EECE233 Signals and Systems

Lecture Note #1

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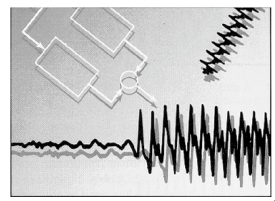
(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

1) Administrative details
2) Signals



* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

Signals and Systems

EECE233 is about using mathematical techniques to help analyze and synthesize systems which process signals.

- Signals are variables that carry information.
- Systems process input signals to produce output signals.

Today: Signals, next lecture: Systems.

Examples of signals

- Electrical signals --- voltages and currents in a circuit
- Acoustic signals --- audio or speech signals (analog or digital)
- Video signals --- intensity variations in an image (e.g. a CAT scan)
- Biological signals --- sequence of bases in a gene

⋮

- In EECE233, we will treat noise as unwanted signals

signal
- scalar
- vector
- matrix
- sequence
- waveform
- field
- polynomial
- tensor
- graph
- ⋮

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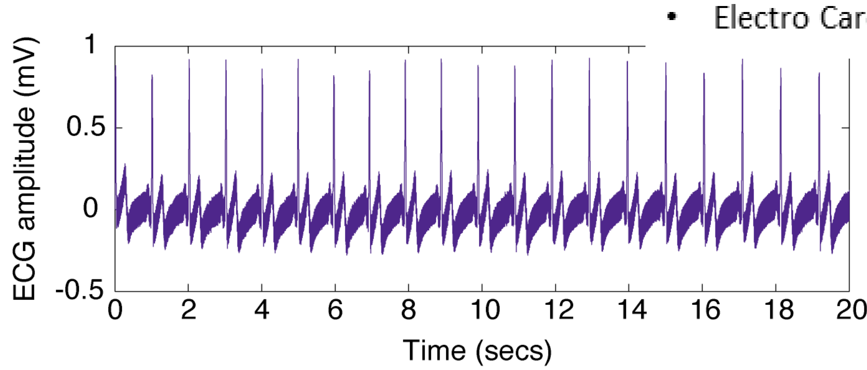
Signal Classification

Type of Independent Variable

Time is often the independent variable. Example: the electrical activity of the heart recorded with chest electrodes — the electrocardiogram (ECG or EKG).

- Electro Cardio Gram 심전도



The variables can also be spatial

Eg. Cervical MRI

In this example, the signal is the intensity as a function of the spatial variables x and y.

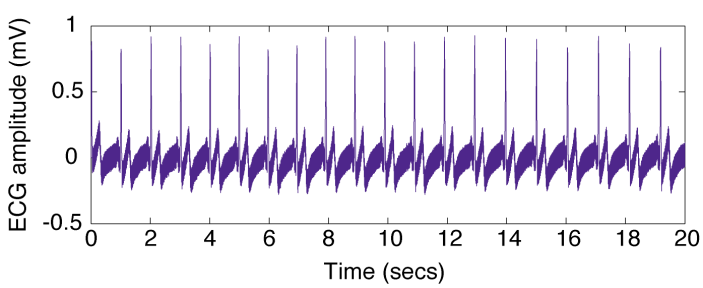
- cervical: 목의
- The left signal can be modeled as, e.g.,

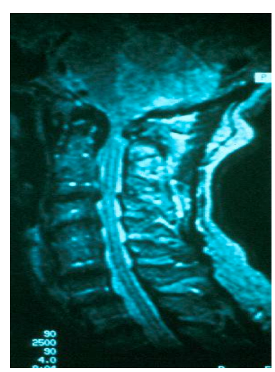
x: {1,2,...,800}x{1,2,...,600} -> [0,1]

Since the domain is 2-D, this signal is a 2-D signal.

Independent Variable Dimensionality

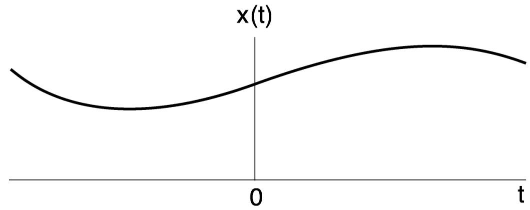
An independent variable can be 1-D (t in the EKG) or 2-D (x, y in an image).





In EECE233, focus on 1-D for mathematical simplicity but the results can be extended to 2-D or even higher dimensions. Also, we will use a generic time t for the independent variable,whether it is time or space.

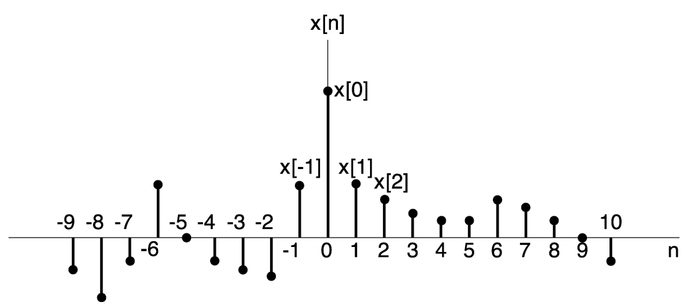
Continuous-time (CT) Signals



- Most of the signals in the physical world are well modeled by CT signals, where the time scale is infinitesimally fine, so are the spatial scales. E.g. voltage & current, pressure, temperature, velocity, etc.

Discrete-time (DT) Signals

- $x[n]$, n — integer, time varies discretely



- Examples of DT signals in nature:
  — DNA base sequence
  — Population of the nth generation of certain species

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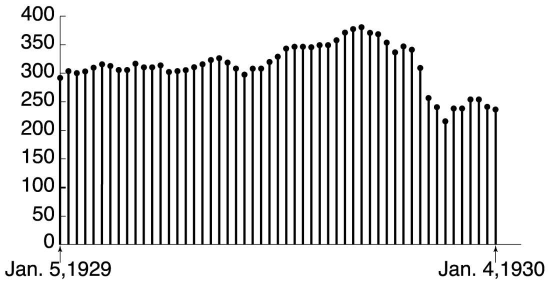
- Notation in EECE233: $x(t)$ — CT, $x[n]$ — DT

Many human-made Signals are DT

Ex.#1 Weekly Dow-Jones industrial average



Ex.#2 digital image

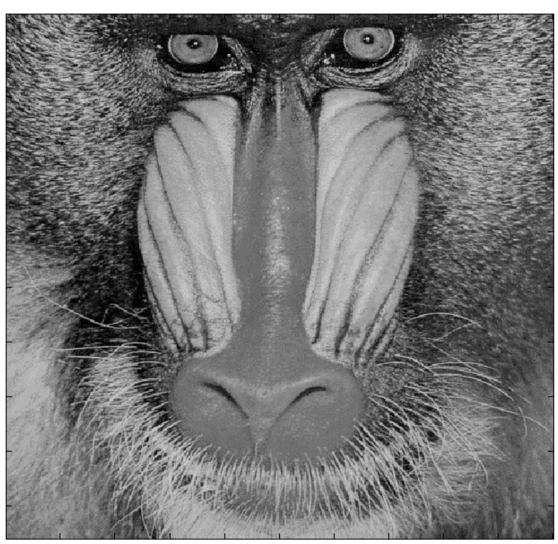
Ex#3. ticking clock

⋮

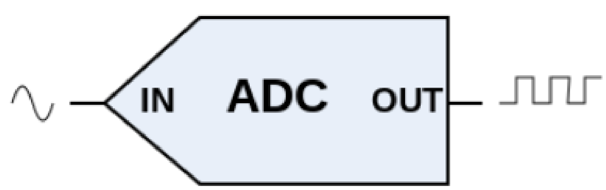
Why DT? — Can be processed by modern digital computers and digital signal processors (DSPs).

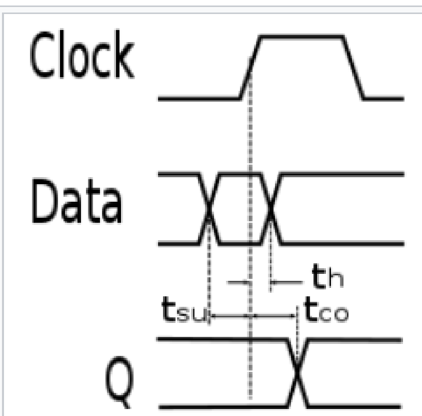
Mandrill Example

Unblurred Image & No Noise

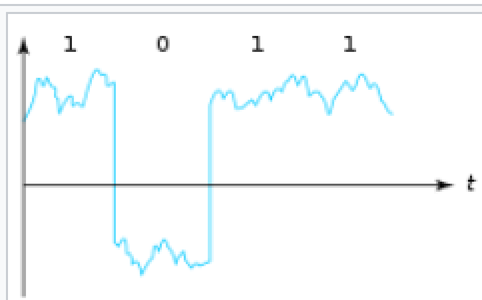


- a scalar: a 0–D signal because x: {1}->R
- a vector: a 1-D signal because x: {1,…,300} ->R
- a matrix: a 2-D signal because x: {1,…,M}x{1,…,N} ->R
- a 60 FPS BW video of 600x300 pixels
  - a (2-D matrix-valued) 1-D signal X[n] because x: Z-> a set of 1-bit 600x300 BW images
  - a (scalar-valued) 3-D signal X_n[k,l] because x: Zx {1,…,300} x{1,…,600}-> {0,1}





Clocking digital signals through a clocked flip-flop



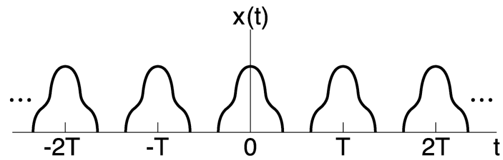
A received digital signal may be impaired by noise and distortions without necessarily affecting the digits

Signals with symmetry

definition, existence, uniqueness; a, an, the
정의, 존재성, 유일성

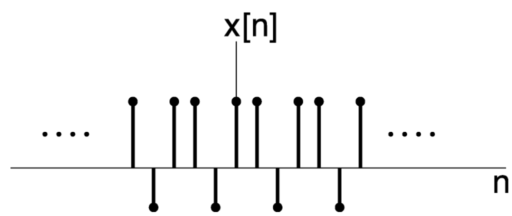
- Periodic signals

CT $\quad x(t) = x(t + T)$ for all t.



- **Ex.** 60-Hz power line, computer clock, etc.

DT $\quad x[n] = x[n + N]$ for all n.



Def. A CT signal x(t) is periodic, if there exists T>0 s. t. x(t) = x(t+T), for all t.
Lemma. If periodic, then there exists T>0 s.t. x(t) = x(t+nT), for all t and all integer n.

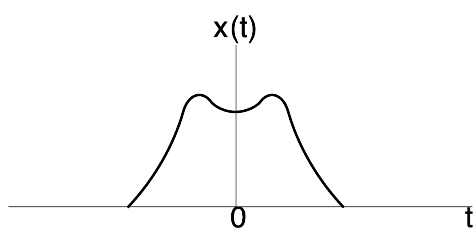
Def. For a periodic signal, the smallest period is called the fundamental period, if exists.
Lemma. For a periodic signal with fundamental period T_0, any period is an integer multiple of T_0.

Signals with symmetry (continued)

- Even and odd signals

Even $\quad x(t) = x(-t)$ or $x[n] = x[-n]$

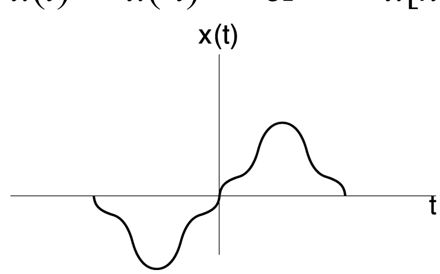
짝우,
우함수



Odd $\quad x(t) = -x(-t)$ or $x[n] = -x[-n]$

$x(0) = 0$

or $\quad x[0] = 0$



The zero function is even and odd at the same time.

Signals with symmetry (continued)

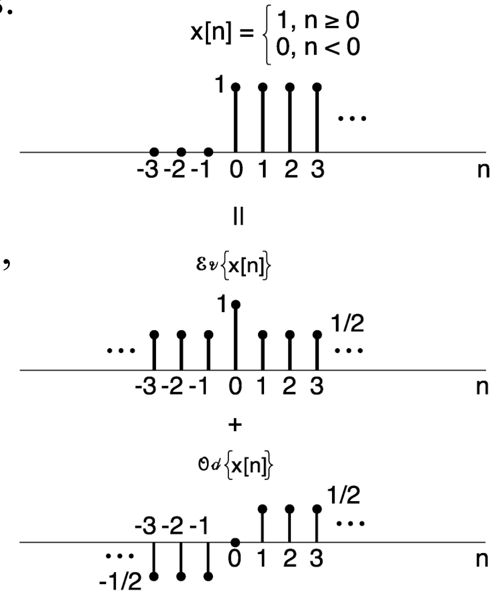
Lemma. Any signals can be expressed as a sum of *Even* and *Odd* signals. That is:

$$x(t) = x_{even}(t) + x_{odd}(t) \ ,$$

where

$$x_{even}(t) = [x(t) + x(-t)]/2 \ ,$$
$$x_{odd}(t) = [x(t) - x(-t)]/2 \ .$$

Ex. DT unit-step function.



u[n]은 the unit step function이라 불리며
argument n이 음수이면 u[n]=0,
나머지 모든 경우는 u[n]=1이다.

when A is square.

$$A = \frac{A+A^T}{2} + \frac{A-A^T}{2}$$

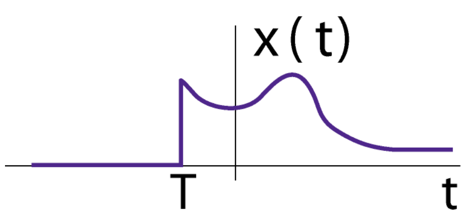
$$A = \frac{A+A^H}{2} + \frac{A-A^H}{2}$$

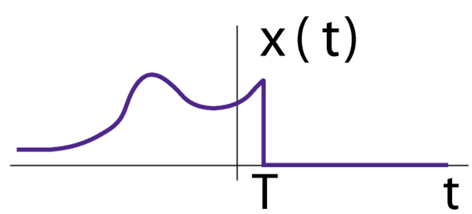
$$x(t) = \underbrace{\frac{x(t)+x^*(-t)}{2}}_{\text{conjugate symmetric}} + \underbrace{\frac{x(t)-x^*(-t)}{2}}_{\text{conjugate anti-symmetric}}$$

Right- and Left-Sided Signals

Def.

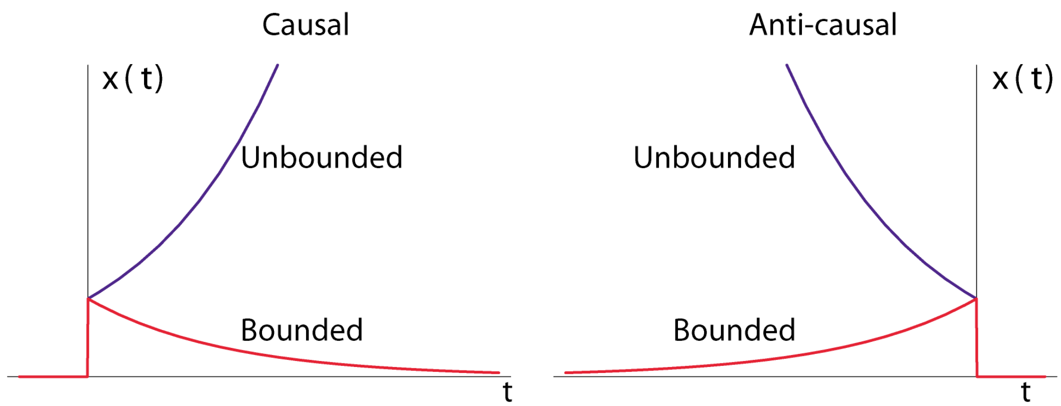
A right-sided signal is zero for $t < T$ and a left-sided signal is zero for $t > T$, where T can be positive or negative.





[commonality
difference] → 압축
compress

Bounded and Unbounded Signals



Whether the output signal of a *system* is bounded or unbounded determines the stability of the system.
Eg. **Demo**, the inverted pendulum, unstable but can be stabilized using a feedback control system.

Def. $x(t)$ is bounded if $\exists\, M \in \mathbb{R}_+$ s.t. $|x(t)| \le M,\ \forall t$,

Def. $x(t)$ is causal if $x(t)=0,\ \forall t<0$. $\lfloor x(t) = O(1))$

Real and Complex Signals

A very important class of signals is:

- CT signals of the form $x(t) = e^{st}$
- DT signals of the form $x[n] = z^n$

Q Find x such that $\sin x = 2$.

where z and s are complex numbers. For both *exponential* CT and DT signals, x is a complex quantity and has:

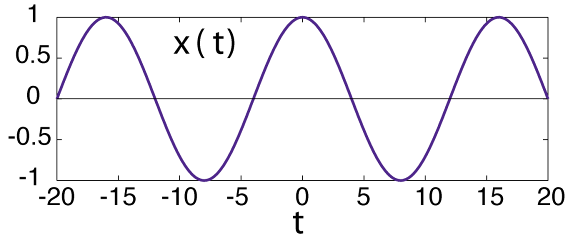
- a real and imaginary part, or equivalently
- a magnitude and a phase angle.

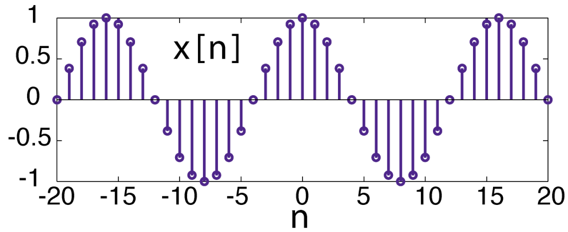
We will use whichever form that is convenient.

For example, suppose $s = j\pi/8$ and $z = e^{j\pi/8}$, then the real parts are

$$\mathcal{R}\{x(t) = e^{st}\} = \mathcal{R}\{e^{j\pi t/8}\} = cos(\pi t/8),$$

$$\mathcal{R}\{x[n] = z^n\} = \mathcal{R}\{e^{j\pi n/8}\} = cos[\pi n/8].$$





Summary

- We are awash in a sea of signal — sounds, visual, electrical, thermal, mechanical, etc.

- Signals can be time-varying or spatially varying, can be a function of multiple variables. However, in EECE233, we will mostly deal with 1D signals and use a generic time t to represent the variable,whether it is time, space, or something else.

- DT signals and systems become more and more important for signal processing, and will be a major part in EECE233.

Next lecture covers:

O & W pp. 39-56

$e^{j\theta} = \cos\theta + j\sin\theta$

$z = a + jb = re^{j\theta}, \quad a, b, r, \theta \in \mathbb{R}, \quad r \geq 0$

$|z| = r$ modulus

$z^* = a - jb = re^{-j\theta}$

$z^{-1} = \frac{1}{r}e^{-j\theta}$

$z^{-*} = (z^*)^{-1} = \frac{1}{r}e^{j\theta}$

정보통신 시대를 연 천재적인 공학자 클로드 섀넌의 "문제해결의 6단계"

단순화에서 시작하여 다시 복잡화를 통해 본래에 문제를 해결할 뿐만 아니라 새로운 문제를 찾아내고 해결함.

Step 1: Simplify. Simplify. Simplify.

Step 2: Fill your 'mental matrix' with solutions to similar problems.

Step 3: Approach the problem from many different angles.

Step 4: Break a big problem down into small pieces.

Step 5: Solve the problem 'backwards.'

Step 6: If you've solved the problem, extend that solution out as far as it will go.