16.02
     - 



real $x(t) \to \boxed{h(t)} \to$ real $y(t)$, real

$H(j\omega) \triangleq \int_{-\infty}^{\infty} \boxed{h(t) e^{-j\omega t}} dt$



17:13 sampling using impulse train
19:40 - Product of CTFS coefficients of two periodic signals and Periodic convolution
22:55 - Average power of a periodic signal and CTFS coefficients: a Parseval's relation
30:48 - Concept of Frequency Domain
36:23 Properties of DTFS/CTFS
42:43 LN 07
49:57 complex gain of an LTI system as a function of frequency = frequency response
53:34 Examples of filter design using LTI systems; notch filter
1:02:55 magnitude and phase responses of an LTI system
1:04:27 non-ideal LPF, HPF,BPF
1:08:07 Ideal LPF, HPF, BPF and cut off frequencies
1:10:50 DT Averager/Smoother
1:15:28 DT edge detector, DT differentiator
Show less

$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$H^*(j\omega) = \left(\int_{-\infty}^{\infty} \quad \right)^*$$

$$= \int_{-\infty}^{\infty} h^*(t) e^{j\omega t} dt$$

$$= \int_{-\infty}^{\infty} h(t) e^{j\omega t} dt$$

$$= H(-j\omega)$$

- Graded Exam will be returned on this Thursday.

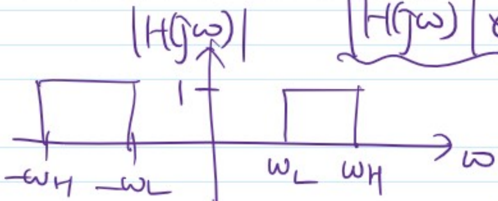
4. 강의 자료
    - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
    - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

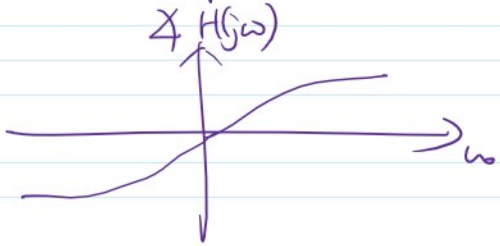
5. 출석
    - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

$$|H(j\omega)| e^{j\angle H(j\omega)} = H(j\omega) = H^*(j\omega), \forall\omega$$

$$|H(-j\omega)| = |H(j\omega)|, \forall\omega \quad \text{even}$$

$$\angle H(-j\omega) = -\angle H(j\omega), \forall\omega \quad \text{odd}$$





6. Overview (The numbers are the chapter numbers of the textbook)
    1. Signals and Systems
    2. Linear Time-Invariant Systems
    3. Fourier Series Representation of Periodic Signals
    4. Continuous-time Fourier Transform
    5. Discrete-time Fourier Transform
    6. Time and Frequency Characterization of Signals and Systems
    9. Laplace Transform
    10. Z-transform
    7. Sampling
    8. Communication Systems

7. Review
    - Def. Impulse train = sampling function
    - Theorem. Parseval's relation for DTFS/CTFS and its implication
    - Def. of the periodic convolution motivated by the product of Fourier coefficients
    - Def. of the Frequency Domain motivated by DTFS and FTFS
    - Response of an LTI system to a weighted sum of complex sinusoidal functions
        - system function evaluated at ..., eigenfunction, complex gain
    - Def. Frequency/magnitude/phase responses of a CT/DT LTI system
    - Lemmas. Properties of a frequency response
        - periodicity of DT response
        - real impulse response: conjugate symmetry of freq. response; even symmetry of mag. response; odd symmetry of phase response
        - Def. LPF, HPF, BPF, notch filters
        - Def. ideal(ized) LPF, HPF, BPF, and cut-off frequencies
    - Examples
        - DT averager (DT smoother)
        - DT edge detector (DT differentiator)
8. Preview
    - Q . Frequency response of an LTI system, commutativity of convolution, and Frequency Domain representation of the input?
    - Fourier's derivation of

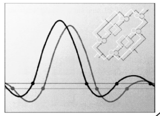
        - Synthesis equation
        - Analysis equation
          from CTFS
    - Def. of CTFT
        - existence
        - sufficient condition
    - Def. of CT inverse Fourier Transform
        - existence
        - sufficient condition
    - Def. Fourier-Plancherel Transform
        - Def. L^2 functions
    - Examples
        - Dirac delta, delayed Dirac delta
        - right-sided exponential function
        - rectangular pulse
        - sinc pulse
        - Gaussian pulse
        - CTFT and CTFS = CTFT of periodic signals
    - Properties of CTFT

앞서 Lecture Note #7에서 LPF, BPF, HPF등의 개념을 배웠는데, 문제는 이들은 입력이 (다른 주기도 허용하여) complex sinusoids의 linear combination일때만 의미가 있었다. 그렇다면 이제는
Q. 주기함수 가 아닌 입력 신호가 들어가면 어떻게 될까?
라는 의문이 들 것이다. 이 Lecture note에서는 이 문제에 대해 배운다.

EECE233 Signals and Systems

Lecture Note #8
Prof. Joon Ho Cho
(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

- Covers O & W pp. 284-309
- Derivation of the CT Fourier Transform pair
- Examples of Fourier Transforms
- Fourier Transforms of Periodic Signals
- Properties of the CT Fourier Transform



* These lecture notes are based on the lecture notes used in 6.003 taught b Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

1

Fourier Transform

- We have shown that Fourier series are useful in analyzing periodic signals, but many (most) signals are aperiodic. Need a more general tool — *Fourier transform*.

Fourier's own derivation of the CT Fourier transform

heuristic (발견적인, 보통 not rigorous but correct in some limited sense의 의미로 쓰임)

- $x(t)$ – an aperiodic signal
  – view it as the limit of a periodic signal as $T \to \infty$
- The harmonic components are spaced $\omega_o = 2\pi/T$ apart, as $T \to \infty$, and $\omega_o \to 0$, then $\omega = k\omega_o$ becomes continuous

$$\Downarrow$$

Fourier series ⟶ Fourier integral

이것은 countably infinite 한 것들의 극한(?)이 어떤 조건하에서는 uncountably infinite한 것이라는 주장으로 엄밀한 수학적 증명이 필요하다. 그러나....공학적으로는 큰 문제없다.

2

$x(t) \to \boxed{h(t)} \to y(t) = x(t) * h(t)$
$= h(t) * x(t)$

$x(t) = \sum_k a_k e^{j\omega_k t}$

$\to \boxed{H(j\omega)} \to y(t) = \sum_k H(j\omega_k) a_k e^{j\omega_k t}$

where $H(j\omega) \triangleq \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$

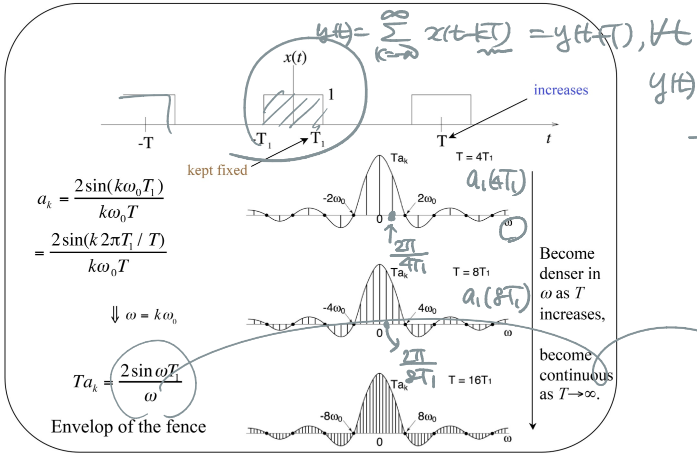
$h(t) \to \boxed{X(j\omega)} \to y(t)$

$X(j\omega) \triangleq \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

TGIBT

Motivating Examples: Rectangular pulse & Square wave

Def. The periodic extension with period T of a signal x(t) is given by
y(t) = \sum_{k=-\infty}^{\infty} x(t-kT).



3

$y(t)=\sum_{k=-\infty}^{\infty} a_k(t) e^{j\frac{2\pi k t}{T}}$

$T \gg 1.$

Def. FT of x(t)

$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$

T=4T_1에서의 k=4에서의 a_k에 associated되어 있는 complex sinusoidal function의 주파수와

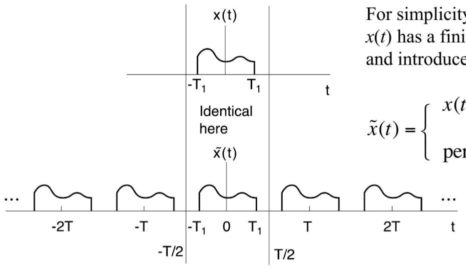
T=8T_1에서의 k=8에서의 a_k에 associated되어 있는 complex sinusoidal function의 주파수가 서로 같다.

따라서 그림들과 같이 같은 주파수에 해당하는 a_k들이 세로로 align이 되도록 그리면

T가 무한대로 감에 따라 envelope는 일정하나 단위 주파수 내에 점점 더 많은 a_k들이 나타나게 된다.

그리고 T를 곱하지 않았지만 성분의 크기들은 점점더 작아진다. 왜냐하면 한 주기 동안의 에너지는 일정하고 Parseval's relation에 의해서 a_k들의 제곱합이 일정해야 하기 때문이다.

So, on with the derivation of FT ...



For simplicity, assume $x(t)$ has a finite duration, and introduce a periodic $\tilde{x}(t)$

$$\tilde{x}(t) = \begin{cases} x(t) & \frac{-T}{2} < t < \frac{T}{2} \\ \text{periodic} & |t| > \frac{T}{2} \end{cases}$$

$$\text{As } T \to \infty, \qquad x(t) = \tilde{x}(t) \text{ for all } t$$

4

다음 세 페이지의 슬라이드에서와 같은 유도가 있고
straightforward한 유도가 있다.

Q. x(t) = \int a_\omega e^(j\omega t)d\omega가 된다고 가정하면

x(t)와
a_\omega의 관계는?

A. a_omega = \frac{1}{2\pi}X(j=omega)

where
X(s)는 x(t)의 Laplace transform.

We call X(j=omega) the Fourier transform of x(t).

이 결과를 얻기 위한 Fundamental assumption in FT 은

\int_{-\infty}^{\infty}e^{j(\omega-\omega_0)t}dt
=
2\pi \delta(\omega-\omega_0)

2\pi \delta(\omega) = \delta(f)
where f=2\pi \omega

Derivation (continued)

$$\tilde{x}(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_o t} \qquad \omega_o = \frac{2\pi}{T}$$

$$a_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \tilde{x}(t) e^{-jk\omega_o t} dt = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) e^{-jk\omega_o t} dt$$

$\uparrow$

$\tilde{x}(t) = x(t)$ in this interval

$$= \frac{1}{T}\int_{-\infty}^{+\infty} x(t) e^{-jk\omega_o t} dt \qquad (1)$$

Since $\quad X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$

then Eq. (1) $\Rightarrow \quad a_k = X(jk\omega_0)/T$

5

which is the sampled value of X(j omega) at 2\pi k/T times 1/T.

Derivation (continued)

radian-frequency 외에 Herz단위로 쓴 synthesis 및 analysis equation이 있으며 통신에서는 주로 이를 사용한다. 특징은 synthesis equation에서 1/(2\pi)가 나타나지 않는다는 것!

Thus, for $-\frac{T}{2} < t < \frac{T}{2}$

$$x(t) = \tilde{x}(t) = \sum_{k=-\infty}^{+\infty} \underbrace{\frac{1}{T} X(jk\omega_0)}_{a_k} e^{jk\omega_0 t}$$

$$= \frac{1}{2\pi} \sum_{k=-\infty}^{+\infty} \omega_0 X(jk\omega_0) e^{jk\omega_0 t}$$

$\Downarrow$

여기서 \omega_0 = 2\pi /T로 놓는다. 그러면 구분구적법에 의해

As $T \to \infty$, $\omega_0 \to 0$, $\sum \omega_0 \to \int d\omega$, and $k\omega_0 = \omega$, we get the CT FT pair

$x(t) = \boxed{\frac{1}{2\pi}} \int_{-\infty}^{+\infty} X(j\omega) e^{j\omega t} d\omega$	Synthesis equation
	— "sum" of $e^{j\omega t}$
$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt$	Analysis equation

6

L^1 functions

$\int_{-\infty}^{\infty} |x(t)| dt < \infty$

the set of all CT signals

L^2 functions

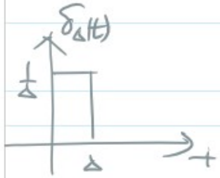
$\int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$

Fourier-Plancherel FT.

$\lim_{\Delta\to\infty} \int_{-\Delta}^{\Delta}$ $d\omega$ / dt

convergence in mean-square sense

$$\left[\begin{aligned} x(t) &= 1 \int_{-\infty}^{\infty} X(f) e^{j2\pi f t} df \\ X(f) &\triangleq \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt \end{aligned}\right] \quad f \text{ (Hz)}$$



$\left(\frac{1}{\Delta}\right)^2 \times \Delta \to \infty$

With CTFT, now the Frequency Response of an LTI System makes complete sense

$e^{j\omega t} \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega)e^{j\omega t}$

$x(t)$ $\qquad$ $y(t)$ $= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega) e^{j\omega t} d\omega$

$= \int_{-\infty}^{\infty} \ldots e^{j\omega t} \ldots j\omega)$ $\qquad$ $= \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega) X(j\omega) e^{j\omega t} d\omega$

$$e^{j\omega t} \longrightarrow \boxed{H(j\omega)} \longrightarrow H(j\omega)e^{j\omega t}$$

$x(t)$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$

$= \frac{1}{2\pi}\int_{-\infty}^{\infty} Y(j\omega) e^{j\omega t} d\omega$

$y(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H(j\omega)X(j\omega) e^{j\omega t} d\omega$

CT Frequency response: $H(j\omega) = \int_{-\infty}^{\infty} h(t)e^{-j\omega t}dt$ *FT* of $h(t)$

$$\left[\begin{array}{l} y(t) = x(t) * h(t) \\ Y(j\omega) = X(j\omega)\,H(j\omega) \end{array}\right.$$

$$e^{j\omega n} \longrightarrow \boxed{H(e^{j\omega})} \longrightarrow H(e^{j\omega})e^{j\omega n}$$

$x[n]$

$(z = e^{j\omega})$

DT Frequency response: $H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$

7

So far, the arguments have been heuristic. From now, more rigorous.

Q. Existence of FT, 즉 The set of all CT signals의 부분집합으로 aperiodic이면서 complex sinusoidal functions 의 linear combination으로 나타낼 수 있는 것들의 집합에 우리는 관심이 있다. 이 부분 집합은???

For what kinds of signals can we do *FT*?

(1) It works also even if $x(t)$ is infinite duration, but satisfies:

a) Finite energy $\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$ $x(t) \in L^2(\mathbb{R})$

In this case, there is *zero* energy in the error

$e_{\Delta}(t) = x(t) - \frac{1}{2\pi}\int_{-\Delta}^{\Delta} X(j\omega)\, e^{j\omega t} d\omega$ Then $\int_{-\infty}^{\infty} |e_{\Delta}(t)|^2 dt \to 0$ as $\Delta \to \infty$

b) Dirichlet conditions (including $\int_{-\infty}^{+\infty} |x(t)|^2 dt < \infty$)

(i) $\frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t} d\omega = \frac{1}{2}[x(t-0) + x(t+0)]$

$= \begin{cases} x(t) & \text{at points of continuity} \\ \text{midpoint at discontinuity} & \end{cases}$

(ii) Gibb' s phenomenon

c) By allowing impulses in $x(t)$ or in $X(j\omega)$, we can represent even *more* signals

E.g. It allows us to consider *FT* for *periodic* signals

8

Q. Analysis equation이 모든 omega 에서 정의된다고 해서 자동으로 이들을 이용한 synthesis equation이 (almost) everywhere x(t)와 같은가?

A. No. CTFT가 존재한다는 것과 synthesis equation이 잘 정의된다는 것과는 별개이다.

우리는 CTFS에서과 마찬가지로

Analysis equation에 의해 구해진 CTFT로 다시 x(t)는 almost everywhere equal로 synthesize할 수 있는 집합의 두 부분집합을 배운다.

1. the set of all finite-energy CT aperiodic signals.

2. the set of CT aperiodic signals that satisfies 3 Dirichlet conditions

CTFT가 존재한다는 것은
analysis equation이 모든 값에서 잘 정의된다는 것, 즉 적분이 수렴한다는 의미이다. 이럴 sufficient conditions를 알아보면...

1. the set of absolutely integrable CT aperiodic signals

2. the set of square integrable CT aperiodic signals: 하지만 여기서는 Fourier Plancherel transform으로 Fourier transform의 정의를 확장해야 성립한다.

3. ...

CTFT와 inverse CTFT를 구별하여 생각해야 함.

Fact 1. x(t)가 absolutely integrable하면 X(j\omega)는 언제나 존재한다. 그러나 inverse CTFT인 \hat x(t)가 존재하는 것은 아니다.
absolutely integrable하다는 것을 x(t) \in L^1(R)이라 표현한다.
Fact 2. x(t)와 X(j\omega)가 모두 absolutely integrable하면 inverse CTFT \hat x(t)가 존재하고 연속이며,x(t)와 \hat x(t)가 almost every where equal이다.

Fact3. 문제는 x(t)는 L^1 function인데 X(j\omega)가 L^1 function이 아닌 중요한 x(t)가 있다는 것이다. 예를 들어 square pulse. 다행히 이들 function중 상당수가 L^2 function인데, L^2 function에 대해서는
Fourier-Plancherel transform이라는 것이 정의되어 (간단히 이것도 구별없이 Fourier transform 이라 부른다.) L^2 function인 Fourier-Plancherel transform (FPT)이 존재하고
inverser Fourie-Plancherel transform (iFPT)이 또한 존재하여

FPT X(j\omega)와 FT에서 -T부터 T까지 적분한것의 오차의 에너지가 0으로 수렴하며

FT에서 -T부터 T까지 적분한것의 iFT와 x(t)와의 오차의 에너지도 0으로 수렴한다.

Example #1

(a) $x(t) = \delta(t)$

X(f) = X(j2\pi f)로 정의하면,

\delta(t) <-> 1
e^{j2\pi f_0t} <-> \delta(f-f_0)

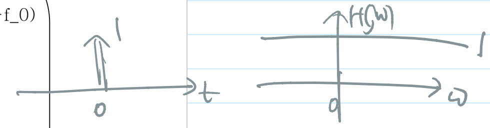
이다.

$$X(j\omega) = \int_{-\infty}^{+\infty} \delta(t) e^{-j\omega t} dt = 1, \ \forall \omega \in \mathbb{R}$$

$$\Downarrow$$

$$\delta(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{j\omega t} d\omega \quad \text{— Synthesis equation for } \delta(t)$$

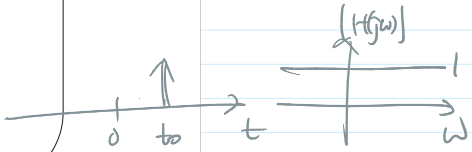
이것이 Fundamental assumption in Fourier transform to handle a delta ft이다.



(b) $x(t) = \delta(t - t_0)$

$$X(j\omega) = \int_{-\infty}^{+\infty} \delta(t - t_0) e^{-j\omega t} dt$$

$$= e^{-j\omega t_o} \quad \text{— Linear phase shift in } \omega$$



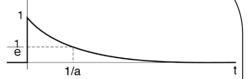
주의. e^{j=omega} 의 어떤 harmonic 들을 모두 countable sum하면 impulse train이 나오고

여기서는 e^{j=omega} 를 모든 주파수에서 uncountable sum (=integrate)하면 impulse가 나온다는 말씀.

Example #2: Exponential function

$x(t) = e^{-at}u(t), \quad a > 0$

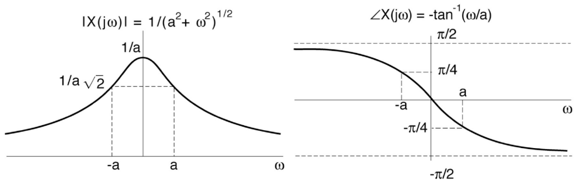
$-a<0$



right−sided exponential function 은 circuit theory, control에서 아주 중요한 신호이다.
이의 CTFT를 반드시 기억하자.

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t) e^{-j\omega t} dt = \int_{0}^{+\infty} \underbrace{e^{-at} e^{-j\omega t}}_{e^{-(a+j\omega)t}} dt$$

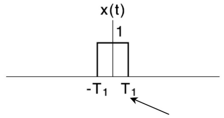
$$= -\left(\frac{1}{a + j\omega}\right) e^{-(a+j\omega)t} \Big|_0^{\infty} = \frac{1}{a + j\omega}$$

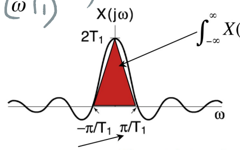


Even *Odd*

Example #3: A square pulse in the time-domain

$$X(j\omega)=\int_{-T_1}^{T_1} e^{-j\omega t}\,dt = \frac{T_1\,2\sin(\omega T_1)}{(\omega T_1)} = \mathrm{sinc}(\ )$$





$\int_{-\infty}^{\infty} X(j\omega)d\omega$

Note the inverse relation between the two widths ⇒ Uncertainty principle

Useful facts about CTFT's

$$X(0)=\int_{-\infty}^{+\infty} x(t)dt \qquad \text{Example above: } \int_{-\infty}^{\infty} x(t)dt = 2T_1 = X(0)$$

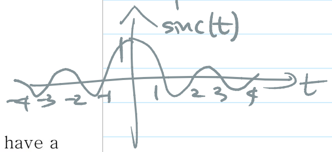
$$x(0)=\frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)d\omega \qquad \text{Ex. above: } x(0)=1=\frac{1}{2\pi}\underbrace{\int_{-\infty}^{\infty} X(j\omega)d\omega}_{2\pi}=\frac{1}{2\pi}\cdot\frac{1}{2}\left(2T_1\times\frac{2\pi}{T_1}\right)$$

$$=\frac{1}{2\pi}\times(\text{Area of the triangle})$$

X(0) is not the DC component. 2X(0)Delta_f is the DC component around |f| \leq Delta_f.

We have a duality. To obtain the value of x(t) at t=0, integrate the CTFT and divide it by 2\pi .

$$\mathrm{sinc}(t) \triangleq \begin{cases} \dfrac{\sin \pi t}{\pi t}, & t \neq 0 \\ 1 & t = 0 \end{cases}$$



strictly time−limited signal은 무한대까지 주파수 성분을 갖는다.

strictly band−limited signal은 무한대까지 time 성분을 갖는다.

Q. True or False: A CT signal is either time−limited or band−limited.
A. No.
There are CT signals neither time−limited nor band−limited. Ex. Gaussian pulse in the next slide.