2023년 12월 6일 수요일 22:54

CM30 2312012T on Design of DT LTI system using ZT, UZT and LCCDE, Fibonacci sequence
Announcements

1. Messages from Instructor
   - Goal of Life: Self-Fulfillment and Contribution to Collective (자아실현, 공동체 기여)
     - Attitude: Exceed the expectations! (기대를 넘어서라!)
     - Drive yourself by fear and dream.
   - "아는 것이 힘이다. 배우고야 무슨 일이든 한다." -심훈, 상록수-
   - 학습 (배우고 익히기)
     - Theory Learning (이론 배우기)
     - Problem Solving (문제 풀기)
     - 배우기와 익히기의 균형
   - How to Learn
     - Stop and Think; 복기가 chess master 를 만든다; Find commonality, similarity, and difference
   - How to Practice

     Purposeful practice with 3F: Time, Focused Action (무엇을 향상시키려는 훈련인가?; Push oneself little bit out of the COMFORTABLE ZONE), Experience, Lesson, Immediate Feedback (코치의 중요성), Fix, Growth
   - How to solve? How to understand? ((S))implify and ((V))isualize:
     - essence, simplify; generalize; consider extremes; mental representation, representational examples;
     - Venn diagram, necessity and sufficienciency, existence and uniqueness/plurality
       - An imperfect classification rule is better than no classification rule as long as you recognize the imperfection and update your rule.
     - block diagram, G(V,E); graph of a function, algebraic vs. geometric interpretations, mathematical vs. physical/engineering interpretations/implications

2. 지난 강의 수정/보완 사항

3. HW, Quiz, Voluntary Project, and Exam
   - Final Exam is approaching. Unless basic mandatory courses have conflicting schedules, it will be taken 7 pm to 11pm on Thursday Dec. 21 at LG 102. (No one has conflict with General Chemistry?)
   - Voluntary Project (due by 12/28 18:00)
     - Topic must be approved before final exam. Topics related to this course material is acceptable.
     - Presentation file and presentation mp4 of length >20 minutes must be submitted.
     - No reuse, double use, etc.
     - To send, you must use POSTECH webmail and attach as Overzised Attachments.
     - Please check my confirmation reply. (I will be out of town from 26-28, so that my replay can be delayed.)
   - There may be a supplementary CM. No attendance required.
   - HW15 will NOT be assigned.

4. 강의 자료
   - Handouts and annotated pdfs are downloadable from the non-PLMS class homepage at http://cisl.postech.ac.kr/class/eece233/schedule.htm.
   - Recorded CMs are available from cisl.postech YouTube channel. Please place clickable time stamps.

5. 출석
   - 출결상태에 대해 전자출결시스템(https://wpsc.postech.ac.kr/eams/login)을 확인하세요. 정정사유가 있는 학생은 증빙를 제출해 주세요.

6. Overview (The numbers are the chapter numbers of the textbook)

1. Signals and Systems
TD 2. Linear Time-Invariant Systems
3. Fourier Series Representation of Periodic Signals
FD 4. Continuous-time Fourier Transform
5. Discrete-time Fourier Transform
6. Time and Frequency Characterization of Signals and Systems
S 9. Laplace Transform

5. Discrete-time Fourier Transform
6. Time and Frequency Characterization of Signals and Systems
9. Laplace Transform
10. Z-transform
7. Sampling
8. Communication Systems

7. Related Courses offered in 24-1
   - EECE302 전자수학 A (확률 및 랜덤프로세스); 월수 2시; 신호및시스템
   - EECE361/DISU361 전자파응용; 화목 11시; 전자기학 개론
   - EECE441 디지털통신개론; 월수 11시; 신호및 시스템, 전자수학
   - EECE442/NGCN301 통신및네트워크개론; 화목 2시
   - EECE443/NGCN302 통신및네트워크실험; 월5시-10시, 1 seat available now
   - EECE451 디지털신호처리개론; 화목 3시반; 신호및시스템
   - 이번 학기말 통신 및 네트워크 융합부전공 신청 기간은 12월 4일부터 12일까지 (신청이전 1과목 이상 수강 필수)

8. Review
- ZTF
  - Bilateral vs. Unilateral
  - Def. X(z) = ... z \in ROC
    - examples
      - a^n u[n] vs -a^n u[-n-1]
      - FIR filter
      - causal signal
  - Caution. z^{-1} = 1/z does not allow z = 0
  - Relation b/w BZT and DTFT
    - unit circle
    - differentiation property
  - Properties of ROC
    - the set of complex numbers vs. the extended set of complex numbers
    - rational function of z, z^{-1} and poles at infinity, zeros at infinity
  - Inverse BZT
  - Properties of BZT

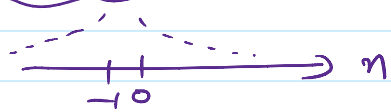
9. Preview

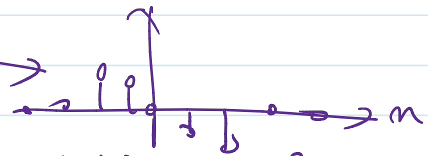
$$\boxed{H(z) = \sum_{n=-\infty}^{\infty} h[n] z^{-n}}$$

$z \in ROC \subset \mathbb{C}, \ \bar{\mathbb{C}}$

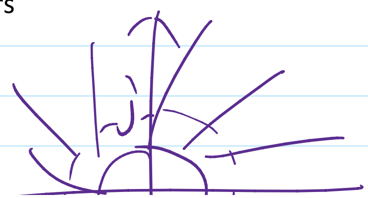
$$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$$

$z \neq 0$





$$H(z) = h[-2]z^{2} + h[-1]z^{1} + h[0] + h[1]z^{-1} + h[2]z^{-2}$$

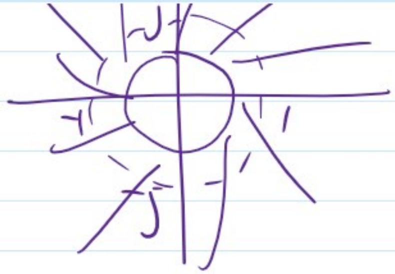


$$H(z)\big|_{z=e^{j\omega}} = H(e^{j\omega})$$

- Properties of BZT

9. Preview

- Design of a DT LTI system by using the BZT
  - stable system and ROC
    - causal rational stable
    - DTFT and ZTF
      - 1st-order and 2nd-order system
  - an example design of a feedback system
- UZT
  - Def.
  - UZT of a delayed signal
  - UZT and solving an LCCDE with initial conditions
    Fibonacci sequence F_{n+2} = F_{n+1}+F_{n}, for n\geq 0, F_{0}=0, F_{1}=1
    - ZIR로서의 Fibonacci sequence
    - ZSR로서의 Fibonacci sequence
- Sampling, Interpolation, Anti-Aliasing Filter
  - sampling
    - uniform vs. non-uniform sampling
    - uncountably infinite pairs vs. countably infinite pairs vs. finitely many pairs of (sampling time instant, function value)
  - ADC (analog-to-digital converter)
    - AAF,
    - CDC (continuous-time to discreter-time converter = sampler),
      - linear time-varying system
    - quantizer,
    - encoder
      - overflow vs. clipping
  - Sampling in the TD vs FD
  - Interpolation
    - ZOH, FOH
    - Perfect reconstruction
    - Nyquist's sampling theorem
  - DT processing of CT signals



$H(z)\big|_{z=e^{j\omega}} = H(e^{j\omega})$

$\underbrace{nx[n]}_{a^n u[n]} \overset{Z}{\longleftrightarrow}$

Stability

- LTI System Stable ⇔ $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$ ⇔

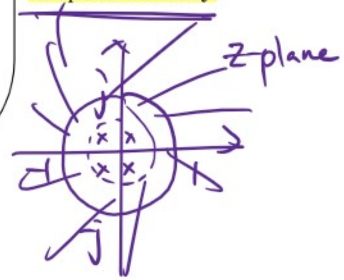
  ROC of $H(z)$ includes the unit circle $|z| = 1$

  ⇒ Frequency Response $H(e^{j\omega})$ (DTFT of $h[n]$) exists.

$\Sigma\, h[n] z^{-n}$ $\quad h[-1]z + h[-2]z^2 + \cdots$

- A causal LTI system with rational system function is stable ⇔ all poles are inside the unit circle, i.e. have magnitudes < 1.

no pole at infty.



$5 - 1 = 4$

Geometric Evaluation of a Rational z-Transform

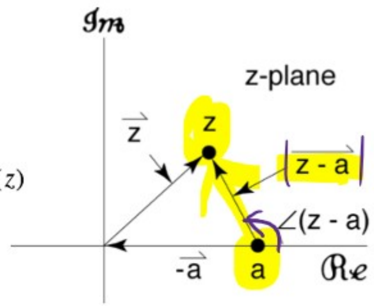
LT에서와 완전히 parallel 하다.

Example #1:
$X_1(z) = z - a$ — A first-order zero

Example #2
$X_2(z) = 1/(z - a)$ — A first-order pole

$|X_2(z)| = 1/|X_1(z)|, \quad \angle X_2(z) = -\angle X_1(z)$

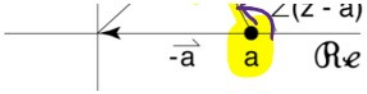
Example #3: $X(z) = M\dfrac{\prod_{i=1}^{R}(z-\beta_i)}{\prod_{j=1}^{P}(z-\alpha_j)}$



$H(e^{j\omega})$

$|H(z)|_{z=e^{j\omega}}$

$\angle H(z)|_{z=e^{j\omega}}$



$\angle H(z)\big|_{z=e^{j\omega}}$

Example #3: $X(z) = M\dfrac{\prod_{i=1}^{R}(z-\beta_i)}{\prod_{j=1}^{P}(z-\alpha_j)}$

$$|X(z)| = |M|\frac{\prod_{i=1}^{R}|z-\beta_i|}{\prod_{j=1}^{P}|z-\alpha_j|}$$

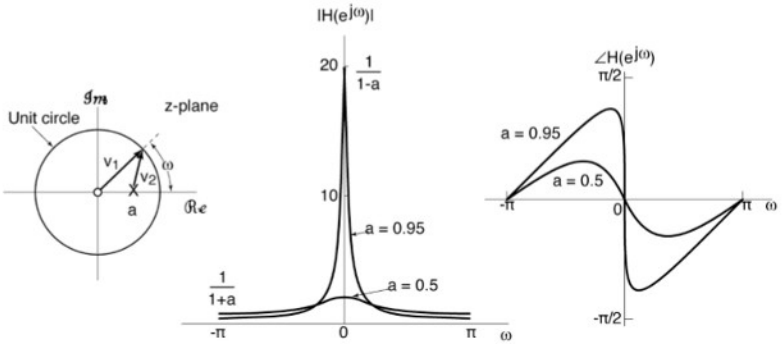
$$\angle X(z) = \angle M + \sum_{i=1}^{R}\angle(z-\beta_i) - \sum_{j=1}^{P}\angle(z-\alpha_j)$$

Geometric Evaluation of DT Frequency Responses

First-Order System
— one *real* pole

$$H(z) = \frac{1}{1-az^{-1}} = \frac{z}{z-a}, \quad |z| > |a| > 0$$

$$h[n] = a^n u[n], \quad |a| < 1$$



$$H(e^{j\omega}) = \frac{v_1}{v_2}, \quad |H(e^{j\omega})| = \frac{|v_1|}{|v_2|} = \frac{1}{|v_2|}, \quad \angle H(e^{j\omega}) = \angle v_1 - \angle v_2 = \omega - \angle v_2$$

pole zero의 위치를 파악할 때와 마찬가지로, DTFT를 ZT를 이용해 이해할때도 이 모양 (분자분모 모두 z의 rational function꼴)으로 바꾼다.

$H(z) = \sum \underbrace{h[n]}_{\text{real}} z^{-n}$

Second-Order System

real

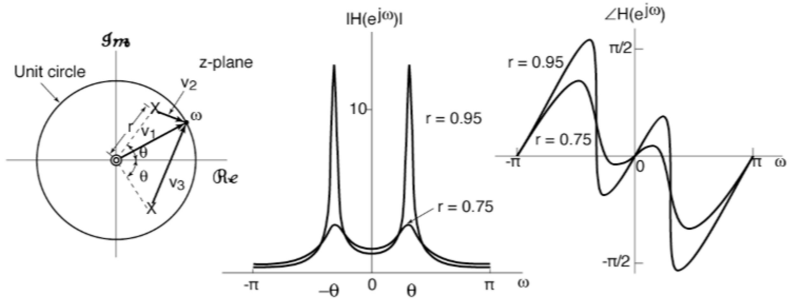
Second-Order System

Two poles that are a complex conjugate pair ($z_1 = re^{j\theta} = z_2^*$)

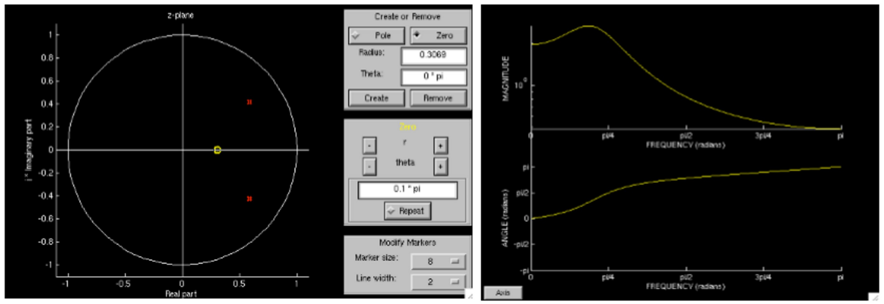
$$H(z) = \frac{z^2}{(z-z_1)(z-z_2)} = \frac{1}{1-(2r\cos\theta)z^{-1}+r^2z^{-2}}, \qquad 0 < r < 1, \quad 0 \le \theta \le \pi$$

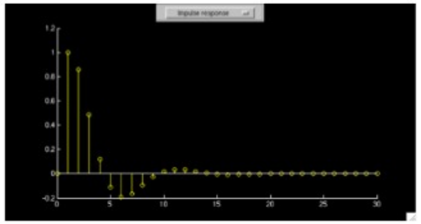
$$|H(e^{j\omega})| = \frac{1}{|(e^{j\omega}-re^{j\theta})(e^{j\omega}-re^{-j\theta})|}, \qquad h[n] = r^n \frac{\sin[(n+1)\theta]}{\sin\theta} u[n]$$

Clearly, $|H|$ peaks at $\omega = \pm\theta$ (remember the tent analogy).



Demo: DT pole-zero diagrams, frequency response, vector diagrams, and impulse- & step-responses





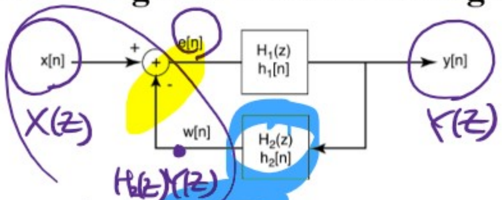
Q. Find $H(z)$ in terms of $H_1(z)$ & $H_2(z)$

A. $Y(z) = \cdots X(z)$

주어진 H1(z) system이 있는데 이 시스템이 stable하지 않다거나 하여 좀 변형을 하고 싶다. 이때 쓰는 방법이 보조 시스템 H2(z)를 만들어 왼쪽과 같이 피드백을 거는 방법이다. 전체 system function이 전과 후로 어떻게 바뀌나 확인하라.

System Function Algebra and Block Diagrams

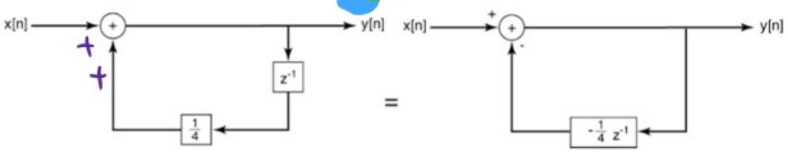
Feedback System
(Causal systems)



Black's formula:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{H_1(z)}{1 + H_1(z)H_2(z)}$$

Example #1:



$$H(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$\Downarrow$$

$$y[n] = \frac{1}{4}y[n-1] + x[n]$$

A delay in the feedback branch results in an exponential $h[n]$ — echo.

1/4은 다름아닌 pole의 위치이다.

$Y(z) = (X(x) - Y(z)H2(z))H2(z)$에서 구하면 ...

$E(z) = X(z) - H_2(z)Y(z)$

$H_1(z)\{X(z) - H_2(z)Y(z)\} = Y(z)$

feedback filter가 0이면 분자만 살아 남는다.--> 당연.

feedback의 역할은 H1(z)였던 system function을

H1(z)/ (1+ H1(z)H2(z))로 바꾼다.

제어에서 많이 사용된다.

impulse response의 ZTF이 first order pole system이면, 이는 delay component가 하나인 feedback LCCDE로 나타난다.

Example #2:

$Y(z) = (1 - 2z^{-1})V(z)$

$y[n] = v[n] - 2v[n-1]$

$Y(z) = (1-2z^{-1})V(z)$

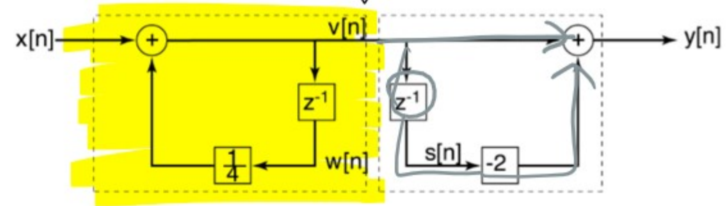
$y[n] = v[n] - 2v[n-1]$

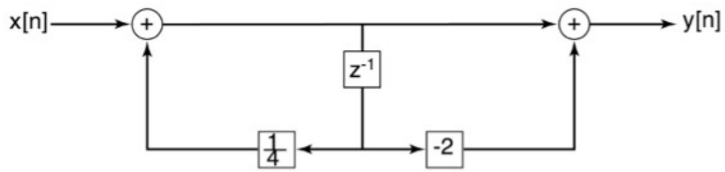
Example #2:

$$H(z) = \frac{1-2z^{-1}}{1-\frac{1}{4}z^{-1}} = \left(\frac{1}{1-\frac{1}{4}z^{-1}}\right)(1-2z^{-1})$$ — Cascade of two systems

$y[n] - \frac{1}{4}y[n-1] = x[n] - 2x[n-1]$

$\Downarrow$





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이제 UZT를 도입하여, 고등학교때 점화식 중에서 linear 했던 것들을 일반적으로 푸는 법을 배워보자.

Unilateral *z*-Transform – Many Parallels with $\mathcal{ULT}$

$$\mathcal{X}(z) = \sum_{n=0}^{+\infty} x[n]z^{-n} = x[0] + x[1]z^{-1} + x[2]z^{-2} \cdots$$

$\frac{1}{z}$ $\frac{1}{z^2}$ $\frac{1}{z^3}$

Note:

(1) If $x[n] = 0$ for $n < 0$, then $\mathcal{X}(z) = X(z)$

$x[n] = x[n]u[n]$

(2) $\mathcal{UZT}$ of $x[n]$ = $\mathcal{BZT}$ of $x[n]u[n]$

$\Rightarrow$ ROC *always* outside a circle and *includes* $z = \infty$

(3) For causal LTI systems

$$\mathcal{H}(z) = H(z)$$

Properties of Unilateral *z*-Transform

Many properties are the same with $\mathcal{BZT}$ ***E.g.***

$x[n] \overset{UZ}{\longleftrightarrow} X(z)$

$y[n]=x[n-1] \overset{UZ}{\longleftrightarrow}$?

- Convolution property (for $x_1[n<0] = x_2[n<0] = 0$)

$$x_1[n] * x_2[n] \overset{\mathcal{UZ}}{\longleftrightarrow} \mathcal{X}_1(z) \cdot \mathcal{X}_2(z)$$

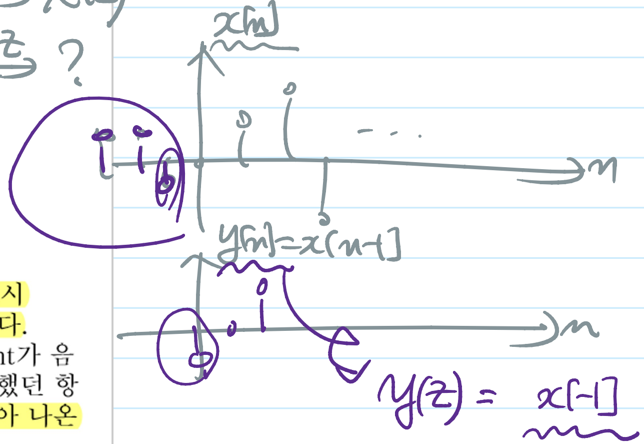
- But there are important differences. For example, *time-shift*

$$y[n] = x[n-1] \overset{\mathcal{UZ}}{\longleftrightarrow} \mathcal{Y}(z) = x[-1] + z^{-1}\mathcal{X}(z)$$

Derivation:

$$\mathcal{Y}(z) = \sum_{n=0}^{+\infty} y[n]z^{-n} = \sum_{n=0}^{+\infty} x[n-1]z^{-n} = x[-1] + \sum_{n=1}^{+\infty} x[n-1]z^{-n}$$

$$= x[-1] + z^{-1}\underbrace{\sum_{n=1}^{+\infty} x[n-1]z^{-(n-1)}}_{\mathcal{X}(z)} \qquad \text{QED}$$



여기서 x[n]은 반드시 causal인 것은 아니다. 그전에는 arguement가 음수라서 살아남지 못했던 항이 delay를 주면 살아 나온다.

ULT에서 미분하면 초기값이 나타나는 것과 유사하다.

Q. 왼쪽과 같이 LCCDE로 입출력관계가 모든 n에 대해 성립할때, 만약 입력이 causal하여 우변이 causal하면 항등관계에 의해 좌변도 causal하다. 따라서 양변을 unilateral ZT해도 양변이 같아야 한다. 이를 이용하여 y[n]의 nonnegative n에서의 값을 구하라.

Use of $\mathcal{UZT}$'s in Solving Difference Equations with Initial Conditions

$$y[n] + 2y[n-1] = x[n]$$

Q. Find y[n]. n≥0

$$y[-1] = \beta,\ x[n] = \alpha u[n] \longleftrightarrow \boxed{\frac{\alpha}{1-z^{-1}}}$$

$$y[-1]=\beta,\ x[n]=\alpha u[n] \longleftrightarrow \boxed{\frac{\alpha}{1-z^{-1}}}$$

$\mathcal{UZT}$ of Difference Equation

$$\mathcal{Y}(z)+2\underbrace{\left[\beta+z^{-1}\mathcal{Y}(z)\right]}^{\mathcal{UZ}\{y[n-1]\}}=\frac{\alpha}{1-z^{-1}}$$

Y(z) = ...

$$\Downarrow$$

$$\mathcal{Y}(z)=\underbrace{-\frac{2\beta}{1+2z^{-1}}}_{ZIR}+\underbrace{\frac{\alpha}{(1+2z^{-1})(1-z^{-1})}}_{ZSR} = \frac{A}{1+2z^{-1}}+\frac{B}{1-z^{-1}}$$

$\overset{UZ}{\leftrightarrow} + A(-2)^n u[n] + B u[n]$

ZIR – Output purely due to the initial conditions,

ZSR – Output purely due to the input.

...이 같아야 한다. 이를 이용하여 y[n]의 nonnegative n에서의 값을 구하라.

주의: initial condition은 이 경우 n=−1에서의 값으로 주어진다!!!

Example (continued)

β = 0 ⇒ System is initially at rest:

$$ZSR \quad \mathcal{Y}(z)=\mathcal{H}(z)\mathcal{X}(z)=\underbrace{\frac{1}{1+2z^{-1}}}_{\mathcal{H}(z)}\underbrace{\frac{\alpha}{1+z^{-1}}}_{\mathcal{X}(z)}$$

$$\mathcal{H}(z)=H(z)=\frac{1}{1+2z^{-1}}$$

α= 0 ⇒ Get response to initial conditions

$$ZIR \quad \mathcal{Y}(z)=-\frac{2\beta}{1+2z^{-1}}$$

$$\Rightarrow \quad y[n]=-2\beta(-2)^n u[n]$$

이와 같은 방식으로 피보나치 수열의 일반항을 구해보라.

라.

□ Fibonacci Sequence

$y[n] = y[n-1] + y[n-2], \quad \forall n \geq 0 \qquad y[-1]=1,\ y[-2]=0$

Q. $a_1 = 0,\ a_2 = 1,\ a_{n+2} = a_{n+1} + a_n, \quad n \geq 1$

Given,

find a_n as a function of n.

$z^2 - z - 1 = 0$

$z = \frac{1 \pm \sqrt{5}}{2}$

A1. $y[n] \triangleq a_{n-2}$

$$y[-1] = 1,\ y[-2] = 0$$

$$y[n] = y[n-1] + y[n-2] + x[n] \qquad x[n] = 0,\ \forall n.$$

UZT $\Rightarrow$

$$Y(z) = \boxed{z^{-1}Y(z) + y[-1]} + z^{-2}Y(z) + z^{-1}y[-1] + y[-2]$$

$$\Rightarrow \boxed{(1 - z^{-1} - z^{-2})Y(z) = 1 + z^{-1}}$$

$$\Rightarrow Y(z) = \frac{1 + z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z^2 + z}{z^2 - z - 1} = \cancel{\frac{\bigcirc}{(z-p)} + \frac{\bigcirc}{(z-)}} = \frac{A}{1 - p_1 z^{-1}} + \frac{B}{1 - p_2 z^{-1}}$$

$$\updownarrow$$

$$A(p_1)^n u[n] + B(p_2)^n u[n], \quad n \geq 0$$

$$\Rightarrow y[n] = A\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + B\left(\frac{1-\sqrt{5}}{2}\right)^n u[n]$$

$y[0] = A + B = 1$

$y[1] = \frac{A+B}{2} + \frac{(A-B)\sqrt{5}}{2} = 2$

$\therefore \begin{cases} A + B = 1 \\ A - B = \frac{2}{\sqrt{5}} \end{cases}$

$\Rightarrow A = \cdots$

$B = \cdots$

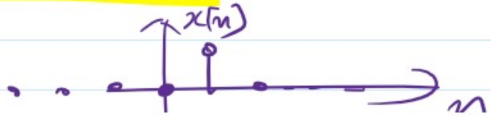
A2. $y[n] = a_{n+1}$

$= F_n$

$$y[n] - y[n-1] - y[n-2] = x[n], \quad n \geq 0$$

$y[n] = y[n]u[n], \quad x[n] = x[n]u[n],$

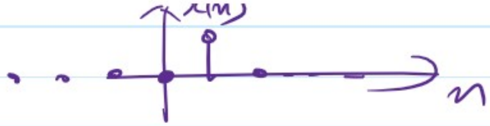
$y[0] = x[0] = 0$



$y[0] = x[0] = 0$

$y[1] = x[1] = 1$

$x[n] = 0, \quad n \geq 2$



$$Y(z)\left(1 - z^{-1} - z^{-2}\right) = z^{-1}$$

$$\Rightarrow Y(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}}$$

$$\Rightarrow y[n] = A\left(\frac{1+\sqrt{5}}{2}\right)^n u[n] + B\left(\frac{1-\sqrt{5}}{2}\right)^n u[n]$$

$y[0] = A + B = 0$

$y[1] = \dfrac{A+B}{2} + \dfrac{(A-B)\sqrt{5}}{2} = 1$

$$\begin{cases} A + B = 0 \\ A - B = \dfrac{2}{\sqrt{5}} \end{cases}$$

$$\Rightarrow A = \frac{1}{\sqrt{5}}, \quad B = -\frac{1}{\sqrt{5}}$$

$$\therefore y[n] = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n\right\} u[n] \doteq F_n$$

$$\therefore a_n = \frac{1}{\sqrt{5}}\left\{\left(\frac{1+\sqrt{5}}{2}\right)^{n-1} - \left(\frac{1-\sqrt{5}}{2}\right)^{n-1}\right\}, \quad n \geq 1$$

Definition [edit]

The Fibonacci numbers may be defined by the recurrence relation[6]

$$F_0 = 0, \quad F_1 = 1,$$

and

$$F_n = F_{n-1} + F_{n-2}$$

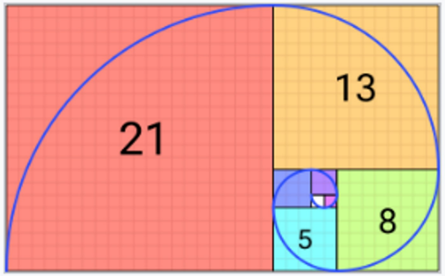
for $n > 1$.

Under some older definitions, the value $F_0 = 0$ is omitted, so that the sequence starts with $F_1 = F_2 = 1$, and the recurrence $F_n = F_{n-1} + F_{n-2}$ is valid for $n > 2$.[7][8]

The first 20 Fibonacci numbers F_n are:[1]

F_0	F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}
0	1	1	2	3	5	8	13	21	34	55	89	144	233	377	610	987	1597	2584	4181

$$F_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$



The Fibonacci spiral: an approximation of the golden spiral created by drawing circular arcs connecting the opposite corners of squares in the Fibonacci tiling (see preceding image)

Closed-form expression [edit]

Like every sequence defined by a linear recurrence with constant c
known as **Binet's formula**, named after French mathematician Jac
Daniel Bernoulli:[22]

$$F_n = \frac{\varphi^n - \psi^n}{\varphi - \psi} = \frac{\varphi^n - \psi^n}{\sqrt{5}},$$

where

$$\varphi = \frac{1+\sqrt{5}}{2} \approx 1.61803\,39887\ldots$$

is the golden ratio, and ψ is its conjugate:[23]

$$\psi = \frac{1-\sqrt{5}}{2} = 1 - \varphi = -\frac{1}{\varphi} \approx -0.61803\,39887\ldots.$$

Since $\psi = -\varphi^{-1}$, this formula can also be written as

$$F_n = \frac{\varphi^n - (-\varphi)^{-n}}{\sqrt{5}} = \frac{\varphi^n - (-\varphi)^{-n}}{2\varphi - 1}.$$

Limit of consecutive quotients [edit]

Johannes Kepler observed that the ratio of consecutive Fibonacci numbers converges. He wrote that "as 5 is to 8 so is 8 to 13, practically, and as 8 is to 13, so is 13 to 21 almost", and concluded that these ratios approach the golden ratio φ: [27][28]

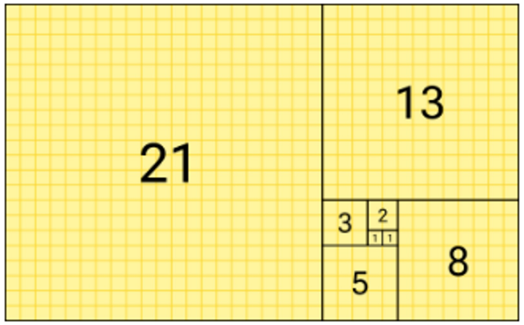
$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \varphi.$$

This convergence holds regardless of the starting values U_0 and U_1, unless $U_1 = -U_0/\varphi$. This can be verified using Binet's formula. For example, the initial values 3 and 2 generate the sequence 3, 2, 5, 7, 12, 19, 31, 50, 81, 131, 212, 343, 555, ... The ratio of consecutive terms in this sequence shows the same convergence towards the golden ratio.

In general, $\lim_{n\to\infty} \frac{F_{n+m}}{F_n} = \varphi^m$, because the ratios between consecutive Fibonacci numbers approaches φ.

$$\sum_{i=1}^{n} F_i^2 = F_n F_{n+1}$$

or in words, the sum of the squares of the first Fibor
begin with a Fibonacci rectangle of size $F_n \times F_{n+1}$
comparing areas:



Generating function [edit]

The generating function of the Fibonacci sequence is the power series

$$s(z) = \sum_{k=0}^{\infty} F_k z^k = 0 + z + z^2 + 2z^3 + 3z^4 + \dots .$$

This series is convergent for any complex number z satisfying $|z| < 1/\varphi$, and its sum has a simple closed form:[35]

$$s(z) = \frac{z}{1 - z - z^2}.$$

This can be proved by multiplying by $(1 - z - z^2)$:

$$\begin{aligned}
(1 - z - z^2)s(z) &= \sum_{k=0}^{\infty} F_k z^k - \sum_{k=0}^{\infty} F_k z^{k+1} - \sum_{k=0}^{\infty} F_k z^{k+2} \\
&= \sum_{k=0}^{\infty} F_k z^k - \sum_{k=1}^{\infty} F_{k-1} z^k - \sum_{k=2}^{\infty} F_{k-2} z^k \\
&= 0z^0 + 1z^1 - 0z^1 + \sum_{k=2}^{\infty} (F_k - F_{k-1} - F_{k-2}) z^k \\
&= z,
\end{aligned}$$

EECE233 Signals and Systems

Lecture Note #18

Prof. Joon Ho Cho

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

Digital 시대가 도래하여 눈부신 발전(가격과 성능–processing speed, power consumption, size)을 이루고 있는데,
CT signal의 processing을 위해서는 계속 CT system을 만들어야만 할까?

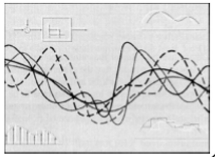
CT signal을 이의 sample들로 나타내어 DT system

CT signal $x(t)$ $x: \mathbb{R} \to \mathbb{C}$

(Slides thanks to Qing Hu*, D. Boning, D. Freeman, J.
Weiss, J. White, and A. Willsky)

Covers O & W pp. 514-527

- Sampling of a CT Signal
- Analysis of Sampling in the Frequency Domain
- The Sampling Theorem — the Nyquist rate
- In the Time Domain: Interpolation
- Undersampling and Aliasing



* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

1

CT signal을 이의 sample들로 나타내어 DT system으로 처리하면 어떨까?

sample들이 CT signal의 모든 정보를 손실없이 가질 수 있을까?

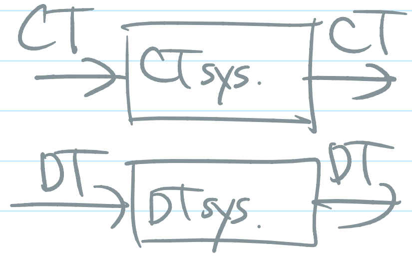
music CD가 나왔을때,

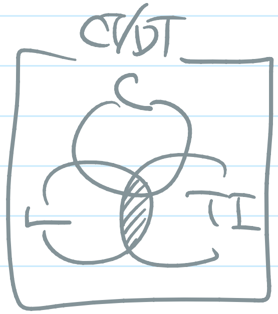
LP판과 CD의 '음질 차이를 알 수 있다'고 주장하는 사람들이 있었는데 이 주장은 옳은 주장일까?

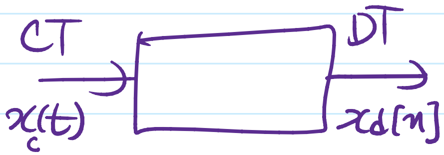
예를 들어 CD quality인 44 kHz sampling rate란 무슨 소리일까?

CT signal $x(t)$ $x: \mathbb{R} \to \mathbb{C}$

DT signal $x[n]$ $x: \mathbb{Z} \to \mathbb{C}$.







uniform sampling

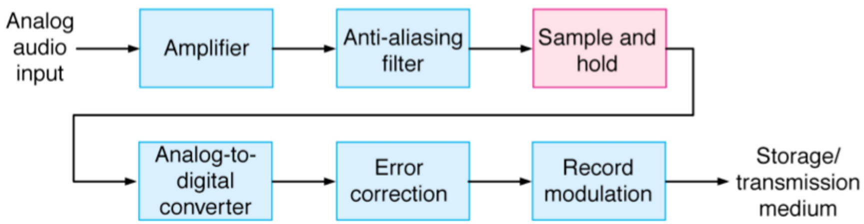
$x_d[n] \triangleq x_c(nT_s+\Delta), \forall n$

TD processing 인 sampling을 FD에서 이해해 보자.
핵심은
sampling전의 CT 신호의 CTFT와
sampling후의 DT 신호의 DTFT와의 관계를 규명하는 것이다.

SAMPLING

— A crucial step in converting CT signals to DT, so that we can use versatile digital computers or DSPs to process them.

Example: Digital recording of sounds



ADC output에 error가 있는 것이 아니고, 앞으로의 processing에서 error가 발생하면 correction할 수 있도록 redundancy를 추가하는 과정이다.

이 AAF가 왜 필요한지는 차차 설명한다.

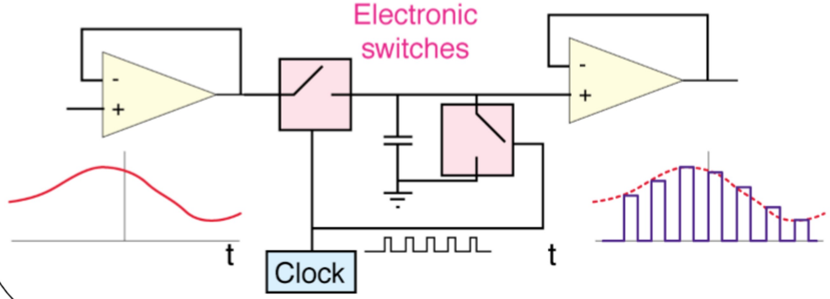
sample and hold는 CT signal을 piecewise constant function으로 approximate하는 CT−input CT−output 시스템이다.

.cda format
sampling rate 44.1.kHz
number of bits 16

How do we perform sampling?

Taking snap shots of $x(t)$ every T second. That is, taking $x(nT)$, and later convert $x(nT)$ to $x[n]$.

Example: A sample-and-hold circuit that produces such periodic snap shots.



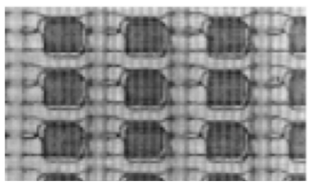
두 스위치는
하나가 닫히면 하나는 열린다.
충전과 방전을 반복한다.
첫번째 사이클에서 왼쪽의 스위치는 충전하고 오른쪽의 스위치는 open되어 있다. ADC 출력으로 신호가 적분되어 나온다.

두번째 사이클에서는 오른쪽의 스위치가 close되고 왼쪽이 open된다. 이에 따라 충전된 전하가 방전되고 출력에는 0이 나온다.

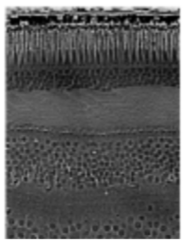
앞서 학기초에 잠시 설명했듯,
discrete-time signal processing이라고 하면, amplitude의 quantization과 finite resolution을 생각지 않는 경우가 많고
digital signal processsing이라하면 quatization과 finite resolution을 포함하여 말하는 경우가 많다.
이 과목에서는 다루지 않지만, quantization effect와 finite resolution에서 오는 한계는 매우 중요한 토픽이다.

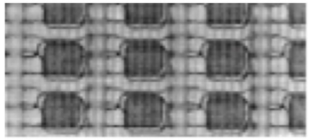
The issue of sampling also applies to spatially varying signals

Examples: Light-sensing part of a digital camera — a CCD (Charge-Coupled-Device) device. The spacing between the adjacent elements determines the *Megapixel*. Also, human's (and all animals') light sensing is discrete.

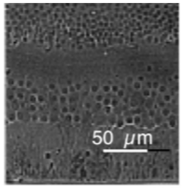


Rods & Cones





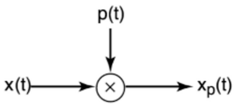
CCD (charge-coupled-device) at the focal plane of a digital camera



Cross section view of a human retina 망막

Mathematical Model of *Idealized* Sampling

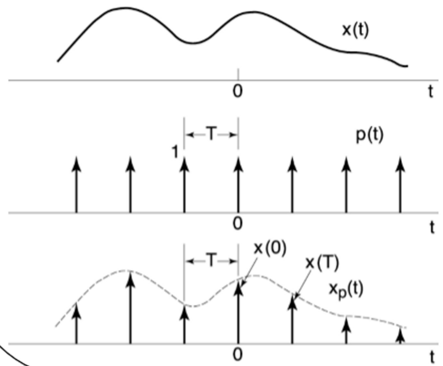
Impulse Train Approach — Multiplying $x(t)$ by the sampling function



$$p(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT) \quad – \text{ sampling function}$$

$$x_p(t) = x(t) \cdot p(t) = \sum_{n=-\infty}^{+\infty} x(t)\delta(t - nT)$$

$$= \sum_{n=-\infty}^{+\infty} x(nT)\delta(t - nT)$$



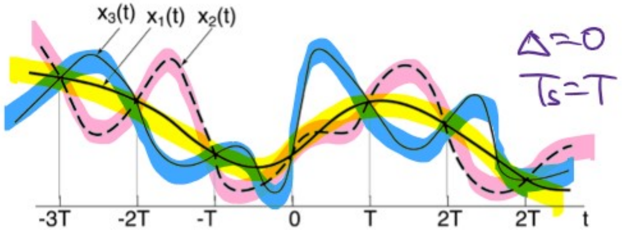
여기서 sampling은 CT signal을 CT modulated impulse train으로 바꾸는 것을 의미한다.

Why/When Would a Set of Samples Be Adequate?

Q.

Why/When Would a Set of Samples Be Adequate?

- Observation: $x_1(t)$, $x_2(t)$, $x_3(t)$ and many other signals have the same samples



$\Delta=0$
$T_s=T$

- Obviously, by doing sampling we throw out some (a lot) information (all values of $x(t)$ between sampling points are lost).

- **Key Question for Sampling**:
Under what conditions can we reconstruct the original CT signal $x(t)$ from the sampled signal $x_p(t)$? — $x(t)$ cannot change too fast.

Q.
rate 1/T인 sampler를 사용할때,
임의의 CT신호에 대해
sampling후의 DT신호가
sampling전의 CT신호의 모든 정보를 그대로 유지하고
있을 필요충분 조건
on sampling rate?
H.
uncountably infinite개의 정보를 요구하는 CT signal을
countably infinite개의 정보로 바꾸는 sampling은
일반적으로 가역적인 처리가 아니다.
왼쪽 그림에서처럼, 서로다른 CT signals가 sampling
후에 같은 신호로 바뀐다.