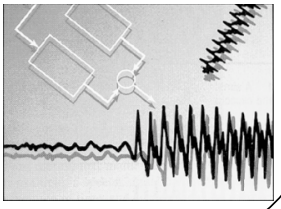
EECE233 Signals and Systems

Lecture Note #2

앞서 lecture note 1에서는 본 과목에서 다루는 signal에 대해 여러가지 방법으로 분류해 보았고
여기 lecture note 2 에서는 본 과목에서 다루는 system에 대해 여러가지로 분류해 본다

- Covers O & W pp. 39-56
- Several examples of systems
- System properties and examples
  a) Cauality
  b) Linearity
  c) Time-Invariance

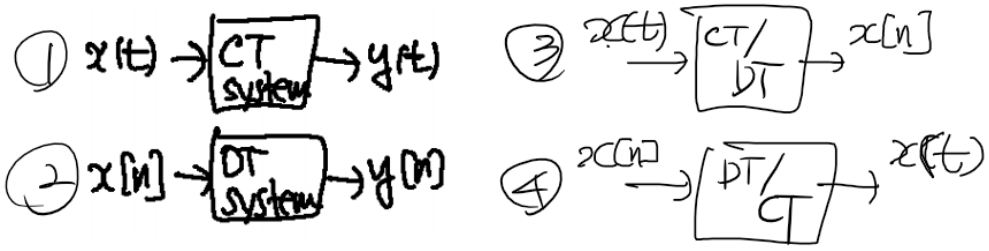


* These lecture notes are based on the lecture notes used in 6.003 taught by Prof. Qing Hu at M.I.T. These notes are adopted for EECE233 under the permission of Prof. Qing Hu.

SYSTEM EXAMPLES

In EECE233, *systems* are described from input/output perspective,that is, input to the system x causes the output y

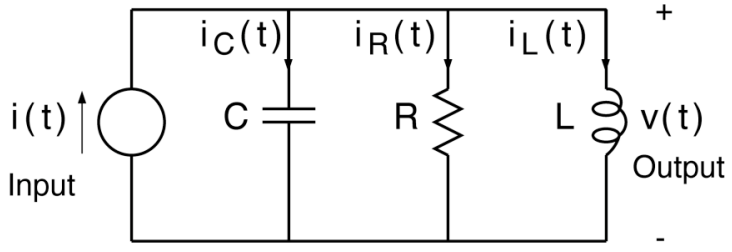
$x(t) \rightarrow$ CT System $\rightarrow y(t)$ $\quad x[n] \rightarrow$ DT System $\rightarrow y[n]$



- A classification of systems in this course:
  - ○ a CT system
  - ○ a DT system
  - ○ a CT/DT converter
  - ○ a DT/CT converter
  - ○ etc.

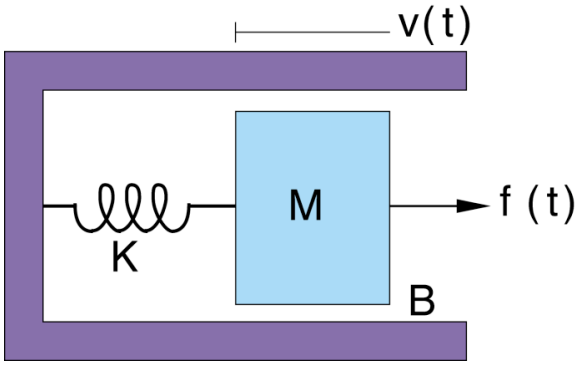
- Description of the mapping rule of a system = Input/Output (I/O) relation
  - How to roughly visualize an I/O relation?
    - ○ a block diagam vs. a graph
  - How to precisely describe an I/O relation?
    - ○ examples
      - § differential equation for CT system
      - § difference equation for DT system

Ex. #1 RLC circuit — an electrical system



$$i(t) = \underbrace{C\frac{dv(t)}{dt}}_{\text{capacitance}} + \underbrace{\frac{v(t)}{R}}_{\text{resistance}} + \underbrace{\frac{1}{L}\int_{-\infty}^{t} v(\tau)d\tau}_{\text{inductance}} .$$

Ex. #2 A shock absorber – a mechanical system

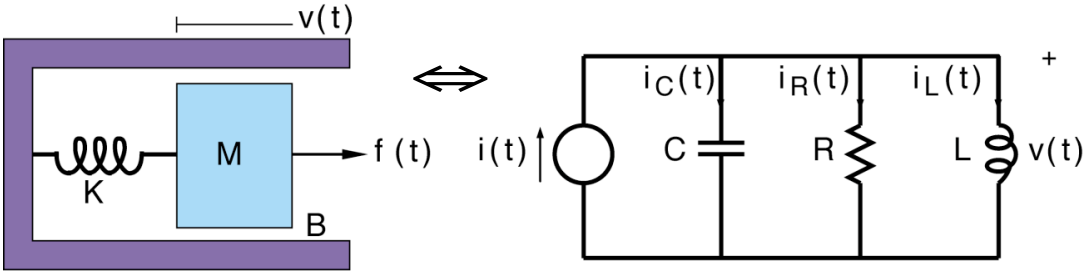


Force Balance: f — input, v — output

$$f(t) = \underbrace{M\frac{dv(t)}{dt}}_{\text{inertial force}} + \underbrace{Bv(t)}_{\text{friction}} + \underbrace{K\int_{-\infty}^{t} v(\tau)d\tau}_{\text{spring force}}.$$

This equation looks quite familiar, we just saw it earlier.

Observation: different systems could be described by the same input/output relations — they are the same as far as EECE233 is concerned



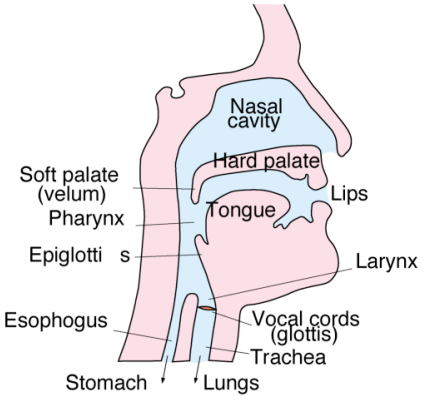
Largely for this reason, EECE233 is the most general among all the EECS subjects, and perhaps the most important.

Example #3 — Human vocal system

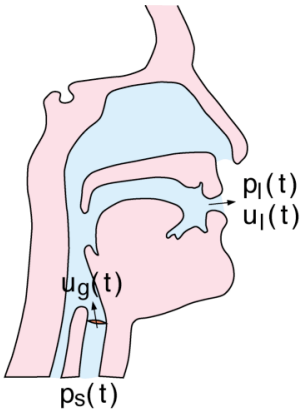
vocal cord: 성대

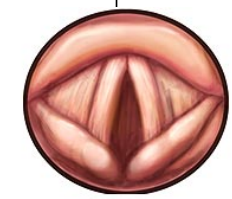
vocal tract: 구강, 혀, 비강, (치아, 입술) 를 묶어

Med. school description



EECE233 version





Vocal cords open during breathing to allow air into lungs.

Vocal cords close when speaking so air from the lungs presses between them to cause the vibrations that produce sound.

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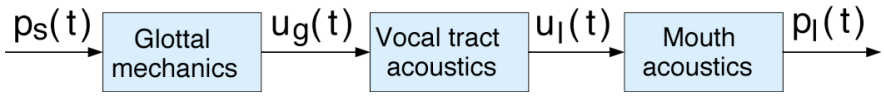
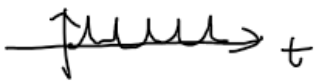




vocal tract가 이 진동적인 공기의 흐름을 여러 가지로 바꾼다.

Q. How to compress a voice signal?

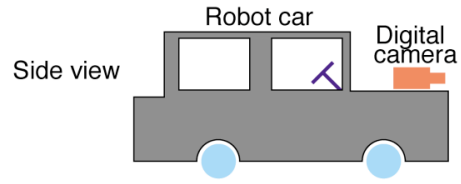
A. LPC linear predictive coding

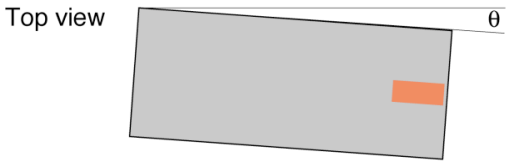
CELP code-excited linear prediction

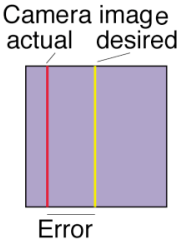
Q-CELP

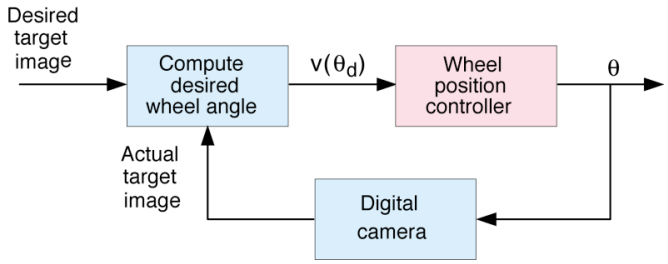
VOCODER

Example #4: A robot car – a close-loop system





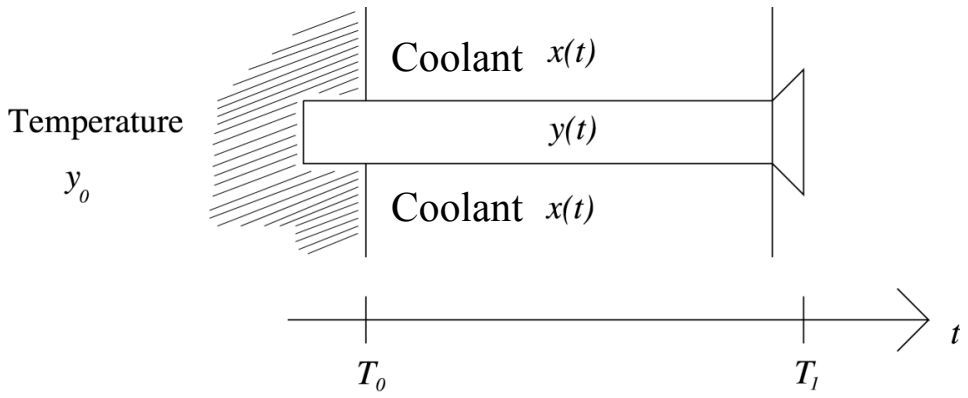




이런 system을 feedback system이라 부름.

Ex. #5 A thermal system

Cooling Fin in Steady State



t = distance along rod

$y(t)$ = Fin temperature as function of position

$x(t)$ = Surrounding temperature along the fin

이 과목에서는 parameter가 시간인 예가 많지만, 이 과목에서 배운 지식은 반드시 시간이 아니더라도 같은 방식으로 모델링 되는 신호와 시스템을 다루는데 이용/확장 가능하다.

Ex. #5 (Continued)

$$\begin{aligned} \frac{d^2y(t)}{dt^2} &= k[y(t) - x(t)] \\ y(T_0) &= y_0 \\ \frac{dy}{dt}(T_1) &= 0 \end{aligned}$$

Observations

- Independent variable can be something other than time, such as space.
- Such systems may, more naturally, have boundary conditions, rather than "initial" conditions.

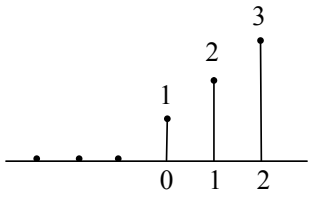
Ex. #6 A DT system

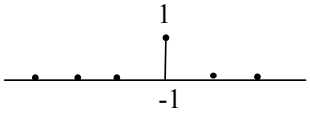
- A rudimentary "edge" detector

$$\begin{aligned} y[n] &= x[n+1] - 2x[n] + x[n-1] \\ &= \{x[n+1] - x[n]\} - \{x[n] - x[n-1]\} \\ &= \text{"Second difference"} \quad \propto \quad \text{"curvature"} \end{aligned}$$

- This system detects changes in signal slope.

(a) $x[n] = n \quad \Rightarrow \quad y[n] = 0$ *for* all n.
(b) $x[n] = nu[n] \quad \Rightarrow \quad y[n] = ?$





(a)의 경우 변하지 않기 때문에 edge 가 없는 것,이 아니라 변하는데 uniform한 rate로 변하기 때문에 edge 가 없는 경우이다.

edge detector는 군사위성이 찍은 사진으로 부터 철도, 도로, 하천 등을 식별하는 데 쓰인다. 또한, 배경화면에서 물체를 분리해 내는 일, 그림file을 압축하는 일 등에 쓰인다.

Observations

1) A very rich class of systems (but by no means all systems of interest to us) are described by differential and difference equations.
2) Such an equation, by itself, does not completely describe the input-output behavior of a system: we need auxiliary conditions (initial conditions, boundary conditions).
3) In some cases the system of interest has time as the natural independent variable and is causal. However, that is not always the case.
4) Very different physical systems may have very similar mathematical descriptions.

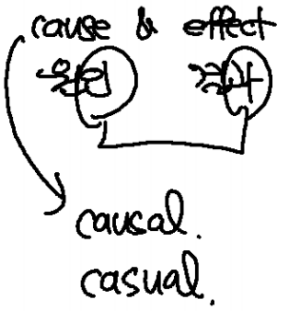
SYSTEM PROPERTIES
(Causality, Linearity, Time-invariance, etc.)

Why bother with such general properties?

- Important practical/physical implications. We can make many important predictions of the system behaviors without having to do any mathematical derivations.
- They allow us to develop powerful tools (*transformations*, more on this later) for analysis and design.

Singal → Signal
Chanel → Channel

Causal
casual



CAUSALITY

- A system is causal if the output does **not** anticipate future values of the input, i.e., if the output at any time depends only on values of the input up to that time.
- **All** real-time physical systems are **causal**, because time only moves forward. Effect occurs after cause. (Imagine if you own a noncausal system whose output depends on tomorrow's stock price.)
- Causality does **not** apply to spatially varying signals. (We can move both left and right, up and down.)
- Causality does **not** apply to systems processing *recorded* signals, e.g. taped sports games vs. live broadcast.

CAUSALITY (continued)

Def. Mathematically (in CT): A system $x(t) \rightarrow y(t)$ is causal

if $x_1(t) \rightarrow y_1(t)$ $x_2(t) \rightarrow y_2(t)$

and $x_1(t) = x_2(t)$ for all $t \leq t_{o,}$

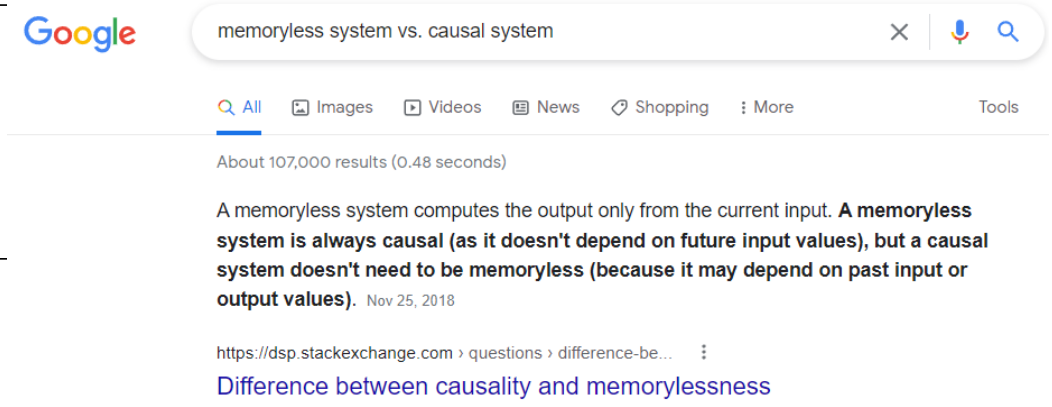
then $y_1(t) = y_2(t)$ for all $t \leq t_o$.

That is, for any t_o, the values of x_1 and x_2 after t_o have no influence on y at $t \leq t_o$ for a causal system.

- More explicit mathematical definition of causality later.

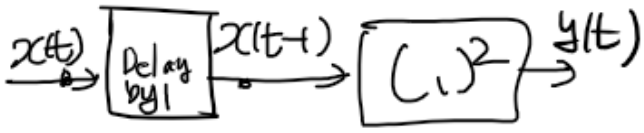
Visualize

Simplify



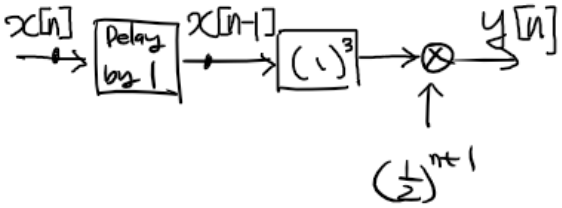
CAUSAL OR NONCAUSAL

(a) $y(t) = x^2(t-1)$



(b) $y(t) = x(t+1)$

(c) $y[n] = x[-n]$



(d) $y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$

LINEAR AND NONLINEAR SYSTEMS

- Many systems are nonlinear.
  Ex. Economic system.
  Input: fiscal and monetary policies, labor, resources, etc.
  → Output: GDP, inflation, etc.
  System behavior is very unpredictable because it is highly nonlinear.

- In EECE233, we will deal with only linear systems, which are good approximations of nonlinear systems in certain ranges.
  Ex. Small-signal conductance of a nonlinear diode.
- Linear systems can be analyzed accurately.

LINEARITY

Def. A (CT) system is linear if it has the superposition property, i.e.,

If $x_1(t) \rightarrow y_1(t)$ and $x_2(t) \rightarrow y_2(t)$,

then $ax_1(t) + bx_2(t) \rightarrow ay_1(t) + by_2(t)$.

Lemma. If a system is linear, then zero input → zero output.

Question: Is the converse true?

"Proof" $x[n]=0 \cdot x_1[n]=0 \rightarrow y[n]= 0 \cdot y_1[n]=0$.

Question: Is the system $y = A + x$ linear?

Recall the contraposition of L => ZIZO.

PROPERTIES OF LINEAR SYSTEMS

Proposition. (superposision of more than 2 signals)

If $x_k[n] \rightarrow y_k[,n]$

then $\sum_k a_k x_k[n] \rightarrow \sum_k a_k y_k[n]$.

This property seems to be almost trivial now, but it is one of the most important ones used in EECE233.

Q How to handle nonlinearity?
A. affine ≃ linear. (0) $y=ax+b$
- linearize. "taylor"
- Volterra series
- Busgang's theorem, etc

Properties of Linear Systems (Continued)

Def. A CT system has the initial rest property if ZI until t_0 implies ZO until t_0, i.e.,

$$x(t) = 0 \quad for\ t \le t_o \quad \rightarrow \quad y(t) = 0 \quad for\ t \le t_o \quad (*).$$

Theorem. A linear system is causal if and only if it satisfies the conditions of initial rest: (both necessary and sufficient)

"Proof"

a) Suppose a CT L system is causal. Show that (*) holds.

b) Suppose a CT L system has (*) hold. Show that the system is causal.

TIME-INVARIANCE (TI)

Informally, a system is time-invariant (TI) if its behavior does not depend on the choice of $t = 0$. Then two identical experiments will yield the same results, regardless the starting time. cf) stationary RP.

Def. Mathematically (in DT): A system $x[n] \rightarrow y[n]$ is TI if for *any* input $x[n]$ and *any* time shift n_0,

If $x[n] \rightarrow y[n]$,
then $x[n - n_0] \rightarrow y[n - n_0]$ for any integer n_0.

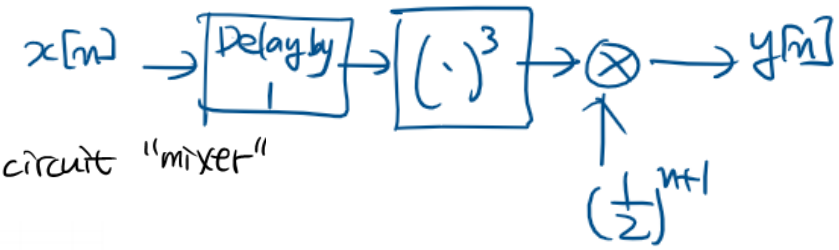
Def. Similarly for CT time-invariant system,

If $x(t) \rightarrow y(t)$,
then $x(t - t_o) \rightarrow y(t - t_o)$ for any real t_0.

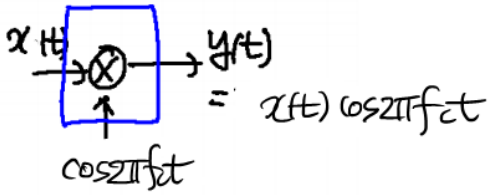
TIME-INVARIANT OR TIME-VARYING ?

$y(t) = x^2(t+1)$

$y[n] = \left(\frac{1}{2}\right)^{n+1} x^3[n-1]$



RF circuit "mixer"



NOW WE CAN DEDUCE SOMETHING!

Theorem. If the input to a TI system is periodic, then the output is also periodic with the same period.

"Proof": Suppose $x(t + T) = x(t)$
and $x(t) \rightarrow y(t)$

Then by TI

$$x(t + T) \rightarrow y(t + T)$$

$$\uparrow \qquad\qquad \uparrow$$

But these are the same input!

So these must be the same output, *i.e.*, $y(t) = y(t+T)$

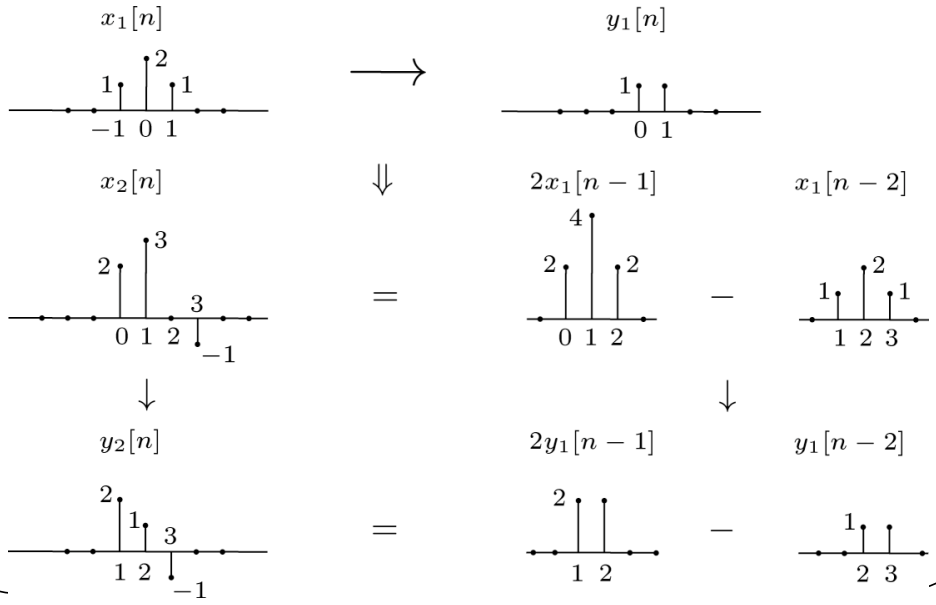
분류를 통해 작은 부분집합을 정의하면
그 부분집합의 원소에 대해 공통되는 성질을 말할 수 있어
이론을 전개해 나갈 수 있게된다.
=> 분류의 힘!!!!

LINEAR TIME-INVARIANT (LTI) SYSTEMS

- Focus of <u>most</u> of this course
  - Practical importance (**Eg.** #1-2 earlier this lecture are both LTI systems.)
  - The powerful analysis tools (*transformation*) associated with LTI systems
- A basic fact: If we know the response of an LTI system to some inputs, we actually know the response to many inputs

Example: DT LTI System

Q: Given x_1[n] -> y_1[n] below, what is y_2[n] when the input is x_2[n]? (Hint. The system is LTI.)



This problem motivates
(i) Representation of an input signal, and
(ii) Representation of an output signal.
-> Representing the input signal in terms of elementary siganls, finding the outputs to the signals, and constructing the output using the outputs of the elementary signals.

cf) eigen-value and eigen-vector of a square matrix?

이상
1. causality
2. linearity
3. time-invariance
를 배웠는데

각각 이분법이므로 이제 8개의 부분집합을 배운것이다.
그런데 이과목에서는 이렇게 8개로 쪼개서 모두 배우는 것이 아니라
linear time-invariant system 이라는 부분집합에 대해서만 주로 배운다.