EECE233 Signals and Systems

Lecture Note #20

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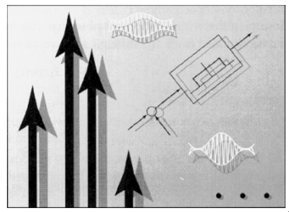
(Slides thanks to Qing Hu*, D. Boning, D. Freeman, T. Weiss, J. White, and A. Willsky)

MODEM, covers O & W pp. 582-599

1. Complex Exponential Amplitude Modulation
2. Sinusoidal AM
3. Demodulation of Sinusoidal AM
4. Single-Sideband (SSB) AM
5. Frequency-Division Multiplexing
6. The Superheterodyne Receivers



* These lecture notes are based on the lecture notes used in 6.003 taught b Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

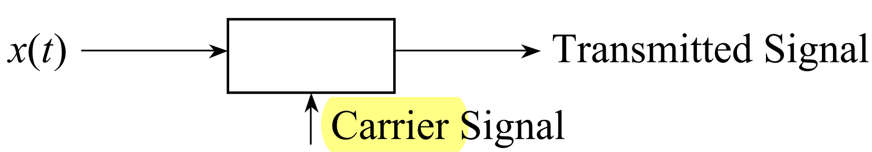
#20, #21에서는 앞서 배운바 있는 modulation property의 응용에 대해 배운다.

여기서 배울 내용은 아날로그 및 디지털 통신의 기초로 매우 중요하다.

기억해야 할 가장 중요한 것은, Radio Communications에서 송신 안테나 포트 입력 또는 수신 안테나 포트 출력 CT signal은 real-valued bandpass signal로 모델링된다는 것이다.

안테나는 전류를 radio wave로 or vice versa하는 기구인데, 효율적으로 송수신하려면 전류 주파수와 광속으로부터 얻어지는 파장의 길이와 비슷한 크기의 안테나가 필요하다.
따라서 안테나의 크기를 줄이려면 높은 주파수의 반송파(carrier)를 사용해야 한다.

The Concept of Modulation (modulating a carrier)

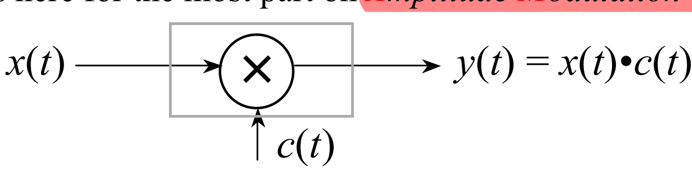


Why?

- More efficient to transmit E&M signals at higher frequencies.
- Transmitting multiple signals through the same medium using different carriers.
- Increase signal/noise ratio in lock-in measurements.
- others...

How?

- *Many* methods
- Focus here for the most part on *Amplitude Modulation* (AM)



예를 들어 WiFi와 5G 셀룰러 주파수의 중간 쯤이 3GHz인데 파장이 10 cm 이다.

낮은 주파수에서는 효율이 좋은 안테나를 작은 크기로 만들기가 어렵다. 보통 파장의 절반 이상 크기는 되어야 효율적이라 한다. 그런데 주파수가 낮으면 파장이 지나치게 길게 된다.

주파수는 한정된 자원이기 때문에 각 국의 regularoty organization에 의해 관리 운영된다. 보다 많은 사용자의 이용을 위해 frequency domain에서 구간 구간 잘라서 할당하는 방법을 쓴다.

이는 two-input one-output system이고 output은 두 inputs의 곱인 소자로 mixer라 부른다.

*A*mplitude *M*odulation (AM) of a Complex Exponential Carrier

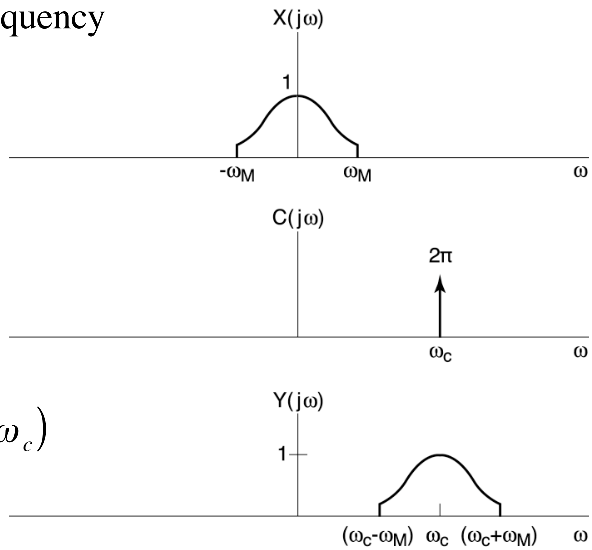
$c(t) = e^{j\omega_c t}$, ω_c — carrier frequency

$y(t) = x(t)\, e^{j\omega_c t}$

$$\mathrm{Y}(j\omega) = \frac{1}{2\pi}\mathrm{X}(j\omega) * \mathrm{C}(j\omega)$$

$$= \frac{1}{2\pi}\mathrm{X}(j\omega) * 2\pi\delta(\omega - \omega_c)$$

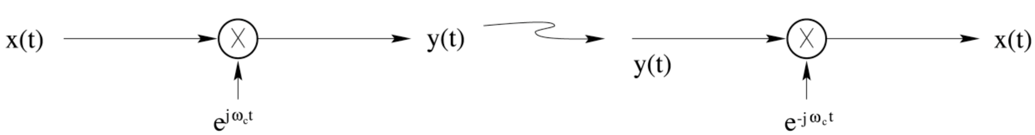
$$= \mathrm{X}(j(\omega - \omega_c))$$



먼저 warming up으로 단순화된 문제부터 해결해 본다.

반드시 기억: \omega_c 주파수의 complex exponential은 곱해지는 신호를 주파수 영역에서 그대로 우측으로 \omega_c만큼 shift 한다.

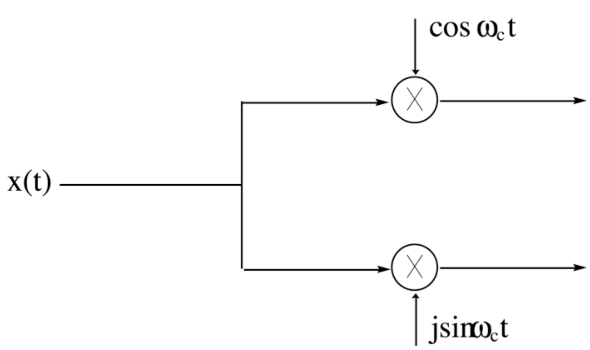
Demodulation of Complex Exponential AM



$$e^{j\omega_c t} = \cos\omega_c t + j\sin\omega_c t$$

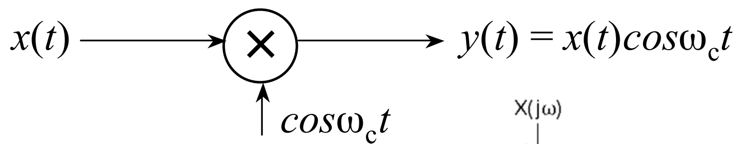
$$\Downarrow$$

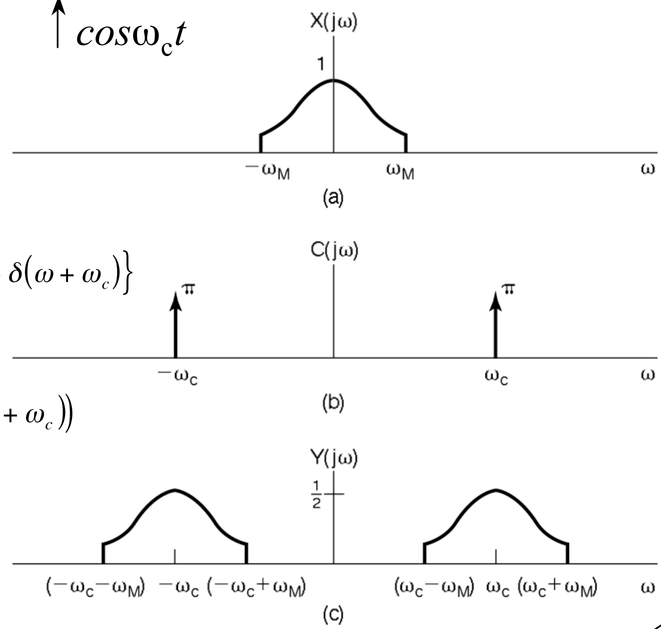
Corresponds to two separate modulation channels (quadratures) with carriers 90° out of phase



모뎀 (MODEM: MOdulator DEModulator 변복조기)

Sinusoidal AM





$$Y(j\omega) = \frac{1}{2\pi} X(j\omega) * \pi\{\delta(\omega - \omega_c) + \delta(\omega + \omega_c)\}$$

$$= \frac{1}{2} X(j(\omega - \omega_c)) + \frac{1}{2} X(j(\omega + \omega_c))$$

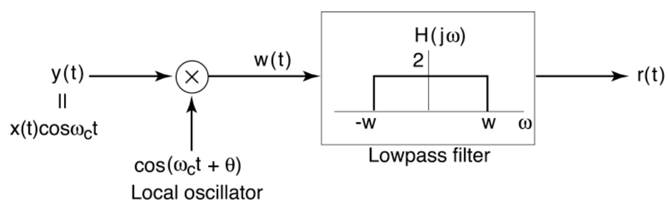
Drawn assume $\omega_c > \omega_M$

이제 real-bandpass signal을 생각해 보자.

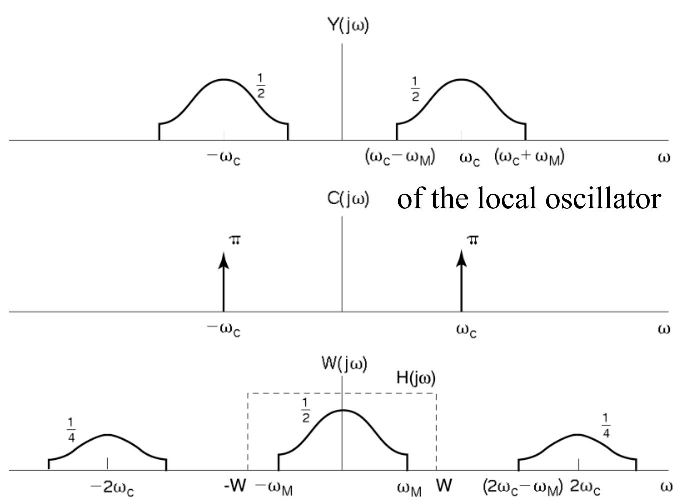
Key idea: Euler's identity을 이용하여 complex exponentials의 합으로 바꾼다.

\omega_c만큼 우측으로 이동하고
또 \omega_c만큼 이동하여 더한후 1/2로 scale한다.

Synchronous Demodulation of Sinusoidal AM



Suppose $\theta = 0$ for now,
⇒ Local oscillator is *synchronized* with the carrier. Implemented in practice by using phase-locked loops (PLL).



AM의 경우 time domain과 frequency domain에서 정확히 이해할 것!!!
그리고 filtering은 그림을 그려서 이해할 것.

Synchronous Demodulation in the Time-Domain

$$w(t) = y(t)\cos\omega_c t = x(t)\cos^2\omega_c t = \frac{1}{2}x(t) + \underbrace{\frac{1}{2}x(t)\cos 2\omega_c t}_{\text{High-frequency signals filtered out by the LPF}}$$

Then $r(t) = x(t)$

Now suppose there is a phase difference, *i. e.* $\theta \neq 0$, then

$$w(t) = y(t)\cos(\omega_c t + \theta) = x(t)\cos\omega_c t\cos(\omega_c t + \theta)$$
$$= \frac{1}{2}x(t)\cos\theta + \underbrace{\frac{1}{2}x(t)\cos(2\omega_c t + \theta)}_{\text{HF signal}}$$

Again $r(t) = x(t)\cos\theta$

Two special cases:

1) $\theta = \pi/2$, the local oscillator is 90° out of phase with the carrier, $\Rightarrow r(t) = 0$, no signals.

2) $\theta = \theta(t)$ — slowly varying with time, $\Rightarrow r(t) = \cos[\theta(t)]\cdot x(t)$, $\Rightarrow$ time-varying "gain".

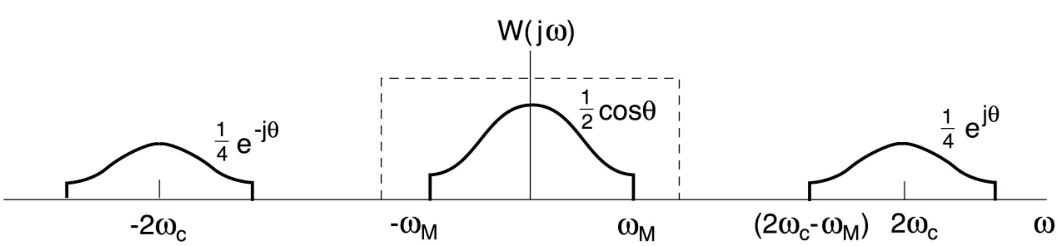
멀리 떨어져 있는 송신기와 수신기의 오실레이터가 같은 phase를 가진다고 가정하는 것은 비현실적이다. 따라서 phase offset이 있다고 가정해 보자. 그러면 이상의 synchronous demodulation에 어떤 문제가 생기는가?

Synchronous Demodulation in the Frequency-Domain

$$\cos(\omega_c t + \theta) = \frac{1}{2}e^{j\theta}e^{j\omega_c t} + \frac{1}{2}e^{-j\theta}e^{-j\omega_c t}$$

$$\updownarrow$$

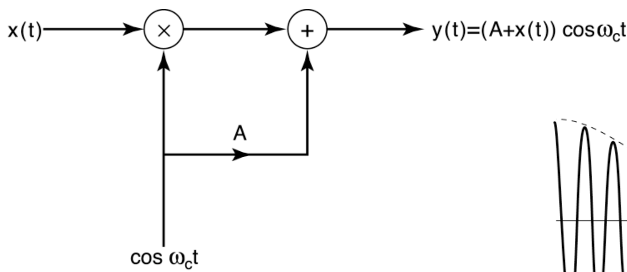
$$\pi e^{j\theta}\delta(\omega - \omega_c) + \pi e^{-j\theta}\delta(\omega + \omega_c)$$

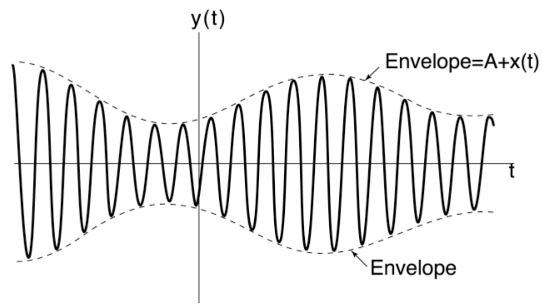


Again, the low-frequency signal ($\omega < \omega_M$) = 0 when $\theta = \pi/2$.

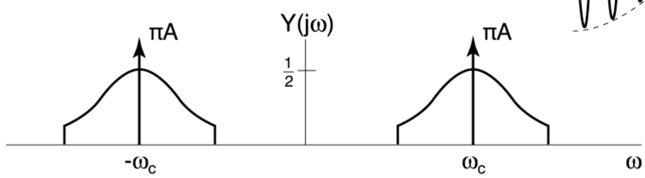
Alternative: Asynchronous Demodulation

- Assume $\omega_c >> \omega_M$, so signal envelope looks like $x(t)$
- Add same carrier





Time Domain



Frequency Domain

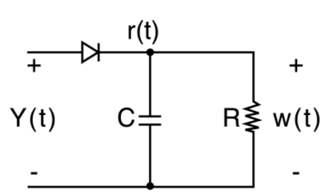
$A = 0 \Rightarrow$ DSB/SC
$A > 0 \Rightarrow$ DSB/WC

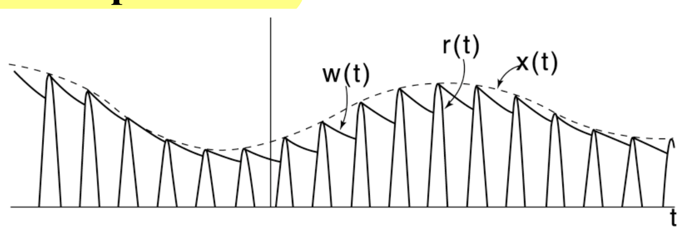
이러한 phase offset 또는 phase drift으로부터 생기는 문제를 해결하기 위해 asynchronous demodulation 기법이 도입되었다. 이를 위해서는 단지 수신기를 바꾸는 것뿐만 아니라 송신기에서도 변경이 있어야 한다. 어떤 변경일까?

DSB: double side band
SC: suppressed carrier

DSB-LC(large carrier)
DSB-TC(transmitted carrier),
DSB-WC(with carrier)

Asynchronous Demodulation (continued)
Envelop Detector





In order for it to function properly, the envelop function must be positive definite, *i.e.* $A + x(t) > 0$.

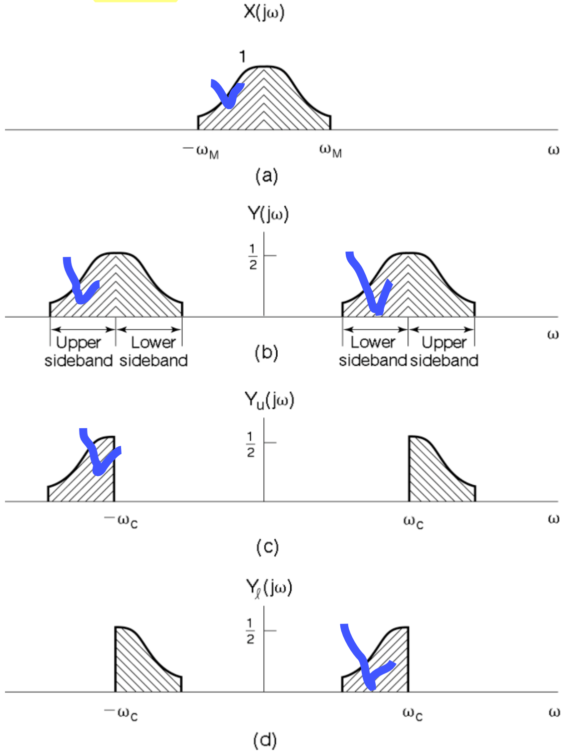
Demo: Envelop detection for asynchronous demodulation.

Advantages of asynchronous demodulation:
— Simpler in design and implementation.

Disadvantages of asynchronous demodulation:
— Requires an extra transmitting power $[A\cos\omega_c t]^2$ to make sure $A + x(t) > 0 \Rightarrow$ Maximum power efficiency = 1/3 (P8.27)

Double-Sideband (DSB) and Single-Sideband (SSB) AM

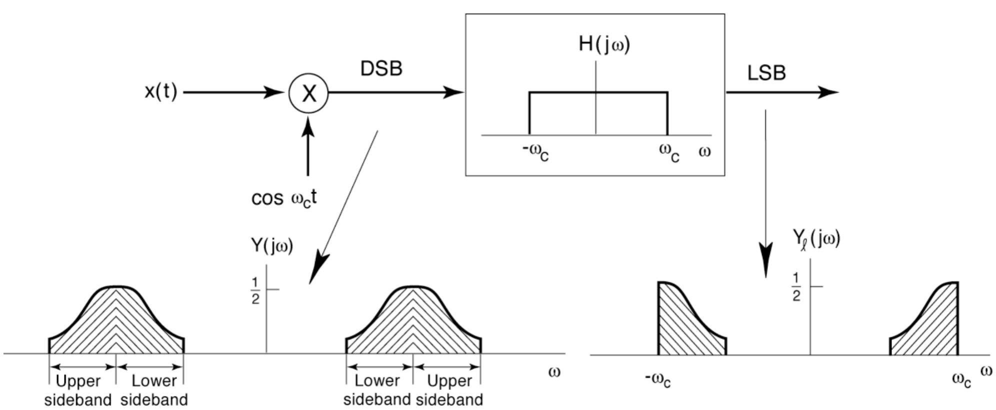
Since $x(t)$ and $y(t)$ are *real*, from conjugate symmetry, both *LSB* and *USB* signals carry exactly the same information.



DSB, occupies **2** ω_M bandwidth in $\omega > 0$, even though *all* the information is contained in ω_M.

Each only occupies ω_M bandwidth in $\omega > 0$.

Single Sideband Modulation

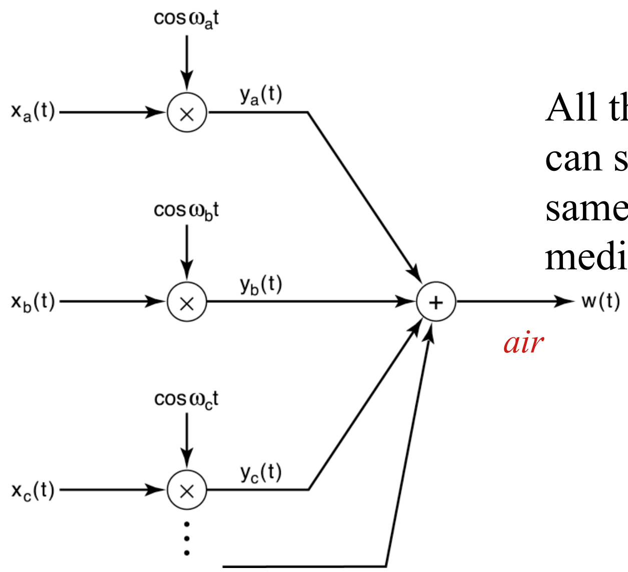


Can also get SSB/SC
or SSB/WC

하지만 현실적으로 이렇게 sharp하게 잘라내는 필터를 만들 수 없다. vestigial side band라는 방법이 해법으로 등장 -> TV신호 전송에 쓰임.

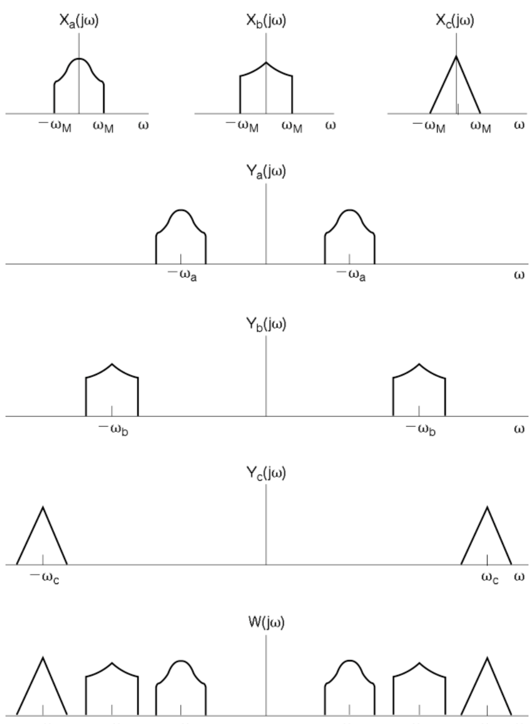
Frequency-Division Multiplexing (FDM)

(Examples: All the radio-station signals and analog cell phones)



All the channels can share the same physical medium.

FDM in the Frequency-Domain



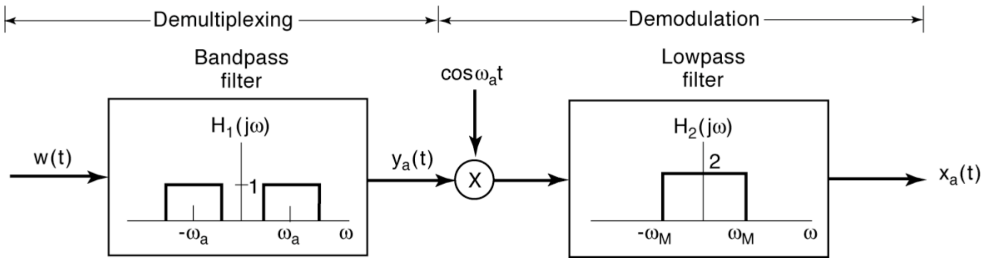
Channel a

Channel b

Channel c

Multiplexed signals

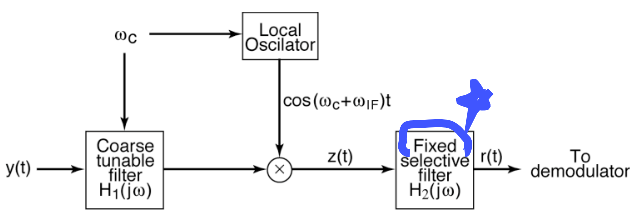
Demultiplexing and Demodulation

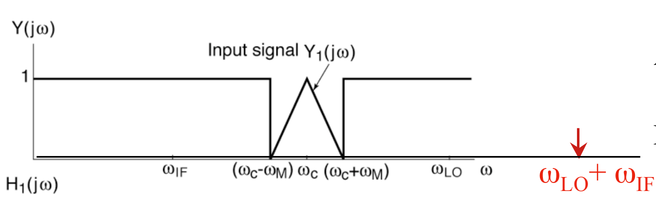


ω_a needs to be tunable

- Channels must not overlap $\Rightarrow$ Bandwidth Allocation.
- It is difficult (and expensive) to design a highly selective bandpass filter with a tunable center frequency (the SAW bandpass filters are fixed).
- Solution — the Superheterodyne Receivers.

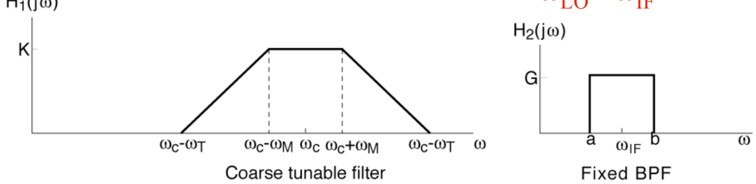
The Superheterodyne Receiver (used in all radios)





AM, $\frac{\omega_c}{2\pi} = 535 - 1605\,kHz$ — RF

FCC: $\frac{\omega_{IF}}{2\pi} = 455\,kHz$ — IF



Operation principle:

— Down convert from ω_c to ω_{IF}, and use a coarse tunable BPF for the front end to block the image signals at $\omega_I = \omega_{LO} + \omega_{IF}$.

— Use a sharp-cutoff *fixed* BPF at ω_{IF} to get rid of other signals.