

LN12 CM23 231121T on Log-Magnitude and Phase, CT/DT Rational Frequency Response

2023년 11월 17일 금요일 18:09

CM23 231121T on Log-Magnitude and Phase, CT/DT Rational Frequency Response

Announcements

1. Messages from Instructor

- Goal of Life: Self-Fulfillment and Contribution to Collective (자아실현, 공동체 기여)
 - Attitude: Exceed the expectations! (기대를 넘어서라!)
- "아는 것이 힘이다. 배우고야 무슨 일이든 한다." -심훈, 상록수-
- 학습 (배우고 익히기)
 - Theory Learning (이론 배우기)
 - Problem Solving (문제 풀기)
 - 배우기와 익히기의 균형
- How to Learn
 - Stop and Think; 복기가 chess master 를 만든다; Find commonality, similarity, and difference
- How to Practice

Purposeful practice with 3F: Time, Focused Action (무엇을 향상시키려는 훈련인가?), Experience, Lesson, Immediate Feedback (코치의 중요성), Fix, Growth
- How to solve? How to understand? ((S))implify and ((V))isualize:
 - essence, simplify; generalize; consider extremes; mental representation, representational examples;
 - Venn diagram, necessity and sufficiency, existence and uniqueness/plurality
 - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
 - block diagram, $G(V,E)$; graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications

2. 지난 강의 수정/보완 사항

- Def. of CT/DT Memoryless System
 - in a narrow sense
 - Def. A CT/DT system is memoryless if the current output depends only on the current input.
 - Consequence. If a CT/DT system is memoryless then it is time-invariant.
 - ◆ There exists a scalar-input scalar-output system such that
 - ◆ Example. $y(t) = \{x(t)\}^2$; Causal, Nonlinear, Time-Invariant; Initial rest, ZIZO; memoryless, $y = x^2$
 - in a wide sense

▪ in a wide sense

□ Def. A CT/DT system is memoryless if the current output does not depend on the past and future inputs.

◆ Example. $y(t) = \{x(t)\}^2 \cos(2\pi f_c t)$; Causal, Nonlinear, Time-Varying; Initial rest, ZIZO; memoryless in a wide sense

3. HW, Quiz, and Exam

- Quiz 2 is taken today.
- There may be a supplementary CM. No attendance required. The recorded lecture will be uploaded as CM21.
- HW12 will be assigned by this Friday. Due will be next Tuesday.

4. 강의 자료

- Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at <http://cisl.postech.ac.kr/class/eece233/schedule.htm>.
- Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석

- 출결상태에 대해 전자출결시스템(<https://wpsc.postech.ac.kr/eams/login>)을 확인하세요. 정정사유가 있는 학생은 증빙을 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)

1. Signals and Systems
2. Linear Time-Invariant Systems
3. Fourier Series Representation of Periodic Signals
4. Continuous-time Fourier Transform
5. Discrete-time Fourier Transform
6. Time and Frequency Characterization of Signals and Systems
9. Laplace Transform
10. Z-transform
7. Sampling
8. Communication Systems

7. Related Courses offered in 24-1

- EECE302 전자수학 A (확률 및 랜덤프로세스); 월수 2시; 신호및시스템
- EECE361/DISU361 전자파응용; 화목 11시; 전자기학 개론
- EECE441 디지털통신개론; 월수 11시; 신호및 시스템, 전자수학
- EECE442/NGCN301 통신및네트워크개론; 화목 2시
- EECE443/NGCN302 통신및네트워크실험; 월5시-10시
- EECE451 디지털신호처리개론; 화목 3시반; 신호및시스템

무엇이든 물어보세요!

차세대 통신 및 네트워크 융합부전공 삼성전자 산학제도

상담부스 이벤트

11.27(월) ~ 12.1(금)

참여대상 학부생이면 (전공무관) 누구나 참여가능



- EECE443/NGCN302 통신및네트워크실험; 월5시-10시
- EECE451 디지털신호처리개론; 화목 3시반; 신호및시스템

8. Review

- DTFT
 - Convolution property (cont.)
 - convolution sum of two causal geometric sequences
 - two different bases => weighted sum of the two sequences
 - same base => weighted sum of the sequence and the linear x the sequence
 - LCCDE
 - frequency response is rational in $e^{-j\omega}$
 - impulse response by PFE
 - ex. first-order recursive system in initial rest
 - Multiplication property
 - multiplication in TD $\Leftrightarrow 1/(2\pi)$ times periodic convolution in FD
 - ex. DTFT of DT sinc squared
- Duality in Fourier Analysis
 - CTFT and inverse CTFT
 - $\text{rect} \leftrightarrow \text{sinc}$, $\text{sinc} \leftrightarrow \text{rect}$.
 - CTFS and DTFT
 - CTFS: periodic in time $\leftrightarrow$ discrete in frequency
 - DTFT: discrete in time $\leftrightarrow$ periodic in frequency
 - DTFS
 - discrete and periodic in time $\leftrightarrow$ discrete and periodic in freq.
- Parseval's relation and Effect of phase
 - Two different time functions can have the same ESD.

$$\alpha^n u[m] * \alpha^n u[m]$$

$$= (n+1) \alpha^n u[m]$$

$$= n \alpha^n u[m] + \alpha^n u[m]$$

참여대상 학부생이면 (전공무관) 누구나 참여가능
설문지만 작성하면 **뽑기 이벤트** 참여 기회!

장소 학생회관 1층 **운영시간** 10:30~16:00

문의사항 LG연구동 416호 | 054-279-5115

홈페이지 www.ngcn.postech.ac.kr

				
갤럭시워치 6	80명	₩ 45,000	₩ 25,000	₩ 10,000
갤럭시워치 5	1명	과자	5명	10명
			25명	

복지회 상품권 (GS25, 버거킹, 카페 등 사용가능)

9. Preview

- Miscellaneous
 - LTI system
 - magnitude and phase responses
 - vs. log-magnitude and phase responses
 - How to read a log-log plot
 - Linear-phase system vs. Nonlinear-phase system
 - Def. A CT/DT LTI system is a linear phase system if Otherwise, the system is of nonlinear phase
 - Caution. In DT LTI, arbitrary linear phase is NOT allowed.
 - All-pass system

- Def. A CT/DT LTI system is an all-pass system if ...
- Def. Given a nonlinear-phase system, the group delay at ω_0 is defined as ...
- Ideal vs. non-ideal LPFs
- Q. How to choose coefficients to design a desirable frequency response?
 - A1. CT/DT rational frequency responses with real coefficients
 - 1st-order system
 - 2nd-order system
 - A2. Laplace and Z-TFs.

EECE233 Signals and Systems

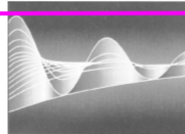
Lecture Note #12

Prof. Joon Ho Cho

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

- Covers O & W pp. 427-472
- Log-Magnitude and Phase
- Linear and Nonlinear Phase
- Ideal and Nonideal Frequency Response
- CT and DT Rational Frequency Responses
- DT First- and Second-Order Systems

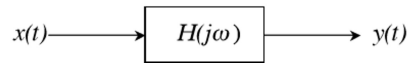
* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.



이 lecture note 후반부에서는 LCCDE로 나타나는 CT LTI system과 DT LTI system에서 coefficient들을 어떻게 선택해야 원하는 response가 나타나는지에 대해 정성적 (qualitatively)으로 배운다.

(qualitatively)으로 배운다.

Log-Magnitude and Phase



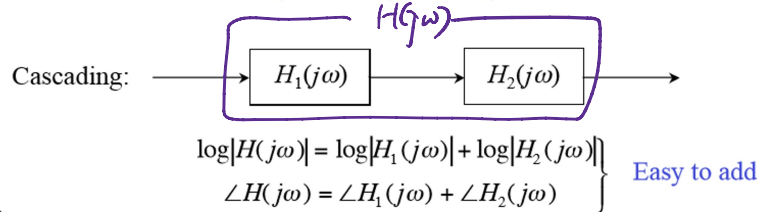
$$Y(j\omega) = H(j\omega)X(j\omega)$$

↓

$$|Y(j\omega)| = |H(j\omega)| \cdot |X(j\omega)|$$

or: $\log|Y(j\omega)| = \log|H(j\omega)| + \log|X(j\omega)|$

and: $\angle Y(j\omega) = \angle H(j\omega) + \angle X(j\omega)$



2

magnitude response 대신 **log magnitude** response를 쓰기도 한다. 장점은, convolution property에 의해 output log magnitude는 input log magnitude에 impulse response의 log magnitude의 '합'이 된다는 것이다. 그리고 **phase** response는 본래부터 '합'이 되었다.

(이렇게 곱을 합으로 나타낼 수 있기 때문에 log가 popular해 졌었다. 지금은 쓰이지 않지만, '계산자'라는 것을 이용하면 곱셈을 간단히 계산할 수 있다. 1950-60년대에는 공대에서 계산자를 사용한 계산대회가 열리기도 했었다.)

Plotting Log-Magnitude and Phase — Bode Plot

a) For **real-valued** signals and systems

$$\left. \begin{aligned} |H(-j\omega)| &= |H(j\omega)| \\ \angle H(-j\omega) &= -\angle H(j\omega) \end{aligned} \right\} \Rightarrow \text{Plot for } \omega \geq 0, \text{ often with a logarithmic scale for frequency in CT}$$

b) In **DT**, need only plot for $0 \leq \omega \leq \pi$ (with **linear** scale)

c) For historical reasons, log-magnitude is usually plotted in units of **decibels (dB)**:

$$(1 \text{ bel} = 10 \text{ decibels} = \frac{\text{output power}}{\text{input power}} = 10)$$

$10 \log_{10} |H(j\omega)|^2$ (circled)

e.g. $|H(j\omega)| = 1 \rightarrow 0 \text{ dB}$ (circled)

$20 \log_{10} |H(j\omega)| = 10 \log_{10} |H(j\omega)|^2$

$|H(j\omega)| = 1 \rightarrow 0 \text{ dB}$ (circled)

'보우디'라고 읽는다.

CT에서는 주파수도 log scale로 나타내기도 한다.

1 meter = 100 centimeters인 것과 같은 원리.

만드시 기억: 3 dB의 power 또는 energy 차이

$$\frac{1}{2\pi} |X(j\omega)|^2$$

meters

$20 \log_{10} |H(j\omega)|$
 e.g. $|H(j\omega)| = 1 \rightarrow 0 \text{ dB}$
 $|H(j\omega)| = \sqrt{2} \rightarrow \sim 3 \text{ dB}$
 $|H(j\omega)| = 2 \rightarrow \sim 6 \text{ dB}$
 $|H(j\omega)| = 10 \rightarrow 20 \text{ dB}$
 $|H(j\omega)| = 100 \rightarrow 40 \text{ dB}$

$= 10 \log_{10} |H(j\omega)|^2$
 $|H(j\omega)|^2 = 1$
 $|H(j\omega)|^2 = 2$
 $|H(j\omega)|^2 = 4$
 $|H(j\omega)|^2 = 100 = 10^2$
 $|H(j\omega)|^2 = 10^4$

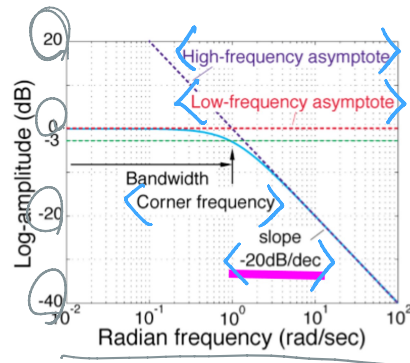
$2 \text{ bels, } 100 \text{ power gain}$

만드시 기억: 3 dB의
 power 또는 energy 차이
 는 2배의 power 또는
 energy 차이이다.
 $10 \log_{10} (2) = 10 * 0.3010 = 3.010$
 따라서 power 2배는 약 3
 dB차이를 만든다.

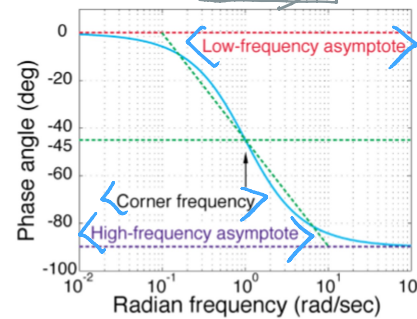
참고로, 음계에서 한 octave차이는 주파수로 2배이다. -> MATLAB 피아노 만들어보자. CTFS해 보면
 약기별로 harmonic 성분들이 다르다.

Approximate Bode plots for insightful analysis

For a frequency response



$$H(j\omega) = \frac{1}{j\omega + 1}$$



Both the amplitude and phase of $H(j\omega)$ change rapidly around the "corner frequency" $\omega = 1$.

linear-linear에서는 쌍곡선
 loglog에서는 기울기가 음수인 직선

$$20 \log_{10} |H(j\omega)| = 20 \log_{10} \frac{1}{\sqrt{\omega^2 + 1}}$$

For small ω , $H(j\omega) \approx 1$

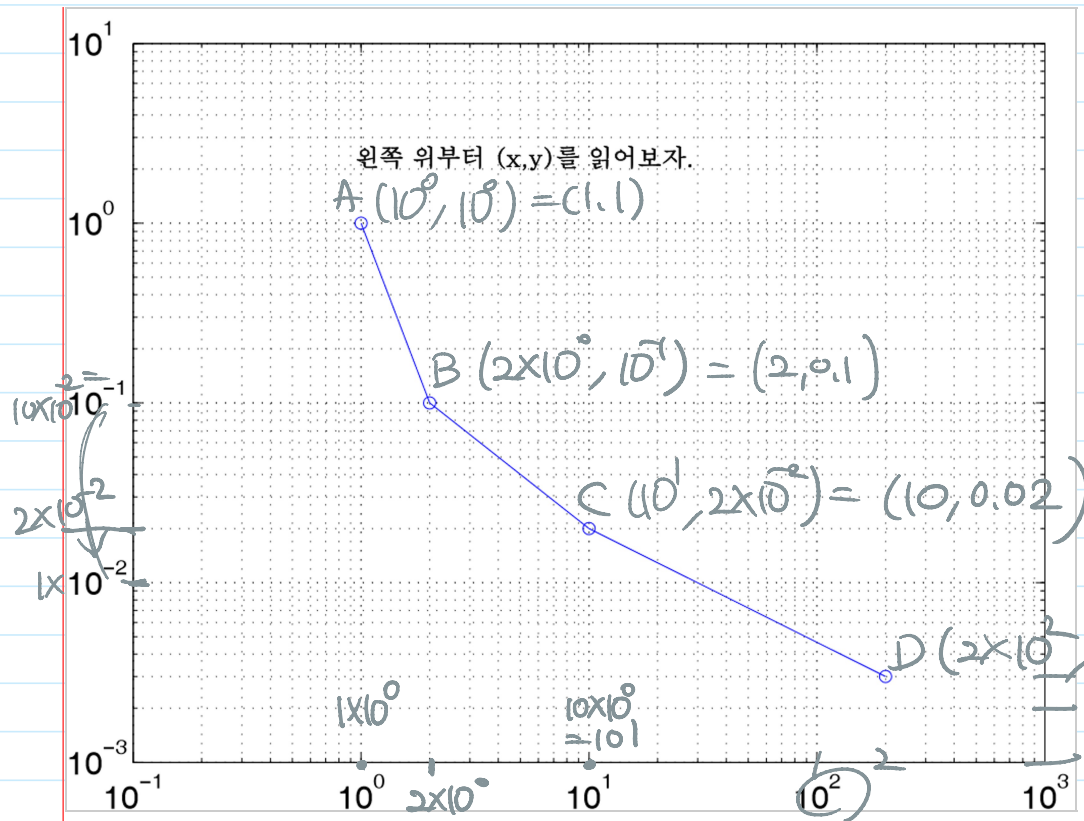
For large ω , $H(j\omega) \approx 1/\omega$

1에 log를 취하면 0.

Q. $h(t)$ 는 ?

MATLAB에서
 plot(x,y) 대신
 semilogx(x,y),
 semilogy(x,y),
 loglog(x,y) 명령을
 쓰면 된다.

10배되면 20dB떨어진다. log magnitude가 20 dB per decade로 감소한다. 즉 주파수 10배마다
 파워 또는 에너지가 20 dB (즉 100배씩) 감소한다.



(24)

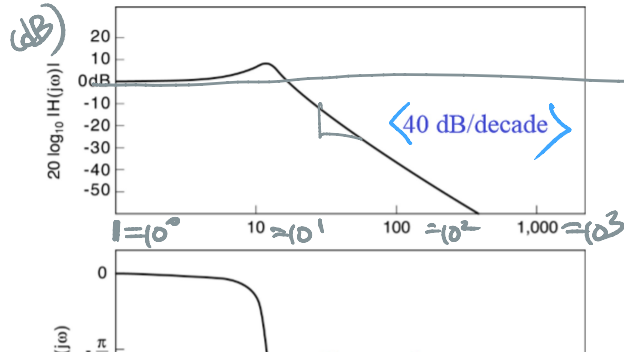


Q. 10^0 과 10^1 사이의 점선의 갯수는? (양끝제외)

A Typical Bode plot for a second-order system

$20 \log |H(j\omega)|$ and $\angle H(j\omega)$ vs. $\log \omega$

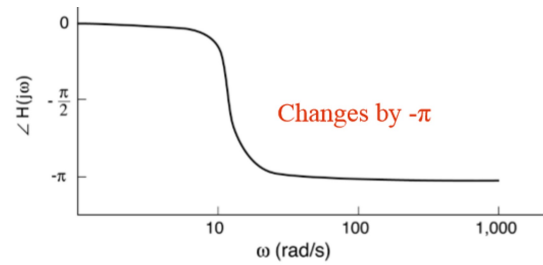
E.g. LCR circuit



$$|H(j\omega)| = \frac{1}{\sqrt{24(j\omega)}}$$

여기서 주목할 것은,
first order
2nd order system들이
각각
-20 dB/decade
-40 dB/decade
등 loglog plot에서
직선으로 나타난다는 것
이다.

그리고 이들 시스템을
cascade하면 log
magnitude는 더해지는



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그리고 이들 시스템을 cascade하면 log magnitude는 더해지는데 직선은 더해도 직선이므로 log-magnitude로 생각하면 합성 시스템의 response를 쉽게 이해할 수 있게 된다는 장점이 있다.

Constant Magn

Linear Phase

CT $x(t) \longrightarrow H(j\omega) \longrightarrow y(t)$

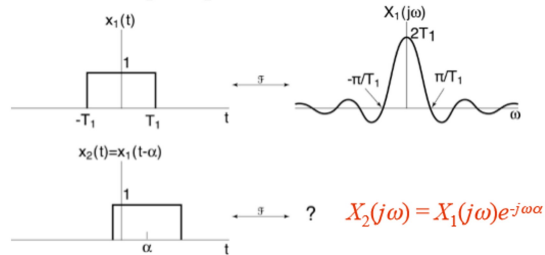
$$H(j\omega) = e^{-j\omega\alpha} \Rightarrow |H(j\omega)| = 1, \angle H(j\omega) = -\alpha\omega \quad \text{— Linear in } \omega$$

$$Y(j\omega) = e^{-j\omega\alpha} X(j\omega) \xleftrightarrow{\text{time-shift}} y(t) = x(t - \alpha)$$

Result: Linear phase $\Rightarrow$ simply a rigid shift in time, no distortion

Nonlinear phase $\Rightarrow$ distortion as well as shift

Example: Shifted square pulse



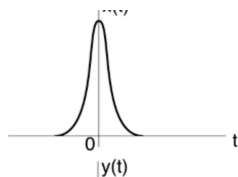
6

Effect of Linear and Nonlinear Phase on the Output Signal

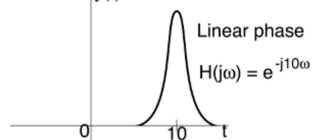
Input



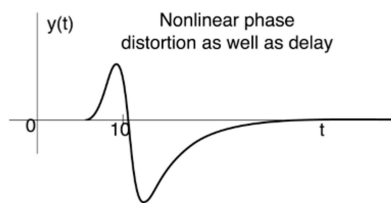
Input



Output from a system with linear phase
E.g. **SAW** devices and telecom systems.



Output from a system with nonlinear phase



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Linear Phase in DT systems

$$\text{DT} \quad y[n] = x[n - n_0] \longleftrightarrow Y(e^{j\omega}) = e^{-j\omega n_0} X(e^{j\omega})$$

$$H(e^{j\omega}) = e^{-j\omega n_0} \Rightarrow |H(e^{j\omega})| = 1, \angle H(e^{j\omega}) = -n_0 \omega$$

Example: **FIR** (Finite-Impulse Response) filters

$$h[n] = \sum_{k=-N_1}^{N_2} a_k \delta[n - k] \quad - \text{Finite duration}$$

A special case: $N_1 = 0, N_2 = 2n_0, a_k = 1, \longrightarrow$ A $2n_0 + 1$ - point averager

$$h[n] = \sum_{k=0}^{2n_0} \delta[n - k] \xrightarrow{FT} \frac{\sin[\omega(n_0 + 1/2)]}{\sin(\omega/2)} e^{-j\omega n_0}$$

$$\text{Note:} \quad h[n + n_0] = \sum_{k=-n_0}^{n_0} \delta[n - k] \xrightarrow{FT} \frac{\sin[\omega(n_0 + 1/2)]}{\sin(\omega/2)}$$

Question: What about $H(e^{j\omega}) = e^{-j\omega\alpha}, \alpha \neq \text{integer?}$

8

Q. TD에서 어떤 신호인지 알기위해 inverse transform해 보면?

A. $H(e^{j\omega})$ 는 반드시 ω 의 주기함수로 주기 2π 여야 한다. 따라서 $e^{-j\omega\alpha}$ 에서 α 가 integer가 아니면 DTFT가 아니다

FIR filter의 antonym은 IIR filter 이다.
IIR = infinite impulse response

$$\text{ex/ } h[n] = \alpha^n u[n]$$

All-Pass Systems

$$(|H(j\omega)| = 1 = |H(e^{j\omega})|)$$

CT

$$H(j\omega) = e^{-j\alpha\omega}$$

– Linear phase

$$H(j\omega) = \frac{a - j\omega}{a + j\omega}$$

– Nonlinear phase

but all-pass $|H(j\omega)| = \sqrt{\frac{a^2 + \omega^2}{a^2 + \omega^2}} = 1$

DT

$$H(e^{j\omega}) = e^{-j\omega n_0}$$

– Linear phase

$$H(e^{j\omega}) = \frac{1 - e^{-j\omega}}{1 - e^{j\omega}}$$

– Nonlinear phase

but all-pass $|H(e^{j\omega})| = \sqrt{\frac{(1 - \cos\omega)^2 + \sin^2\omega}{(1 - \cos\omega)^2 + \sin^2\omega}} = 1$

all-pass system =
frequency flat filters
such as
identity,
linear phase,
nonlinear-phase
systems

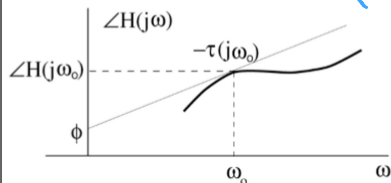
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frequency selective
filters
including LPF, BPF,
HPF

9

How do we handle signal delay when the phase is nonlinear?

Group Delay

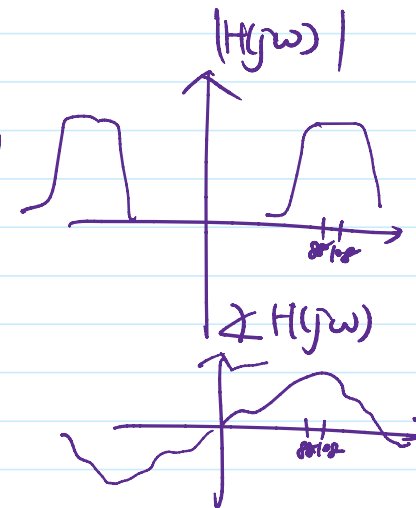
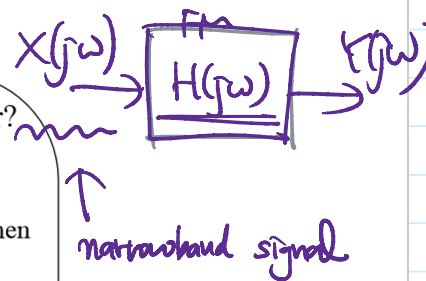


When the signal is **narrowband**, then
 $-\frac{d\angle H(j\omega)}{d\omega}$ instead of $-\frac{\angle H(j\omega)}{\omega}$
reflects the time delay.

For frequencies "near" ω_0

$$\angle H(j\omega) \approx \angle H(j\omega_0) - \tau(\omega_0)(\omega - \omega_0) = \phi - \tau(\omega_0) \cdot \omega$$

$$\tau(\omega_0) = -\frac{d}{d\omega} [\angle H(j\omega)] = \text{Group Delay}$$



where $\phi = \angle H(j\omega_0)$
 $\tau(\omega_0)$

$$\angle H(j\omega) \approx \angle H(j\omega_0) - \tau(\omega_0)(\omega - \omega_0) = \phi - \tau(\omega_0) \cdot \omega$$

$$\tau(\omega_0) = -\frac{d}{d\omega} [\angle H(j\omega)] \Big|_{\omega=\omega_0} = \text{Group Delay}$$

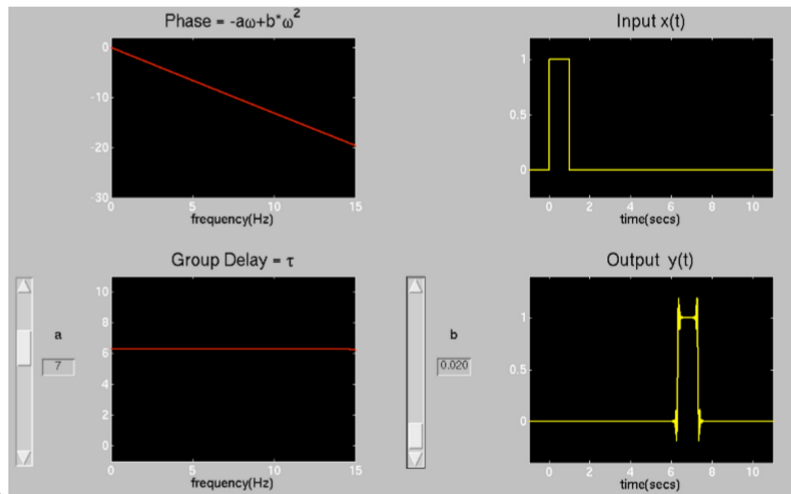
$$y = f(x_0) + f'(x_0)(x - x_0)$$

$$H(j\omega) = \underbrace{|H(j\omega)|}_{\text{Distortion of the signal}} e^{j\phi} = \underbrace{e^{-j\tau(\omega_0)(\omega - \omega_0)}}_{\text{Delay by } \tau(\omega_0)}$$

10

where $\phi = \angle H(j\omega_0)$
 $\tau(\omega_0)$
 이다.

Demo: All-pass system $H(j\omega) = e^{j\angle H(j\omega)}$,
 and $\angle H(j\omega) = -a\omega + b\omega^2$.



11

Ideal Lowpass Filter

CT

$$H(j\omega)$$

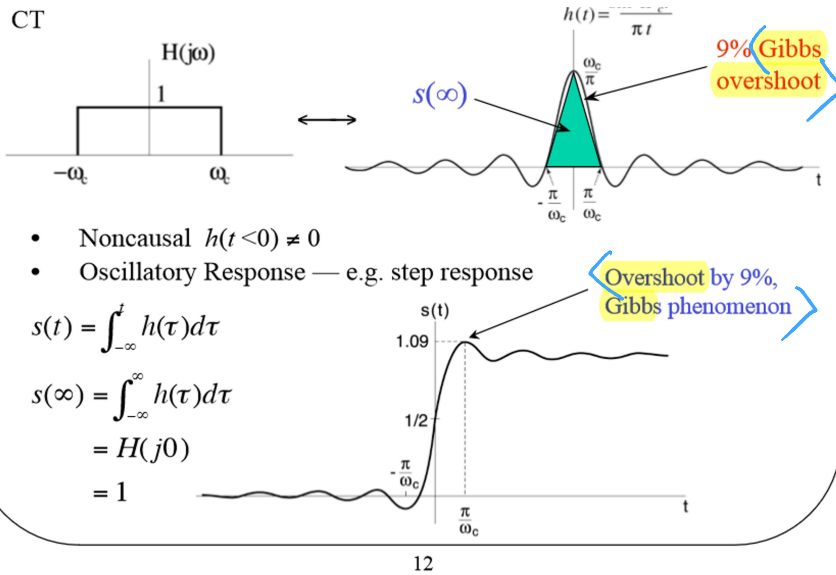
$s(\infty)$

$$h(t) = \frac{\sin \omega_c t}{\pi t}$$

9% Gibbs overshoot

여기서 주의할 것은,
 ω_c 가 달라져도
 $t = -\pi/\omega_c$ 에서
 $t = +\pi/\omega_c$ 까지의
 저보가오 변화가 않는다는

CT



ω_c 가 달라져도
 $t = -\pi/\omega_c$ 에서
 $t = +\pi/\omega_c$ 까지의
 적분값은 변하지 않는다는
 것이다.
 x축 방향으로 줄어드는 만큼
 y축 방향으로 늘어나서
 면적변화를 보상하게 된다.

step response가
 $t = -\infty$ 에서는 0,
 $t = \infty$ 에서는 1이고,
 $h(t)$ 는 $t=0$ 을 중심으로
 symmetrical 하므로,
 step response는 $t=0$ 에
 서 1/2의 값을 갖는다.

ω_c 가 커지면
 maximum overshoot되는 위치는 0에 가까와 지지만,
 maximum overshoot의 양은 변하지 않는다.

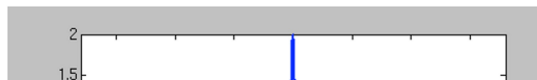
Q. Gibbs phenomenon과의 관계는???

ex/ periodic square wave의 Fourier series를 truncate하는 것과 무슨 관계가???

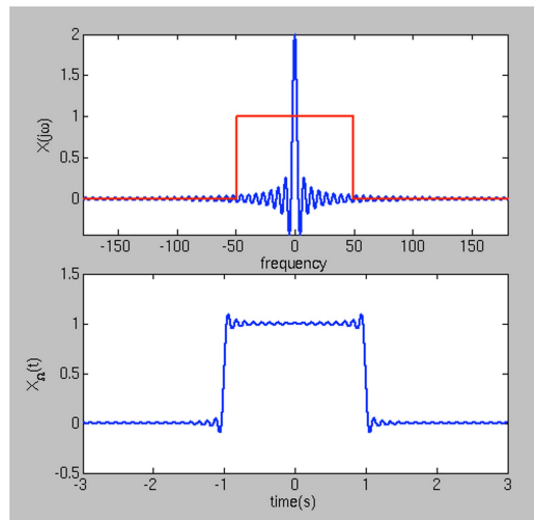
Q. 그렇다면 jump discontinuity가 있는 신호의 Gibbs phenomenon은 어떻게 설명하나?

만약 $x(t)$ 가 t_0 에서 α 크기의 jump discontinuity가 있으면
 $x(t) = x_0(t) + \alpha u(t-t_0)$ 로 나타내어...

Demo: Gibbs phenomenon in FT
 Passing a square pulse through an ideal LPF



Passing a square pulse through an ideal LPF

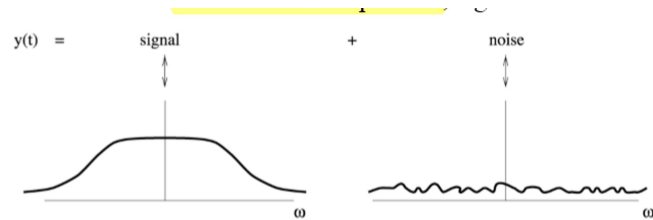


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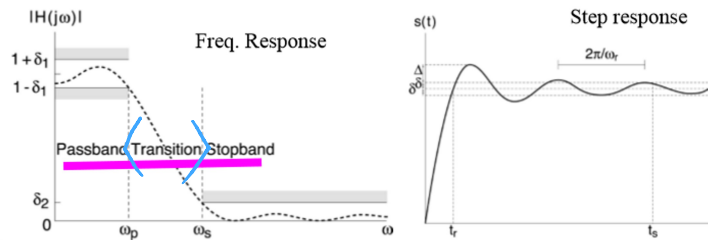
Nonideal Lowpass Filter

- Sometimes we don't want a sharp cutoff, e.g.

$$y(t) = \underset{\substack{\updownarrow \\ \text{signal}}}{\text{signal}} + \underset{\substack{\updownarrow \\ \text{noise}}}{\text{noise}}$$



- Often have specifications in time and frequency domain $\Rightarrow$ Trade-offs



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이제 qualitative하게 LCCDE로 나타나는 LTI system을 설계하는 법의 기초를 배운다. 보다 자세한 것은 Laplace transform과 함께 배운다.

우리는 LCCDE로 나타나는 시스템이 임의의 real-valued signal을 real-valued signal로 mapping하기를 원하는 경우가 많다. 이렇게 하기 위한 sufficient한 방법으로는 $H(j\omega)$ 가 real-valued coefficient를 갖는 rational function들의 product이면 된다.

CT Rational Frequency Responses

If the CT system is described by LCCDEs,

$$\sum a_k \frac{d^k y(t)}{dt^k} = \sum b_k \frac{d^k x(t)}{dt^k}$$

then $\frac{d^k}{dt^k} \longleftrightarrow (j\omega)^k$, and $H(j\omega) = \frac{\sum b_k (j\omega)^k}{\sum a_k (j\omega)^k} = \prod_i H_i(j\omega)$

$H_i(j\omega)$ = First - or Second - order factors

Prototypical Systems $H_1(j\omega) = \frac{1}{j\omega\tau + 1}$ — First - order system, has only **one** energy storing element, i.e. L or C

$$H_2(j\omega) = \frac{1}{\left(j\frac{\omega}{\omega_n}\right)^2 + 2\xi\left(j\frac{\omega}{\omega_n}\right) + 1} = \frac{\omega_n^2}{(j\omega)^2 + 2\xi\omega_n(j\omega) + \omega_n^2}$$

— Second - order system, has **two** energy storing elements i.e. L and C

$\prod$ 대문자 π 이다.
곱 (product)을 나타낸다.
 $\prod$ 뒤에 나오는 것을 multiplicand라고 한다.
cf) summand, integrand

이렇게 하면 system을 구성할때, $h_i(t)$ 를 갖는 작은 system들을 구성하여 serially connect하면 된다. parallel connection으로 구성하

DT Rational Frequency Responses

If the DT system is described by LCCDE's (Linear-Constant-Coefficient Difference Equations,

$$\sum a_k y[n-k] = \sum b_k x[n-k]$$

then from $y[n-k] \longleftrightarrow Y(e^{j\omega})e^{-jk\omega}$, $x[n-k] \longleftrightarrow X(e^{j\omega})e^{-jk\omega}$

We have:

$$H(e^{j\omega}) = \frac{\sum b_k e^{-jk\omega}}{\sum a_k e^{-jk\omega}} = \frac{\sum b_k (e^{-j\omega})^k}{\sum a_k (e^{-j\omega})^k}$$

$$= \prod_i H_i(e^{j\omega})$$

$H_i(e^{j\omega})$ = First - or second - order

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CT에서와 마찬가지로 factoring하면 serial connection으로 구성하는데 유용하고 PFE하면 parallel connection으로 구성하는데 유용하다.

즉, LCCDE로 시스템을 구성하려면 orders M and N과 coefficients a_k , b_k 들을 선정해야 하는데, Fourier transform 만으로는 어떻게 해야 하는지 확실히 보이지 않는다. -> 뭔가 더 필요하다. Laplace transform...

모든 $H_i(j\omega)$ 가 1차 또는 2차의 $j\omega$ 의 rational function이고 real - valued coefficients를 갖는 경우를 공부하자.

DT First-Order Systems

$$y[n] - ay[n-1] = x[n], \quad |a| < 1, \quad \text{initial rest}$$

$$H(e^{j\omega}) = \frac{1}{1 - ae^{-j\omega}} = \frac{1}{(1 - a\cos\omega) + ja\sin\omega}$$

$\Downarrow$

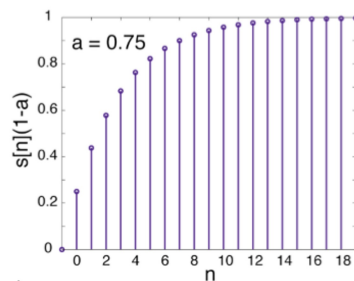
Frequency Domain

$$|H(e^{j\omega})| = \frac{1}{(1 + a^2 - 2a\cos\omega)^{1/2}}$$

and $\angle H(e^{j\omega}) = -\tan^{-1} \left[\frac{a\sin\omega}{1 - a\cos\omega} \right]$

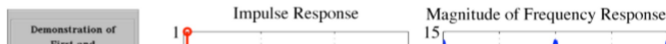
Time Domain $h[n] = a^n u[n]$

$$s[n] = h[n] * u[n] = \sum_{m=0}^n a^m = \frac{1 - a^{n+1}}{1 - a} u[n]$$

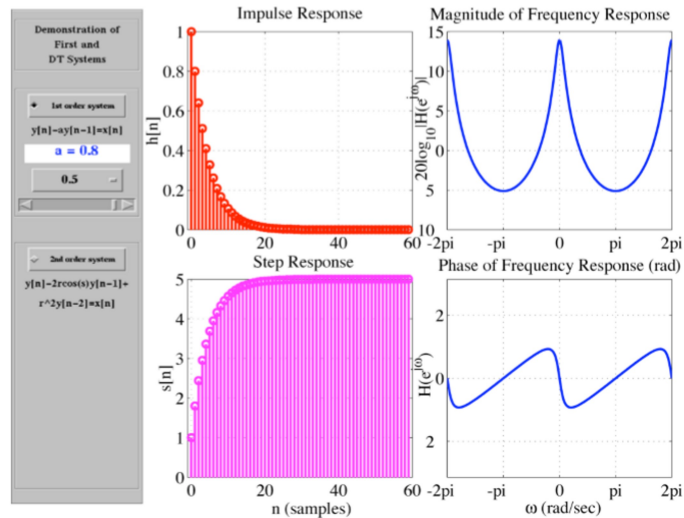


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Demo: Unit-sample, unit-step, and frequency response of DT first-order systems



of DT first-order systems



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DT Second-Order System (O & W pp.465-466)

$$y[n] - 2r \cos \theta y[n-1] + r^2 y[n-2] = x[n], \quad 0 < r < 1 \text{ and } 0 \leq \theta \leq \pi$$

$$\begin{aligned} H(e^{j\omega}) &= \frac{1}{1 - (2r \cos \theta) e^{-j\omega} + r^2 e^{-j2\omega}}, \\ &= \frac{1}{1 - (r e^{j\theta}) e^{-j\omega}} \cdot \frac{1}{1 - (r e^{-j\theta}) e^{-j\omega}}, \\ &= \frac{A}{1 - (r e^{j\theta}) e^{-j\omega}} + \frac{B}{1 - (r e^{-j\theta}) e^{-j\omega}} \end{aligned}$$

$$\begin{aligned} h[n] &= [A r^n e^{jn\theta} + B r^n e^{-jn\theta}] u[n] \\ &= \left[\frac{r^n \sin[(n+1)\theta]}{\sin \theta} \right] u[n] \end{aligned}$$

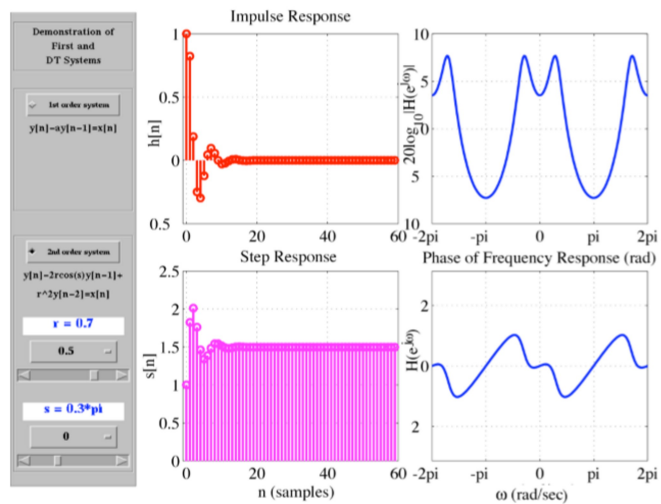
분모를 $e^{-j\omega}$ 로
풀면, 항상 켈레복소근을
갖는다.
켈레복소근의 특징은
mag는 갖고
phase는 반대.

$$= \left[\frac{r \sin((n+1)\theta)}{\sin \theta} \right] u[n]$$

$$s[n] = h[n] * u[n]$$

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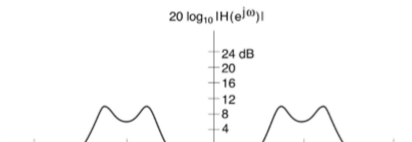
Demo: Unit-sample, unit-step, and frequency response of DT second-order systems

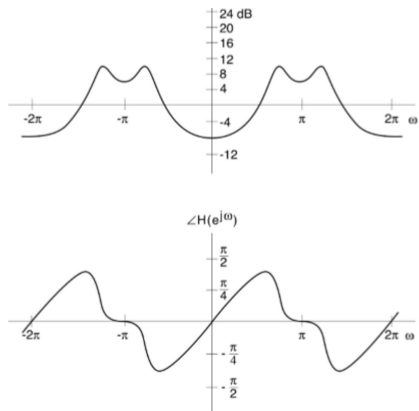


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A typical plot of the log-magnitude and phase of a second-order DT frequency response

$20\log|H(e^{j\omega})|$ and $\angle H(e^{j\omega})$ vs. ω





Next lecture covers O & W pp. 514-527

