□ Ann.

- Exam#2 claim within a week
- HW#11 will be assigned soon

□ Review

$X(t) \rightarrow \boxed{T} \rightarrow Y(t)$

mapping rule < deterministic / random

§9-2-1 Memoryless System

$X(t) \rightarrow \boxed{g(\cdot)} \rightarrow Y(t) \triangleq g(X(t))$

□ Full description of $Y(t)$?

$$\begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} g(X_1) \\ g(X_2) \end{bmatrix}$$

(i) $f_{Y(t)}(y) = \cdots$

(ii) $f_{\underbrace{Y(t_1)}_{Y_1},\underbrace{Y(t_2)}_{Y_2}}(y_1, y_2) = \sum_i \dfrac{f_{X(t_1),X(t_2)}(x_{1i}, x_{2i})}{|J(x_{1i}, x_{2i})|}$

$$\det \begin{bmatrix} \frac{dg}{dx_1} & 0 \\ 0 & \frac{dg}{dx_2} \end{bmatrix}$$

$= \sum_i \dfrac{f_{X(t_1),X(t_2)}(x_{1i}, x_{2i})}{|g'(x_{1i})\, g'(x_{2i})|}$

(iii) $f_{Y(t_1),Y(t_2),\cdots,Y(t_n)}(y_1 \cdots$

Page 02

2018년 2월 19일 월요일 오후 7:54

□ Expectation & Moments

$$\mu_{Y(t)} \triangleq E\{Y(t)\} = \int_{-\infty}^{\infty} y\, f_{Y(t)}(y)\, dy$$

$$= E\{g(X(t))\} = \int_{-\infty}^{\infty} g(x)\, f_{X(t)}(x)\, dx.$$

$R_{XX}(t_1,t_2)$

$$R_{XY}(t_1,t_2) = E\{X(t_1)\, Y(t_2)\} = E\{\underbrace{X(t_1)}_{x_1}\, g(\underbrace{X(t_2)}_{x_2})\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x_1\, g(x_2)\, f_{X(t_1)X(t_2)}(x_1,x_2)\, dx_1\, dx_2$$

$R_{YX}(t_1,t_2)$

$R_{YY}(t_1,t_2)$

□

$$\underset{SSS}{X(t)} \longrightarrow \boxed{\text{Memoryless}} \longrightarrow \underset{SSS\ ?}{Y(t)}$$

$$\underline{t}+\tau \triangleq \underline{t}+\tau\underline{1}$$

$\underline{t} \in \mathbb{R}^N$

$$\underline{Y}(\underline{t}) \triangleq \begin{bmatrix} Y(t_1) \\ \vdots \\ Y(t_N) \end{bmatrix} = \begin{bmatrix} g(X(t_1)) \\ \vdots \\ g(X(t_N)) \end{bmatrix} \triangleq \underline{g}(\underline{X}(\underline{t}))$$

$$\Phi_{\underline{Y}(\underline{t})}(\underline{\omega}) = \Phi_{\underline{Y}(\underline{t}+\tau)}(\underline{\omega}),\ \forall \underline{\omega}$$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$E\{e^{j\underline{\omega}^T \underline{Y}(\underline{t})}\} \qquad E\{e^{j\underline{\omega}^T \underline{Y}(\underline{t}+\tau)}\}$$

$$= E\{e^{j\underline{\omega}^T \underline{g}(\underline{X}(\underline{t}))}\} \qquad = E\{e^{j\underline{\omega}^T \underline{g}(\underline{X}(\underline{t}+\tau))}\}$$

$$LHS = \int_{\underline{x}\in\mathbb{R}^N} e^{j\underline{\omega}^T \underline{g}(\underline{x})}\, f_{\underline{X}(\underline{t})}(\underline{x})\, d\underline{x}$$

$$RHS = \int_{\underline{x}\in\mathbb{R}^N} e^{j\underline{\omega}^T \underline{g}(\underline{x})}\, f_{\underline{X}(\underline{t}+\tau)}(\underline{x})\, d\underline{x}$$

Page 03

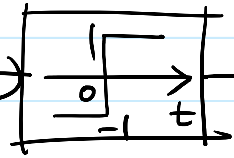
2018년 2월 19일 월요일 오후 7:54

Q. $X(t) \rightarrow$ [Memoryless] $\rightarrow Y(t)$

WSS WSS ?

□ Examples

— square-law device $\quad X(t) \rightarrow \boxed{(\cdot)^2} \rightarrow Y(t) = (X(t))^2$

— hardlimiter $\quad X(t) \rightarrow$  $\rightarrow Y(t) = \underbrace{\text{sgn}}\{X(t)\}$

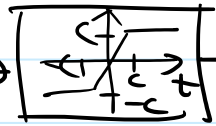
signum

sign

$$E\{Y(t)\} = 1 \times Pr(X(t) > 0) + (-1) \times Pr(X(t) < 0)$$
$$= 1 - Pr(X(t) \le 0) - Pr(X(t) < 0)$$
$$= 1 - 2F_{X(t)}(0).$$

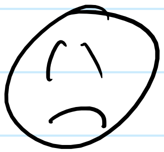
MATLAB

$$\text{sign}(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

— limiter $\quad X(t) \rightarrow$  $\rightarrow Y(t) = \begin{cases} X(t) \\ c, \\ -c, \end{cases}$

§9-2-2 LTI systems

$X(t) \rightarrow \boxed{h(t)} \rightarrow Y(t) = h(t) * X(t)$



X(t) full characterization (O)

Y(t) " " (X)

in general.

2018년 2월 19일 월요일 오후 7:54

□ $X(t)$ is Normal.
$\Rightarrow Y(t)$ is Normal.

$$\underset{\text{Gaussian r.p.}}{X(t)} \longrightarrow \boxed{h(t)} \longrightarrow \underset{\text{Gaussian r.p.}}{Y(t)}$$

$$Y(t) = h(t) * X(t) = \int_{-\infty}^{\infty} h(\tau)\, X(t-\tau)\, d\tau$$
$$\simeq \sum_i h(\tau_i)\, X(t-\tau_i)\, \Delta i$$

□ $$\mu_Y(t) = E\{Y(t)\} = E\{h(t) * X(t)\} = E\left\{\int_{-\infty}^{\infty} h(\tau)\, X(t-\tau)\, d\tau\right\} = \int_{-\infty}^{\infty} h(\tau)\, \mu_X(t-\tau)\, d\tau$$
$$= h(t) * \mu_X(t).$$?!

$$R_{YY}(t_1,t_2) = E\{Y(t_1)\, Y^*(t_2)\} = E[\{h(t_1) * X(t_1)\}\{h(t_2) * X(t_2)\}^*] = h(t_1) * R_{XX}(t_1,t_2) * h(t_2)$$
$$= E\left[\left\{\int_{-\infty}^{\infty} h(\tau_1)\, X(t_1-\tau_1)\, d\tau_1\right\}\left\{\int_{-\infty}^{\infty} h^*(\tau_2)\, X^*(t_2-\tau_2)\, d\tau_2\right\}\right]$$
$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)\, E\{X(t_1-\tau_1)\, X^*(t_2-\tau_2)\}\, h^*(\tau_2)\, d\tau_1 d\tau_2$$
$$= h(t_1) * R_{XX}(t_1,t_2) * h^*(t_2).$$

cf) $(h(t_1) * x(t_1)) \times (h(t_2) * x(t_2))$
$= h(t_1) * (x(t_1)\, x(t_2)) * h(t_2)$.

$$R_{XY}(t_1,t_2) = R_{XX}(t_1,t_2) * h^*(t_2)$$
$$R_{YX}(t_1,t_2) = h(t_1) * R_{XX}(t_1,t_2).$$

Page 05

2018년 2월 19일 월요일 오후 7:54

full cha   full cha.

□ If $X(t)$ is Gaussian w/ $\mu_X(t)$, $R_{XX}(t_1,t_2)$,
then $Y(t)$ "   " w/ $\mu_Y(t) = h(t) * \mu_X(t)$, $R_{YY}(t_1,t_2) = h(t_1) * R_{XX}(t_1,t_2) * h(t_2)$.

□ If $X(t)$ is WSS, $\mu_X(t) = \mu_X, \forall t$. $R_{XX}(t+\tau, t) = R_{XX}(\tau), \forall \tau \forall t$.
then $Y(t)$ is WSS. $\mu_Y(t) = h(t) * \mu_X(t) = \mu_X \int_{-\infty}^{\infty} h(t)\,dt = \mu_X H(0)$

$R_{YY}(t_1,t_2) =$
$R_{YY}(t+\tau,t) = \cdots$ ft of τ.

$$R_{YY}(t_1,t_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1) R_{XX}(t_1-\tau_1, t_2-\tau_2) h(\tau_2)\,d\tau_1 d\tau_2$$ Claim

$$R_{YY}(t+\tau,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1)\underbrace{R_{XX}(t+\tau-\tau_1, t-\tau_2)}_{=R_{XX}(\tau-\tau_1+\tau_2)} h(\tau_2)\,d\tau_1 d\tau_2 = \left(h(\tau) * R_{XX}(\tau)\right) * h(-\tau)$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(\tau_1) R_{XX}(\underbrace{\cancel{t-\tau_1}}_{\tau-\tau_1+\tau_2})\, h(-\tau_2)\, d\tau_1 \,\cancel{* h(-\tau)}\, d\tau_2$$

□ White r.p. $X(t)$ non-WSS $R_{XX}(t_1,t_2) = f(t_1)\,\delta(t_1-t_2)$
  WSS $R_{XX}(\tau) = f\ \delta(\tau)$

2018년 2월 19일 월요일 오후 7:54

§ 9-2-3 Differential Equations.

$$x(t) \rightarrow \boxed{\text{LCCDE}} \rightarrow y(t)$$

$$\sum_{k} a_k y^{(k)}(t) = \sum_{l} b_l x^{(l)}(t)$$

Q1. Is this system LTI?

A1. No. in general.

Yes if initial rest (zero input zero output)

Q2. Any $h(t)$ can be synthesized by an LCCDE?

A2. No, in general.