2025년 5월 20일 화요일 19:52

CM27-250527T

Announcements

- HW28 for CMs 27-28 will be posted on PLMS and due at 14:00 on next Tuesday.
  - Midterm and final exam dates will be counted for the rest 2 CMs. So, we have total 30 CMs.
- Voluntary Project
  - Topic must be approved by the instructor before final exam via Email.
    - Topics related to this course is acceptable.
  - Presentation file and presentation mp4 of length approximately 20 minutes must be submitted.
  - No reuse, double use, etc.
  - Due on Wed. June 11th.
  - If pass, one grade up. For example, F-> D-.
  - No penalty on those who choose not to submit.

Overview

1: Introduction

2: Discrete-Time Signals and Systems

3: The z-Transform

4: Sampling of Continuous- and Discrete-Time Signals

5: Transform Analysis of Linear Time-Invariant Systems

6: Structures for Discrete-Time Systems

7: Filter Design Techniques

8: The Discrete Fourier Transform

9: Computation of the DFT

10: Fourier Analysis of Signals Using DFT

11: Parametric Signal Modeling ← parametric approach

12: Discrete Hilbert Transform

13: Cepstrum Analysis and Homomorphic Deconvolution

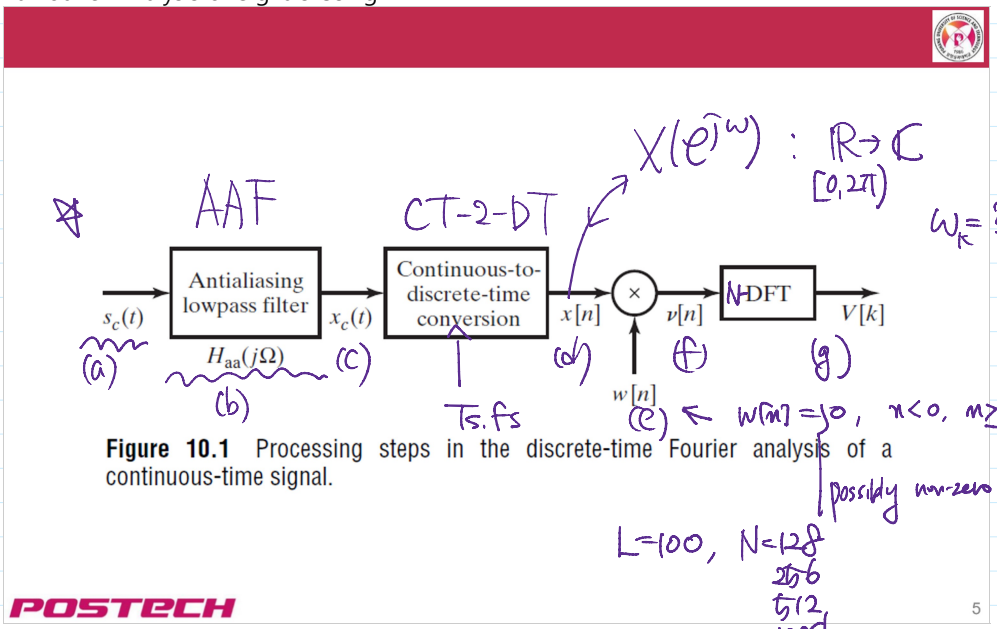
~~14~~: AI/ML and digital signal processing

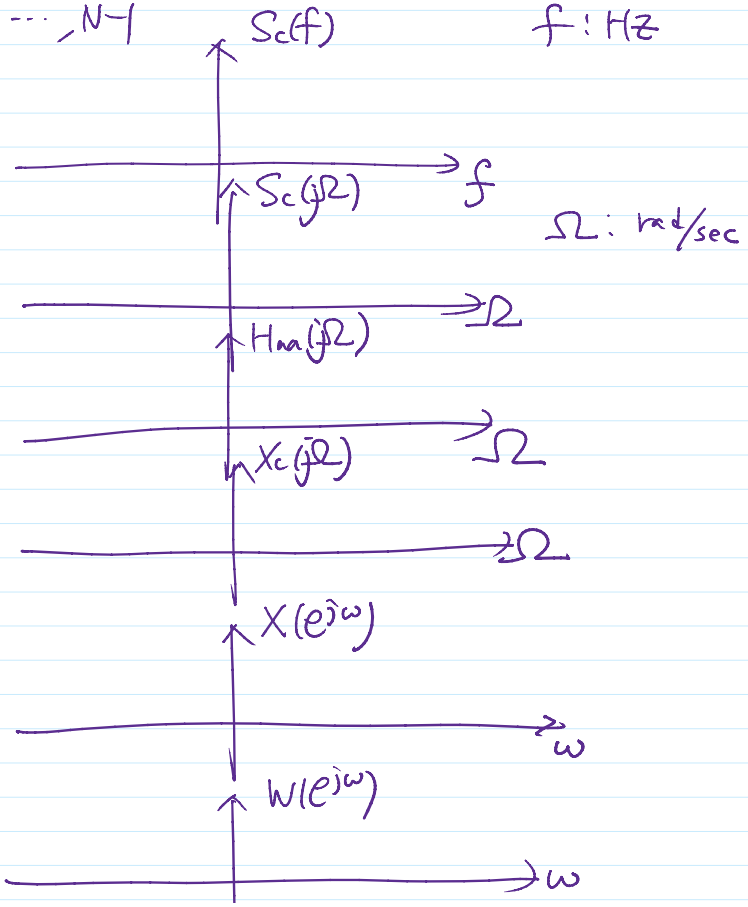
- Messages from Instructor
  - Verbally state mathematical results!
  - Be aware of halluciation of LLMs.
  - Read textbook in order not to become a degraded copy of your professor.
  - Stop and Think!!!

- Something new coming in, then scan your memory to find anything similar, and compare and compress your knowledge.
- Be careful about the difference of a definition from its applications or use cases!!
- 1, 2, ... , many

Review

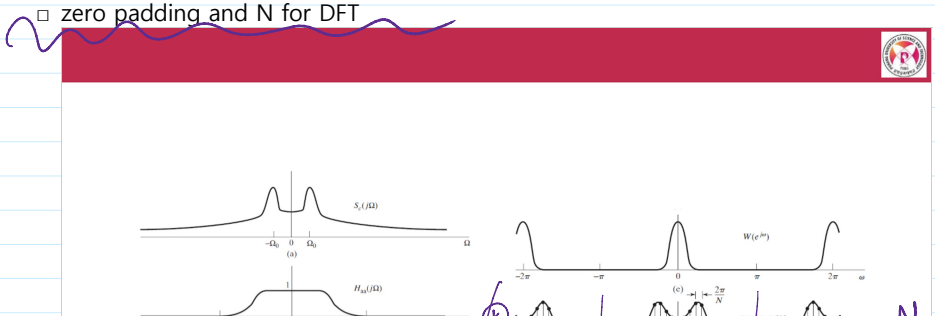
- 10 Fourier Analysis of Signals Using DFT





  - 10.1 Uniform sampling of a CT signal and relation between CTFT and DTFT in terms of the sampling rate
    - CT signal -> AAF -> CT/DT -> rectangular windowing -> DFT -> DFT output
    - CTFT

      CTFT DTFT DTFT DFT
      - f_s and DTFT
      - L and DTFT
      - zero padding and N for DFT



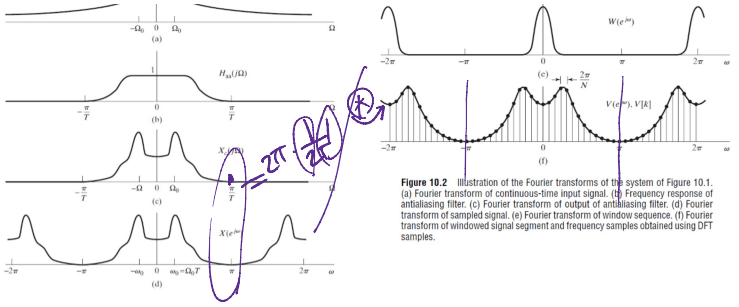
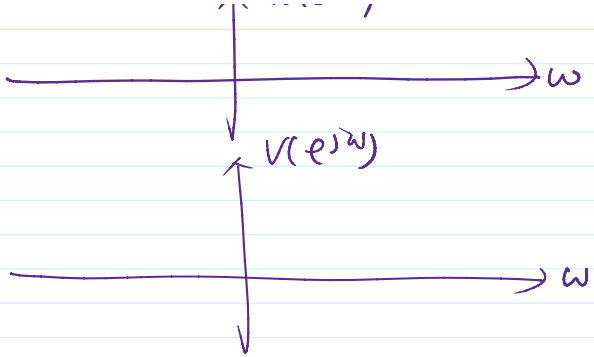
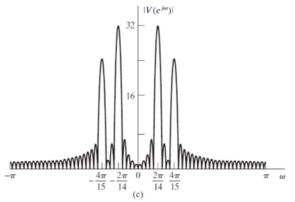


Figure 10.2 Illustration of the Fourier transforms of the system of Figure 10.1. (a) Fourier transform of continuous-time input signal. (b) Frequency response of antialiasing filter. (c) Fourier transform of output of antialiasing filter. (d) Fourier transform of sampled signal. (e) Fourier transform of window sequence. (f) Fourier transform of windowed signal segment and frequency samples obtained using DFT samples.

POSTECH 6



- 10.2 Sinusoidal signals
  - deterministic power signal
    - sinusoidal signal
    - sum of sinusoidal signals
  - rectangular windowing and DTFT, DFT



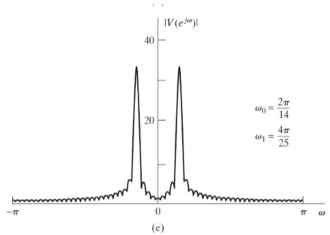
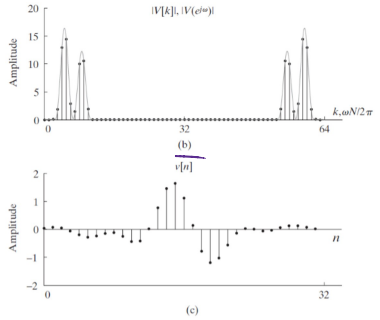


Figure 10.3 (*continued*) (c) $\Omega_0 = (2\pi/14) \times 10^4, \Omega_1 = (4\pi/15) \times 10^4$. (d) $\Omega_0 = (2\pi/14) \times 10^4, \Omega_1 = (2\pi/12) \times 10^4$. (e) $\Omega_0 = (2\pi/14) \times 10^4, \Omega_1 = (4\pi/25) \times 10^4$.

POSTECH 9

  - Kaiser window and effect of L and N



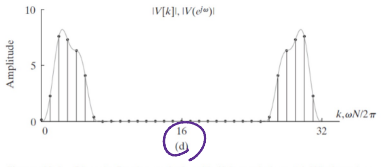


Figure 10.8 Discrete Fourier analysis with Kaiser window. (a) Windowed sequence for $L = 64$. (b) Magnitude of DFT for $L = 64$. (c) Windowed sequence for $L = 32$. (d) Magnitude of DFT for $L = 32$.

Example 10.7 DFT Analysis with 32-point Kaiser Window and Zero-Padding

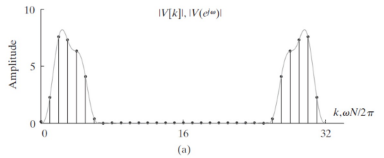


Figure 10.9 Illustration of effect of DFT length for Kaiser window of length $L = 32$. (a) Magnitude of DFT for $N = 32$.

zero-padding, so that the spectrum is sufficiently oversampled

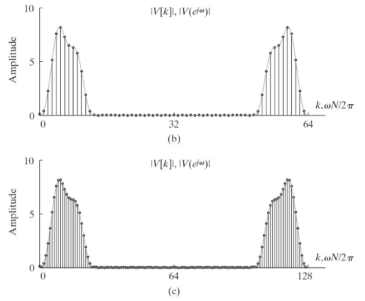
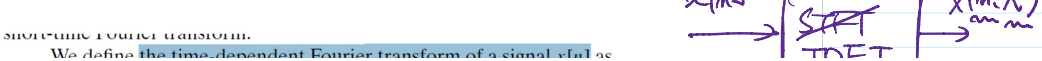


Figure 10.9 (*continued*) (b) Magnitude of DFT for $N = 64$. (c) Magnitude of DFT for $N = 128$.

- 10.3 TDFT or STFT
  - Def.



short-time Fourier transform.

We define the time-dependent Fourier transform of a signal $x[n]$ as

$$X[n,\lambda] = \sum_{m=-\infty}^{\infty} x[n+m]w[m]e^{-j\lambda m}, \qquad (10.18)$$

where $w[n]$ is a window sequence. In the time-dependent Fourier representation, the one-dimensional sequence $x[n]$, a function of a single discrete variable, is converted into a two-dimensional function of the time variable n, which is discrete, and the frequency variable λ, which is continuous.[2] Note that the time-dependent Fourier transform is periodic in λ with period 2π; therefore, we need consider only values of λ for $0 \le \lambda < 2\pi$ or any other interval of length 2π.

the DTFT of the shifted signal $x[n+m]$, as viewed through the window $w[m]$.

$x[n] \to$ STFT TDFT $\to X(n,\lambda)$

$X : \mathbb{Z} \times [0, 2\pi) \to \mathbb{C}$

↑ the Cartesian product

cf) $s \in A \times B$

$s = (a, b)$, $a \in A$, $b \in B$.

- Applications
  - Chirp signal

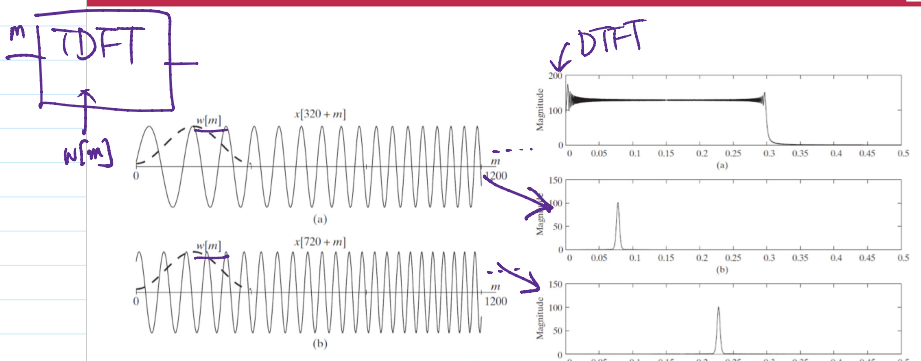


Figure 10.11 Two segments of the linear chirp signal $x[n] = \cos(\alpha_0 n^2)$ for $\alpha_0 = 15\pi \times 10^{-6}$ with a 400-sample Hamming window superimposed. (a) $X[n, \lambda)$ at $n = 320$ would be the DTFT of the top trace multiplied by the window. (b) $X[720, \lambda)$ would be the DTFT of the bottom trace multiplied by the window.

DTFT

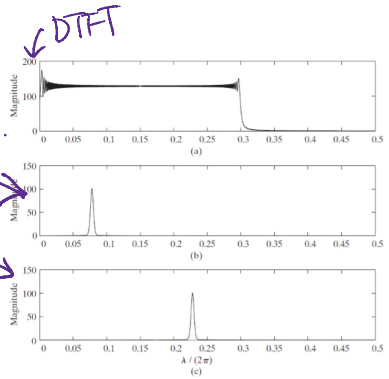
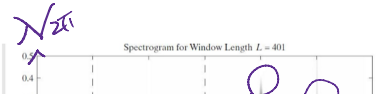


Figure 10.12 DTFTs of segments of a linear chirp signal: (a) DTFT of 20,000 samples of the signal $x[n] = \cos(\alpha_0 n^2)$. (b) DTFT of $x[5000 + m]w[m]$ where $w[m]$ is a Hamming window of length $L = 401$; i.e., $X[5000, \lambda)$. (c) DTFT of $x[15,000 + m]w[m]$ where $w[m]$ is a Hamming window of length $L = 401$; i.e., $X[15,000, \lambda)$.

- Spectrogram

Example 10.10 Plotting $X[n,\lambda)$: The Spectrogram

- This display, which shows 20 log10 |Y [n, λ)| as a function of



- This display, which shows 20 log10 |Y [n, λ)| as a function of λ/2π in the vertical dimension, and the time index *n* in the horizontal dimension is called a *spectrogram*

In Figure 10.13, we show a display as a function of both time index n and frequency $\lambda/(2\pi)$ of the magnitude of the time-dependent Fourier transform, $|Y[n, \lambda)|$, for the signal

$$y[n] = \begin{cases} 0 & n < 0 \\ \cos(\alpha_1 n^2) & 0 \le n \le 20{,}000 \\ \cos(0.2\pi n) & 20{,}000 < n \le 25{,}000 \\ \cos(0.2\pi n) + \cos(0.23\pi n) & 25{,}000 < n. \end{cases} \quad (10.23)$$

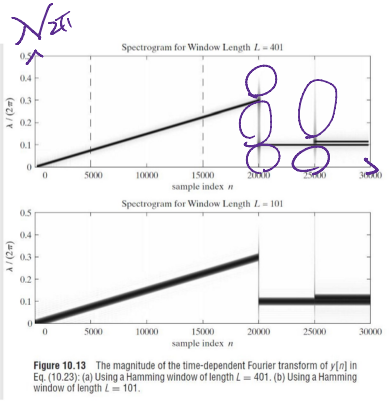


Figure 10.13 The magnitude of the time-dependent Fourier transform of $y[n]$ in Eq. (10.23): (a) Using a Hamming window of length $L = 401$. (b) Using a Hamming window of length $L = 101$.

Preview
- STFT and spectrogram
- PSD estimation
  - parametric vs. non-parametric
    - parametric
      - AR, MA, ARMA, ARIMA, etc.
    - non-parametric
      - periodogram
      - auto-covariance

- 10.3.1 Invertibility of $X[n,\lambda)$
- 10.3.2 Filter Bank Interpretation of $X[n,\lambda)$
- 10.3.3 The Effect of the Window

Since X[n, λ) is the DTFT of x[n + m]w[m], the time-dependent Fourier transform is invertible if the window has at least one nonzero sample.

A rearrangement of the sum in Eq. (10.18) leads to another useful interpretation of the time-dependent Fourier transform. If we make the substitution $m' = n + m$ in Eq. (10.18), then $X[n, \lambda)$ can be written as

$$X[n, \lambda) = \sum_{m'=-\infty}^{\infty} x[m']w[-(n - m')]e^{j\lambda(n-m')}. \tag{10.26}$$

Equation (10.26) can be interpreted as the convolution

$$X[n, \lambda) = x[n] * h_\lambda[n], \tag{10.27a}$$

where

$$h_\lambda[n] = w[-n]e^{j\lambda n}. \tag{10.27b}$$

From Eq. (10.27a), we see that the time-dependent Fourier transform as a function of n with λ fixed can be interpreted as the output of an LTI filter with impulse response $h_\lambda[n]$ or, equivalently, with frequency response

$$H_\lambda(e^{j\omega}) = W(e^{j(\lambda-\omega)}). \tag{10.28}$$

- 10.3.4 Sampling in Time and Frequency

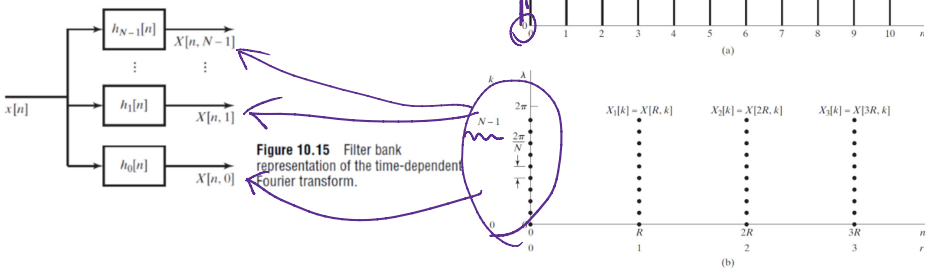


Figure 10.15 Filter bank representation of the time-dependent Fourier transform.

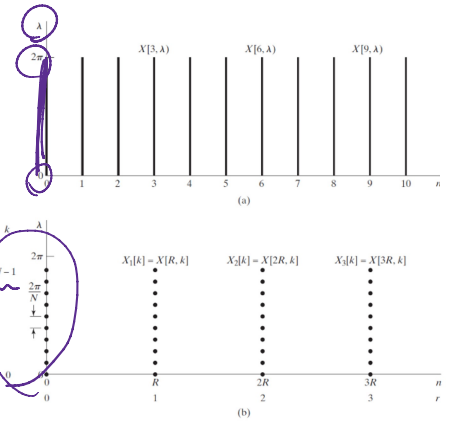


Figure 10.16 (a) Region of support for $X[n, \lambda)$. (b) Grid of sampling points in the $[n, \lambda)$-plane for the sampled time-dependent Fourier transform with $N = 10$ and $R = 3$.

transform sampled in both the frequency and the time dimension. In general, for the region of support of the window as specified in Eq. (10.33), we define this sampled time-dependent Fourier transform as

$$X[rR, k] = X[rR, 2\pi k/N) = \sum_{m=0}^{L-1} x[rR + m]w[m]e^{-j(2\pi/N)km}, \tag{10.39}$$

where r and k are integers such that $-\infty < r < \infty$ and $0 \le k \le N - 1$. To further simplify our notation, we define

10.3.5 The Overlap–Add Method of Reconstruction

While the previous discussion verifies the possibility of theoretically exact reconstruction of the signal from its time- and frequency-sampled time-dependent Fourier transform,
the demonstration proof is not a viable reconstruction algorithm when modifications are made to the time-dependent Fourier transform as is common, for example, in applications such as audio coding and noise reduction.
In these applications division by a tapering window as required in Eq. (10.42) can greatly enhance errors at the edges.
…
time-aliased window sequences
…

10.3.6 Signal Processing Based on the Time-Dependent Fourier Transform

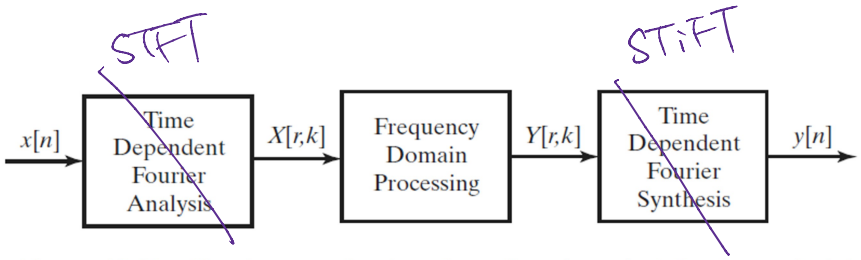


Figure 10.18 Signal processing based on time-dependent Fourier analysis/synthesis.

the sampled time-dependent Fourier transform $X_r[k]$ consists of a set of N-point DFTs of segments of the input sequence

$$x_r[m] = x[rL + m], \qquad 0 \le m \le L - 1. \tag{10.53}$$

Since each sample of the signal $x[n]$ is included, and the blocks do not overlap, it follows that

$$x[n] = \sum_{r=0}^{\infty} x_r[n - rL]. \tag{10.54}$$

that if we construct an output signal

$$y[n] = \sum_{r=0}^{\infty} y_r[n - rL], \tag{10.57}$$

then $y[n]$ is the output of an LTI system with impulse response $h[n]$. The procedure just described corresponds exactly to the *overlap–add* method of block convolution. The overlap–save method discussed in Section 8.7 can also be applied within the framework

- **10.3.7 Filter Bank Interpretation of the Time-Dependent Fourier Transform**

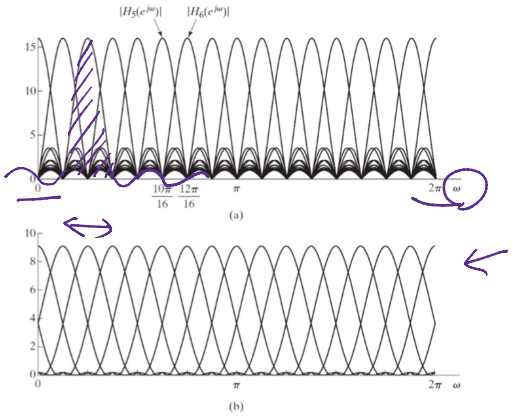


Figure 10.19 Filterbank frequency response. (a) Rectangular window. (b) Kaiser window.

alias cancellation is one of the important concepts
to result from a detailed investigation of the filter bank interpretation.

10.4 EXAMPLES OF FOURIER ANALYSIS OF NONSTATIONARY SIGNALS

RP
- stationary: 'something does not change over time'
- nonstationary

A nonstationary signal is a signal whose properties vary with time …
spectral resolution, time variation, and other issues are the most important.

10.4.1 Time-Dependent Fourier Analysis of Speech Signals

- three basic classes
  - of speech sounds*Voiced sounds* are produced by exciting the vocal tract with quasi-periodic pulses of airflow caused by the opening and closing of the glottis.
  - *Fricative sounds* are produced by forming a constriction somewhere in the vocal tract and forcing air through the constriction so that turbulence is created, thereby producing a noise-like excitation.
  - *Plosive sounds* are produced by completely closing off the vocal tract, building up pressure behind the closure, and then abruptly releasing the pressure
- segment

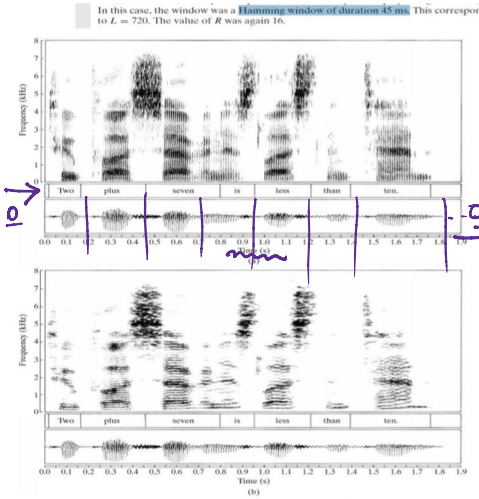


Figure 10.22 (a) Wideband spectrogram of waveform of Figure 10.21. (b) Narrowband spectrogram.

Task
voice-to-text

DTFT.

"Segment."

200msec.

$x(t) \leftrightarrow X(f)$

$x(t-\tau) \leftrightarrow X(f)e^{-j2\pi f\tau}$

10.4.2 Time-Dependent Fourier Analysis of Radar Signals

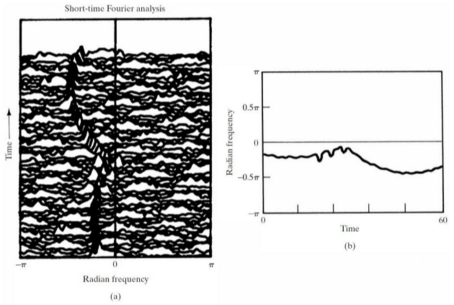


Figure 10.23 Illustration of time-dependent Fourier analysis of Doppler radar signal. (a) Sequence of Fourier transforms of Doppler radar signal. (b) Doppler frequency estimated by picking the largest peak in the time-dependent Fourier transform.

10.4 non-stationary signal

STFT

n

DTFT

DFT

10.5 FOURIER ANALYSIS OF STATIONARY RANDOM SIGNALS: THE PERIODOGRAM

- the estimation of the power spectrum of a random signal using the DFT → Both approximation of the def of PSD.
- there are two basic approaches to estimating the power spectrum.
  - One approach, which we develop in this section, is referred to as periodogram analysis and is based on direct Fourier transformation of finite-length segments of the signal.
  - The second approach, developed in Section 10.6, is to first estimate the autocovariance sequence and then compute the Fourier transform of this estimate.
- In either case, we are typically interested in obtaining unbiased consistent estimators. Unfortunately, the analysis of such estimators is very difficult, and generally, only approximate analyses can be accomplished. Even approximate analyses are beyond the scope of this text.

Defs.

- PSD of CTRP $X(t)$ | - PSD of DTRP $X[n]$

$X(t)$ | $X[n]$

10.5.1 The Periodogram

random signals.) Consequently, a good estimate of $P_{xx}(\omega)$ will provide a useful estimate of $P_{ss}(\Omega)$. The window $w[n]$ in Figure 10.1 selects a finite-length segment (L samples) of $x[n]$, which we denote $v[n]$, the Fourier transform of which is

$$V(e^{j\omega}) = \sum_{n=0}^{L-1} w[n]x[n]e^{-j\omega n}. \tag{10.66}$$

Consider as an estimate of the power spectrum the quantity

$$I(\omega) = \frac{1}{LU}|V(e^{j\omega})|^2, \tag{10.67}$$

where the constant U anticipates a need for normalization to remove bias in the spectrum estimate. When the window $w[n]$ is the rectangular window sequence, this estimator for the power spectrum is called the *periodogram*. If the window is not rectangular,

- **10.5.4 Computation of Average Periodograms Using the DFT**

- Caution!

$x_c(t)$

$x[n] = x_c(nT)$

$V[n] = x[n]\,w[n]$ — $w[n]$: a window function

$$V(e^{j2\pi fT}) = \frac{1}{T}\sum_{k=-\infty}^{\infty} W\left(e^{j2\pi\left(f-\frac{k}{T}\right)T}\right) X_c\left(f-\frac{k}{T}\right)$$

$$V_k = V\left(e^{j2\pi\frac{k}{N}}\right) = V(e^{j2\pi fT})\Big|_{f=\frac{k}{N}\frac{1}{T}}$$

10.6 SPECTRUM ANALYSIS OF RANDOM SIGNALS USING ESTIMATES OF THE AUTOCORRELATION SEQUENCE

- Wiener-Khinchine Theorem

$$S_{xx}(f) = \lim_{\Delta\to\infty}\frac{E\{|V(f)|^2\}}{2\Delta}$$

For a CTRP $X(t)$,

$$S_{xx}(f) = \mathcal{F}_\tau\left[A_t\{R_{xx}(t+\tau,t)\}\right]$$

- Covollary: For a WSS CTRP $X(t)$,

$$S_{xx}(f) = \mathcal{F}\{R_{xx}(\tau)\}$$

Similar for DTRP $X[n]$

Def

$$R_{xx}(t_1,t_2) \triangleq E\{X(t_1)\,X^*(t_2)\}$$

Lemma.

If $X(t)$ is WSS, then $R_{xx}(\tau) = R_{xx}(t+\tau,t)$

- 10.6.1 Computing Correlation and Power Spectrum Estimates Using the DFT
- 10.6.2 Estimating the Power Spectrum of Quantization Noise
- 10.6.3 Estimating the Power Spectrum of Speech

Let us assume, as before, that we are given a finite record of a random signal $x[n]$. This sequence is denoted

$$v[n] = \begin{cases} x[n] & \text{for } 0 \le n \le Q-1, \\ 0 & \text{otherwise.} \end{cases} \tag{10.97}$$

Consider an estimate of the autocorrelation sequence as

$$\hat{\phi}_{xx}[m] = \frac{1}{Q} c_{vv}[m], \tag{10.98a}$$

where, since $c_{vv}[-m] = c_{vv}[m]$,

$$c_{vv}[m] = \sum_{n=0}^{Q-1} v[n]v[n+m] = \begin{cases} \displaystyle\sum_{n=0}^{Q-|m|-1} x[n]x[n+|m|], & |m| \le Q-1, \\ 0 & \text{otherwise,} \end{cases} \tag{10.98b}$$

corresponding to the aperiodic correlation of a rectangularly windowed segment of $x[n]$ of length Q.

To determine the properties of this estimate of the autocorrelation sequence, we consider the mean and variance of the random variable $\hat{\phi}_{xx}[m]$. From Eqs. (10.98a) and (10.98b), it follows that

$$\mathcal{E}\{\hat{\phi}_{xx}[m]\} = \frac{1}{Q} \sum_{n=0}^{Q-|m|-1} \mathcal{E}\{x[n]x[n+|m|]\} = \frac{1}{Q} \sum_{n=0}^{Q-|m|-1} \phi_{xx}[m], \tag{10.99}$$

and since $\phi_{xx}[m]$ does not depend on n for a stationary random process,

$$\mathcal{E}\{\hat{\phi}_{xx}[m]\} = \begin{cases} \left(\dfrac{Q-|m|}{Q}\right) \phi_{xx}[m], & |m| \le Q-1, \\ 0 & \text{otherwise.} \end{cases} \tag{10.100}$$

From Eq. (10.100), we see that $\hat{\phi}_{xx}[m]$ is a biased estimate of $\phi_{xx}[m]$, since $\mathcal{E}\{\hat{\phi}_{xx}[m]\}$ is not equal to $\phi_{xx}[m]$, but the bias is small if $|m| \ll Q$. We see also that an unbiased estimator of the autocorrelation sequence for $|m| \le Q-1$ is

$$\check{\phi}_{xx}[m] = \left(\frac{1}{Q-|m|}\right) c_{vv}[m]; \tag{10.101}$$

i.e., the estimator is unbiased if we divide by the number of nonzero terms in the sum of lagged products involved in computing each value of $c_{vv}[m]$, rather than by the total number of samples in the data record.

values to include when estimating the power spectrum. Thus, we define the power spectrum estimate

$$S(\omega) = \sum_{m=-(M-1)}^{M-1} \hat{\phi}_{xx}[m] w_c[m] e^{-j\omega m}, \tag{10.102}$$

where $w_c[m]$ is a symmetric window of length $(2M-1)$ applied to the estimated autocorrelation function. We require that the product of the autocorrelation sequence and the window be an even sequence when $x[n]$ is real, so that the power spectrum estimate will be a real, even function of ω. Therefore, the correlation window must be an even sequence. By limiting the length of the correlation window so that $M \ll Q$, we include only autocorrelation estimates for which the variance is low.