LN11

2023년 11월 16일 목요일 09:17

CM22 231116T on DTFT Properties, Duality in FS/FT

Announcements

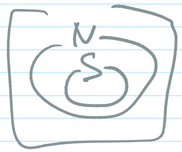
1. Messages from Instructor
   - Goal of Life: Self-Fulfillment and Contribution to Collective (자아실현, 공동체 기여)
     - Attitude: Exceed the expectations! (기대를 넘어서라!)
   - "아는 것이 힘이다. 배우고야 무슨 일이든 한다." -심훈, 상록수-
   - 학습 (배우고 익히기)
     - Theory Learning (이론 배우기)
     - Problem Solving (문제 풀기)
     - 배우기와 익히기의 균형
   - How to Learn
     - Stop and Think; 복기가 chess master 를 만든다; Find commonality, similarity, and difference
   - How to Practice

     Purposeful practice with 3F: Time, Focused Action (무엇을 향상시키려는 훈련인가?), Experience, Lesson, Immediate Feedback (코치의 중요성), Fix, Growth
   - How to solve? How to understand? ((S))implify and ((V))isualize:
     - essence, simplify; generalize; consider extremes; mental representation, representational examples;
     - Venn diagram, necessity and sufficienciency, existence and uniqueness/plurality
       - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications

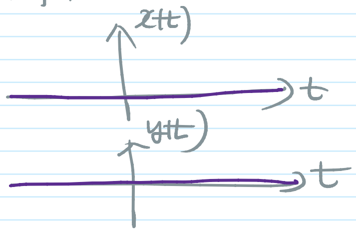
2. 지난 강의 수정/보완 사항
   - LCCDE, ZIZO, Initial rest
     - Review of related definitions and consequences
       - Def1(ZIZO). A CT/DT system satisfies the zero-input zero-output condition if
         - the input is zero for all t implies the output is zero for all t.
       - Lemma1. If a CT/DT system is linear then it satisfies the ZIZO condition.
         - The converse is not true.

       - Def2(Causality). A CT/DT system satisfies the causality condition if
         - two different inputs are equal until time t_0 implies the outputs are equal

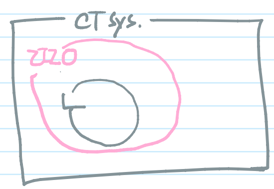
cf) If S, then N



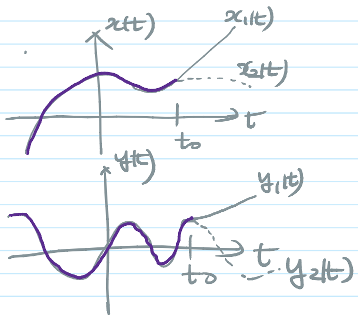
Def1. ZIZO



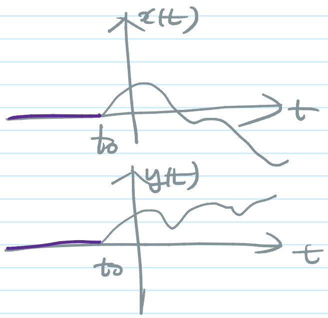
Lemma1.



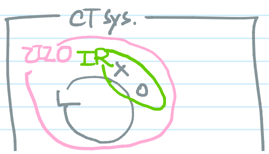
Def2. Causality



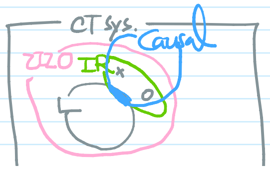
Def3. Initial Rest



Lemma2.



Thm.



- Def2(Causality). A CT/DT system satisfies the causality condition if
  - two different inputs are equal until time t_0 implies the outputs are equal until time t_0.
- Def3(Initial Rest). A CT/DT system satisfies the initial rest condition if
  - an input is zero until time t_0 implies its output is zero until time t_0.
- Lemma2. If a CT/DT satisfies the initial rest condition, then it satisfies the ZIZO condition.
  - The converse is not true.
- Theorem. A linear system is causal iff it satisfies the initial rest condition.
  - i.e. linear ^ causal = linear ^ IR

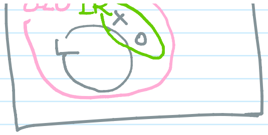
- **A CT/DT system whose I/O relation is described by an LCCDE** is not necessarily a linear system.
  - ex) charged capacitor at time 0, inductor with magnetic field present at time 0
- Lemma. If **a CT/DT system whose I/O relation is described by an LCCDE** satisfies the ZIZO condition, then it is an LTI system.
  - ex) y'(t) + ay(t) = x(t) with ZIZO can be an LTI system with impulse response h(t) = e^(-at)u(t) or an LTI system with impulse response h(t) = -e^(-at)u(-t).
- Lemma. If **a CT/DT system whose I/O relation is described by an LCCDE** satisfies the initial rest condition, then it is a causal LTI system.
  - ex) y'(t) + ay(t) = x(t) satisfying the initial rest condition is an LTI system with impulse response h(t) = e^(-at)u(t).

3. HW, Quiz, and Exam
   - There may be a supplementary CM. No attendance required. The recorded lecture will be uploaded as CM21.
   - HW11 will be assigned by this Friday. Due will be next Tuesday.
   - Quiz 2 will be taken on Tue. 11/21.

4. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정 사유가 있는 학생은 증빙를 제출해 주세요.

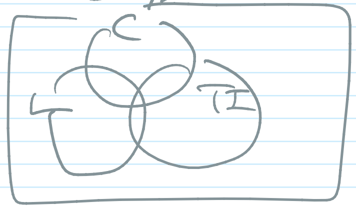
6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems



X : $y(t) = \{x(t)\}^2$
nonlinear
causal, TI
IR, ZIZO
memoryless

O : $y(t) = \{x(t)\}^2 \cos 2\pi f_c t$ $f_c \neq 0$
nonlinear
causal, TV
IR, ZIZO
memoryless

CT sys



$y'(t) + ay(t) = x(t)$. LCCDE CT system.
Q1. Linear?
A1. No.
Q2. ZIZO! Linear?
A2. Yes moreover, LTI
Q3. IR/ Linear?
A3. Yes. " " , Causal.
$h(t) = e^{-at}u(t)$.

6. Overview (The numbers are the chapter numbers of the textbook)
   1. Signals and Systems
   2. Linear Time-Invariant Systems
   3. Fourier Series Representation of Periodic Signals
   4. Continuous-time Fourier Transform
   5. Discrete-time Fourier Transform
   6. Time and Frequency Characterization of Signals and Systems
   9. Laplace Transform
   10. Z-transform
   7. Sampling
   8. Communication Systems

7. Related Courses offered in 24-1
   - EECE302 전자수학 A (확률 및 랜덤프로세스); 월수 2시; 신호및시스템
   - EECE361/DISU361 전자파응용; 화목 11시; 전자기학 개론
   - EECE441 디지털통신개론; 월수 11시; 신호및 시스템, 전자수학
   - EECE442/NGCN301 통신및네트워크개론; 화목 2시
   - EECE443/NGCN302 통신및네트워크실험; 월5시-10시
   - EECE451 디지털신호처리개론; 화목 3시반; 신호및시스템

8. Review
- DTFT
  - Motivation of DTFT
    - Periodic extension of an aperiodic signal
    - For sufficiently large period

$x[n] = \frac{1}{2\pi}\int_{\langle 2\pi\rangle} X(e^{j\omega})e^{j\omega n}\,d\omega$

  - Synthesis and Analysis equations
    - periodicity of DTFT
    - Math. vs. EE

$X(e^{j(\omega+2\pi)}) = X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$

    - Def. discrete-parameter to continuous-parameter Fourier Transformation
      - In EECE233, DTFT.
      - Existence and l^1(Z) sequences, i.e., absolutely summable sequences on Z
    - Def. continuous-parameter to discrete-parameter inverse Fourier Transformation
      - In EECE233, inverse DTFT.
      - Existence and L^1([0,2₩pi]) functions, i.e., absolutely integrable functions on [0,2₩pi)
    - Def. Fourier-Plancherel FT for l^2(Z) sequences

$e^{j\omega_0 n} \longrightarrow \boxed{h[n]} \longrightarrow H(e^{j\omega_0})e^{j\omega_0 n}$

  - Examples
    - delta, delayed delta
    - right-sided exponential
    - rectangular pulse <-> DT Dirichlet kernel
    - sinc <-> ideal LPF

$\delta[n-n_0] \overset{FT}{\longleftrightarrow} e^{-j\omega n_0},\ \forall\omega.$

$a^n u[n] \overset{FT}{\longleftrightarrow} \frac{1}{1-ae^{-j\omega}}$

$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n]e^{-j\omega n}$ ← freq. response

$H(z) = \sum_{n=-\infty}^{\infty} h[n]z^{-n}$ ← system ft

- delta, delayed delta
- right-sided exponential
- rectangular pulse <-> DT Dirichlet kernel
- sinc <-> ideal LPF
  - SAW filter
- periodic in DT <-> impulses
  - impulse train <->impulse train
- carrier modulation <->frequency shifting
- time reversal <->frequency reversal
- real <-> conjugate symmetrical
  - real even <-> real even
  - real odd <-> pure imaginary odd
- linear factor multiplication<->j times differentiation in FD
- Parseval's relation

$$a^n u[n] \xleftrightarrow{FT} \frac{1}{1-ae^{-j\omega}}$$

$$H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n} \leftarrow \text{system ft}$$

$$e^{j\omega_0 n} x[n] \longleftrightarrow X(e^{j(\omega-\omega_0)})$$

$$n x[n] \xleftrightarrow{FT} j\frac{d}{d\omega} X(e^{j\omega})$$

- Convolution property
  - convolution of sinc pulses = sinc pulse
  - DT (down-)sampling
    - Fast forward by removing samples
    - decimation = anti-alias filtering + downsampling
  - DT upsampling
    - slowing down by zero insertion
    - interpolation = upsampling + interpolation filtering
    - upsampling in TD <-> compression in FD

9. Preview
   - LCCDE
     - PFE
   - Multiplication Property
- Duality in FT
- Effect of Phase

EECE233 Signals and Systems

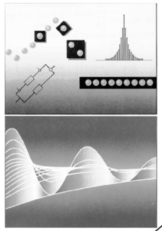
Lecture Note #11

Prof. Joon Ho Cho

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

Covers O & W pp. 385-427

- DTFT Properties and Examples
- Duality
- Magnitude/Phase of Fourier transforms



* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

1

Convolution Property Example

Example 5.13 (O & W p385) $h[n]=\alpha^n u[n], \quad x[n]=\beta^n u[n], \quad |\alpha|,|\beta|<1$

$$H(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}, \quad X(e^{j\omega})=\frac{1}{1-\beta e^{-j\omega}}$$

$$\Downarrow$$

$$y[n]=h[n]*x[n] \longleftrightarrow Y(e^{j\omega})=\frac{1}{1-\alpha e^{-j\omega}}\cdot\frac{1}{1-\beta e^{-j\omega}}$$

– ratio of polynomials in $v=e^{-j\omega}$

$\beta\neq\alpha$: $\quad Y(e^{j\omega}) \overset{PFE}{=} \dfrac{A}{1-\alpha e^{-j\omega}}+\dfrac{B}{1-\beta e^{-j\omega}}$

A, B – determined by partial fraction expansion

$$y[n]=A\alpha^n u[n]+B\beta^n u[n].$$

$\beta=\alpha$: $\quad Y(e^{j\omega})=\dfrac{1}{(1-\alpha e^{-j\omega})^2}$

$$y[n]=\,? \xrightarrow{ny[n]\leftrightarrow j\frac{dY(e^{j\omega})}{d\omega}} y[n]=(n+1)\alpha^n u[n]$$

Q. 왜 이 조건들이 필요한가?
A. 이 조건들이 violate 되면 DTFT들이 존재하지 않는다.

앞서 CT에서는 j\omega의 rational function이었다. 이제는 e^(−j \omega)의 rational function이다.

주의!! e^(+j \omega)의 rational function으로 보지 않는다.

CT에서 parallel한 것을 기억해 보자.

2

Q. 앞서 설명했듯, 주파수영역에서 원하는 시스템을 설계하여 inverse DTFT하여 구한 impulse response를 저장하여 software적으로 DT LTI system을 구현하는 방법은 저가의 빠른 시스템을 구현하는데 적합치않다.
CT에서는 RLC 그리고 op-amp등을 써서 (또는 their equivalents를 써서) approximately 구현했는데 DT에서는 어떻게 하면 될까?
A. We use delay components (shift registers), multipliers, and adders.

DT LTI System Described by LCCDE's

$$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$

From time-shifting property : $x[n-k] \longleftrightarrow e^{-jk\omega} X(e^{j\omega})$

$$\Downarrow$$

$\mathcal{FT}$ on both sides : $\displaystyle\sum_{k=0}^{N} a_k e^{-jk\omega} Y(e^{j\omega}) = \sum_{k=0}^{M} b_k e^{-jk\omega} X(e^{j\omega})$

$$\Downarrow$$

Devide both sides by $\displaystyle\sum_{k=0}^{N} a_k e^{-jk\omega}$: $Y(e^{j\omega}) = \underbrace{\left[\frac{\sum_{k=0}^{M} b_k e^{-jk\omega}}{\sum_{k=0}^{N} a_k e^{-jk\omega}}\right]}_{H(e^{j\omega})} X(e^{j\omega})$

— Rational function of $e^{-j\omega}$, use PFE to get $h[n]$.

주의: LCCDE로 input output relation이 나타난다고 해서 모두 linear system은 아니다. zero input zero output이 만족되도록 initial conditions가 잘 정해져야 한다.

주의: 여기서는 x[n]과 y[n]의 DTFT가 모두 존재한다는 가정하에 변환을 하였다.

모순되는 결과 (예를 들어 h[n]이 absolutely 또는 square summable하지 않게 나오는 경우)가 생기면 가정이 잘못된 것이다.

Q. 우측과 같이 I/O relation이 LCCDE로 나타나는 causal DT LTI system의 impulse response를 구하시오.

Example: First-order *recursive* system

LCCDE에서 과거의 출력이 현재의 y[n]에 영향을 준다. = recursive system이다.

first-order=y[n-1]이 영향을 준다.

$$y[n] - \alpha y[n-1] = x[n], \qquad |\alpha| < 1$$

with the condition of initial rest $\Leftrightarrow$ *causal* LTI

Recall that a linear system is causal iff initial rest.

$\mathcal{FT}$ on both sides : $(1-\alpha e^{-j\omega})Y(e^{j\omega}) = X(e^{j\omega})$

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{1}{1-\alpha e^{-j\omega}}$$

$$\updownarrow$$

$$h[n] = \alpha^n u[n] \quad - \text{ infinite duration}$$

A. h[n] = ...

이렇게 impulse response가 infinite duration을 가지는 filter (DT system)을 IIR (Infinite Impulser Response) filter라 한다.

Q. 위 문제에서 causal 조건이 없으면 어떻게 될까?
A. h[n] = −a^n u[−n−1] 도 impulse response 로서 가능하다.!!!!!

이 impulse response는 여전히 LCCDE를 만족한다. 즉, LCCDE가 주어지면 무조건 causal system 인 것은 아니다.

1. adder, constant multiplier, delay components로 구성된 discrete−time circuit => causal하며 LCCDE로 나타남.

그러나

2. LCCDE로 linear 함 does not necessarilly imply causal system.

Remind that

x(t) and y(t) are periodic with same period

=>

x(t)periodic convolution y(t) has the CTFS coefficients that are term by term products of the CTFS coefficients of x(t) and y(t)

DTFT Multiplication Property

$$y[n]=x_1[n]\cdot x_2[n] \longleftrightarrow Y(e^{j\omega})=\boxed{\frac{1}{2\pi}}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$$

$$=\frac{1}{2\pi}X_1(e^{j\omega})\otimes X_2(e^{j\omega}) \quad - \text{ periodic convolution}$$

각각의 DT 신호의 DTFT는 주기 2\pi인 주기신호이고
곱의 DTFT는 이들 주기신호의 periodic convolution이다.

Proof : $Y(e^{j\omega})=\sum_{n=-\infty}^{\infty} x_1[n]\cdot x_2[n]e^{-j\omega n}$

Use synthesis Eq. for $x_1[n]$: $=\sum_{n=-\infty}^{\infty}\underbrace{\left(\frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})e^{j\theta n}d\theta\right)}_{x_1[n]}x_2[n]e^{-j\omega n}$

Exchange order of $\sum$ and $\int$: $=\frac{1}{2\pi}\int_{2\pi}\left\{X_1(e^{j\theta})\underbrace{\sum_{n=-\infty}^{\infty}x_2[n]e^{-j(\omega-\theta)n}}_{X_2(e^{j(\omega-\theta)})}\right\}d\theta$

$$=\frac{1}{2\pi}\int_{2\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta \qquad \text{QED}$$

5

요약하면, DT의 경우
TD에서의 convolution은 FD에서의 곱이고 TD에서의 곱은 FD에서의 periodic convolution이다.
CT의 경우
TD에서의 convolution은 FD에서의 곱이고 TD에서의 곱은 FD에서의 convolution이다.
공통점과 차이점에 유의!!!

Calculating Periodic Convolutions

Suppose we integrate from -π to π:

$$Y(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$$

can be written as: $=\frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{X}_1(e^{j\theta})X_2(e^{j(\omega-\theta)})d\theta$

Changes a periodic convolution to a regular convolution.

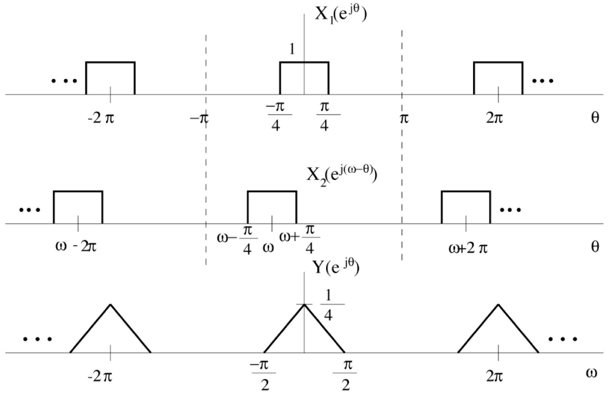
where

$$\hat{X}_1\left(e^{j\theta}\right)=\begin{cases} X_1\left(e^{j\theta}\right), & |\theta|\le\pi \\ 0, & \text{otherwise}\end{cases}$$

6

Example: $y[n] = \left[\dfrac{\sin(\pi n/4)}{\pi n}\right]^2 = x_1[n]\cdot x_2[n]$, $x_1[n] = x_2[n] = \dfrac{\sin(\pi n/4)}{\pi n}$

$$Y(e^{j\omega}) = \frac{1}{2\pi} X_1(e^{j\omega}) \otimes X_2(e^{j\omega})$$



DTFT이야기

이제는 CT/DT FS/FT이야기

duality는 '쌍대성'이라 한다.

Duality in Fourier Analysis

Fourier Transform is highly *symmetric*

CTFT: *Both* time and frequency are continuous and in general *aperiodic*

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}\,d\omega$$

Same operation

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}\,dt$$

$$\Downarrow$$

Suppose $f(\cdot)$ and $g(\cdot)$ are two functions such that they are a Fourier-transform pair (***Eg***. f is a square and g is a *sinc*),

$$x_1(t) = f(t) \leftrightarrow X_1(j\omega) = g(\omega).$$

Then for another time-domain function such that,

$$x_2(t) = g(t)$$

then without doing any math, we should know that

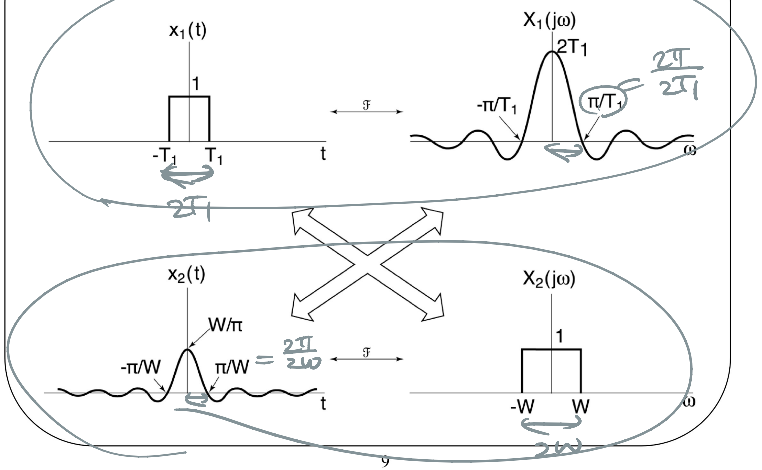
$$X_2(j\omega) = 2\pi f(-\omega)$$

만약에 f(t)의 CTFT가 g(\omega)라면

g(t)의 CTFT는 2\pi f(-\omega)이다.

Example of CTFT duality

Square pulse in either time or frequency domain



Duality between CTFS and DTFT

CTFS

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = x(t+T) \qquad \omega_o = \frac{2\pi}{T}$$

$$a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$$

Periodic in time $\longleftrightarrow$ *discrete* in frequency

DTFT

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n} = X(e^{j(\omega+2\pi)})$$

Discrete in time $\longleftrightarrow$ periodic in frequency

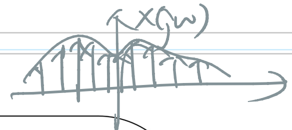
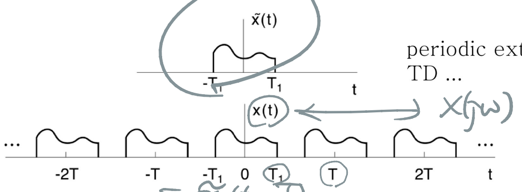


Illustration: *Periodic* in time $\longleftrightarrow$ *discrete* in frequency



periodic extension in TD ...

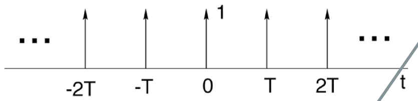
$X(j\omega)$

$T_1 < T$

$= \sum \tilde{x}(t-nT)$

It is clear that $x(t) = \boxed{\tilde{x}(t) * \sum_{n=-\infty}^{\infty} \delta(t-nT)}$ — sampling function

$\sum_{n=-\infty}^{\infty} \delta(t-nT)$



Any periodic signal $x(t)$ is a convolution of $x(t)$ in one period with

$\sum_{n=-\infty}^{\infty} \delta(t-nT)$

sampling function 의 CTFS는 1/T times sum of all harmonics

따라서 CTFT는 2 \pi/T sum of delta functions with period 2\pi/T

$\Downarrow$

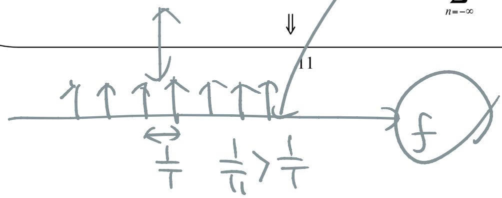


Illustration (continued):

$$\underbrace{X(j\omega)}_{\text{Also discrete}} = \tilde{X}(j\omega) \times \underbrace{\sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \delta\left(\omega - \frac{k2\pi}{T}\right)}_{\text{Discrete}} \quad \text{— also a sampling function}$$

Also discrete

Discrete

$$= \sum_{k=-\infty}^{\infty} \frac{2\pi}{T} \tilde{X}\left(j\frac{k2\pi}{T}\right) \delta\left(\omega - \frac{k2\pi}{T}\right)$$

Poisson sum Formula

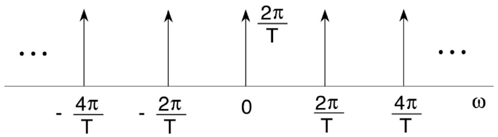
$$\overset{FT}{\longleftrightarrow} \quad x(t) = \sum_{k=-\infty}^{\infty} \frac{\tilde{X}\left(j\frac{k2\pi}{T}\right)}{T} e^{j\frac{k2\pi}{T}t}$$

☆

Illustration (continued):

$$\underbrace{X(j\omega)}_{\text{Also discrete}} = \tilde{X}(j\omega) \times \underbrace{\sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - \frac{k2\pi}{T})}_{\text{Discrete}} \quad \text{— also a sampling function}$$

$$\sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta(\omega - \frac{k2\pi}{T})$$



From the symmetry of Fourier Transform, the reverse is also true:

$$\textit{Discrete} \text{ in time} \xleftrightarrow{\text{DTFT}} \textit{periodic} \text{ in frequency}$$

DTFS

Discrete & periodic in time $\longleftrightarrow$ *periodic & discrete* in frequency

$$x[n] = \sum_{k=<N>} a_k e^{jk\omega_0 n} = x[n+N] \qquad \omega_o = \frac{2\pi}{N}$$

$$a_k = \frac{1}{N}\sum_{k=<N>} x[n]e^{-jk\omega_0 n} = a_{k+N}$$

Same mathematical operation

$$\Downarrow$$

Suppose $f[*]$ and $g[*]$ are two functions such that they are a **FT** pair,

$$x_1[n] = f[n] \leftrightarrow a_k = g[k]. = g[k+N]$$

f[n]의 DTFS coefficient가 g[k]이면

Then for another time-domain function such that,

$$b_k \leftrightarrow x_2[n] = g[n]$$

g[n]의 DTFS coefficient는 f[−k]/N이다.

then without doing any math, we should know that

$$b_k = f[-k]/N$$

확인

Magnitude and Phase of FT, and Parseval Relation

CT:

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = |X(j\omega)|e^{j\angle X(j\omega)}$$

Parseval Relation: $E_{xx} = \int_{-\infty}^{+\infty}|x(t)|^2 dt = \int_{-\infty}^{+\infty}\frac{1}{2\pi}\underbrace{|X(j\omega)|^2}_{\text{Energy distribution in }\omega} d\omega$

DT:

$$x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega})e^{j\omega n}d\omega$$

$$X(e^{j\omega}) = |X(e^{j\omega})|e^{j\angle X(e^{j\omega})}$$

Parseval Relation: $E_{xx} = \sum_{n=-\infty}^{+\infty}|x[n]|^2 = \int_{2\pi}\frac{1}{2\pi}|X(e^{j\omega})|^2 d\omega$

No phase information in the TD and FD.

Effects of Phase

- *Not* on signal energy distribution as a function of frequency

- *Can* have dramatic effect on signal shape/character

  — Constructive/Destructive interference

- Is that important?

  — Depends on the signal and the context, not as important as the amplitude in hearing, but very important for image processing.

전혀 관계없지는 않다. 예를 들어 linear phase shift 가 일어나면 signal이 delay가 되고 이는 우리 귀가 알아챈다.

다만 우리귀는 short time Fourier transform을 하는 것으로 생각할 수 있고 이렇게 짧은 시간동안 서로 다른 phase를 가진 tone신호들이 들어오면 phase는 알아듣지 못한다.

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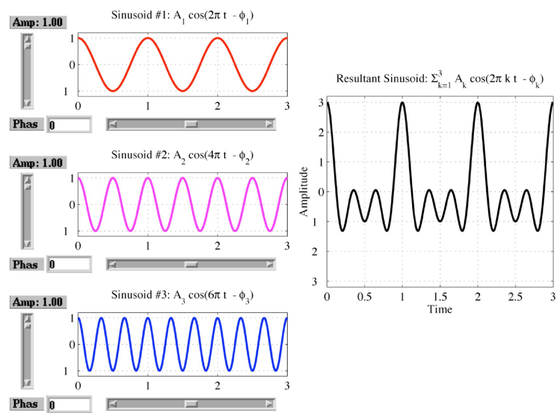
Demo: 1) Effect of phase on Fourier Series



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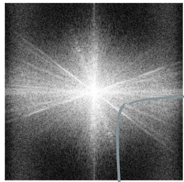
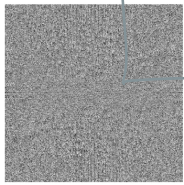
Demo: 2) Effect of phase on image processing

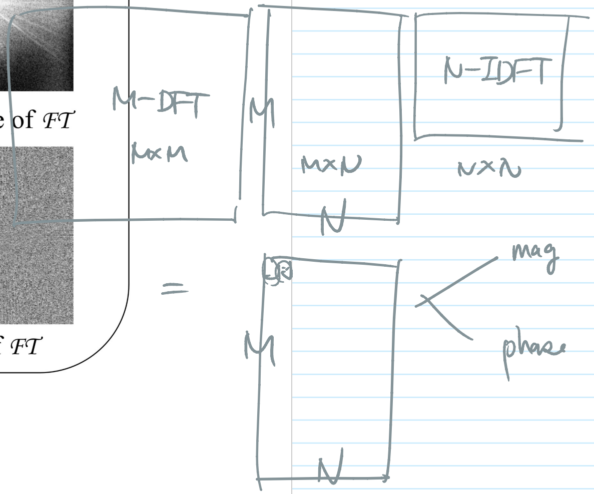
Original image of a boat



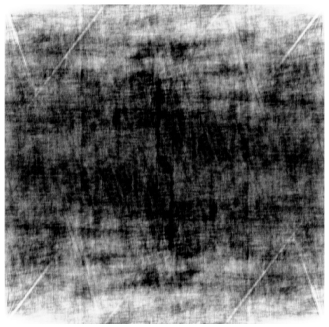
Amplitude of $\mathcal{FT}$



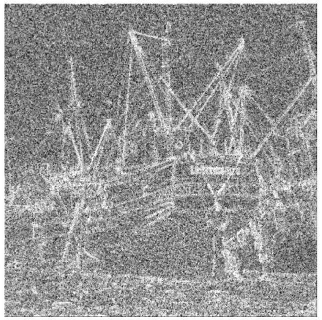
Phase of $\mathcal{FT}$



Demo:2) Effect of phase on image processing (cont.)



Constructed with the amplitude of $\mathcal{FT}$ of the boat, but with a uniform phase.

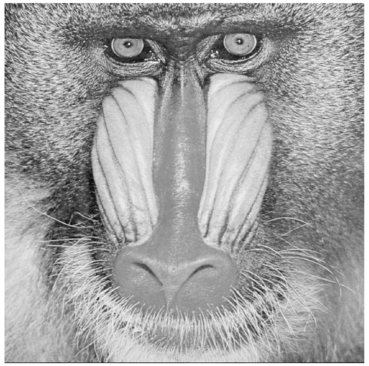


Constructed with the phase of $\mathcal{FT}$ of the boat, but with a uniform amplitude.

Demo:2) Effect of phase on image processing (cont.)

This image was constructed with the phase of $\mathcal{FT}$ of the boat, but with amplitude from a very different image — you wouldn't have guess it.

Demo:2) Effect of phase on image processing (cont.)



The amplitude information for the previous image is from our old friend! Cannot make a monkey out of a boat!
Next lecture covers O & W pp. 427-472.