2023년 4월 11일 화요일 12:50

CM14-230411Tue

Announcements

1. Midterm exam will be taken from 19:00-22:00 on Thursday Apr. 13th at LG102.
2. No class on this Thursday.
3. HW#10 is NOT assigned.
4. Handouts, annotated handouts are available at http://cisl.postech.ac.kr/class/eece574/schedule.htm .
   Passwords are ...
5. Lecture videos are available at cisl.postech YouTube channel. Clickable time stamps, please.

Corrections; Clarifications; Common mistakes,misunderstandings, misconceptions, myth; Frequently Asked Questions; Conventional wisdom

Overview w/ Chapter numbers from the main textbook by Papoulis, (2)-(7) from Peebles

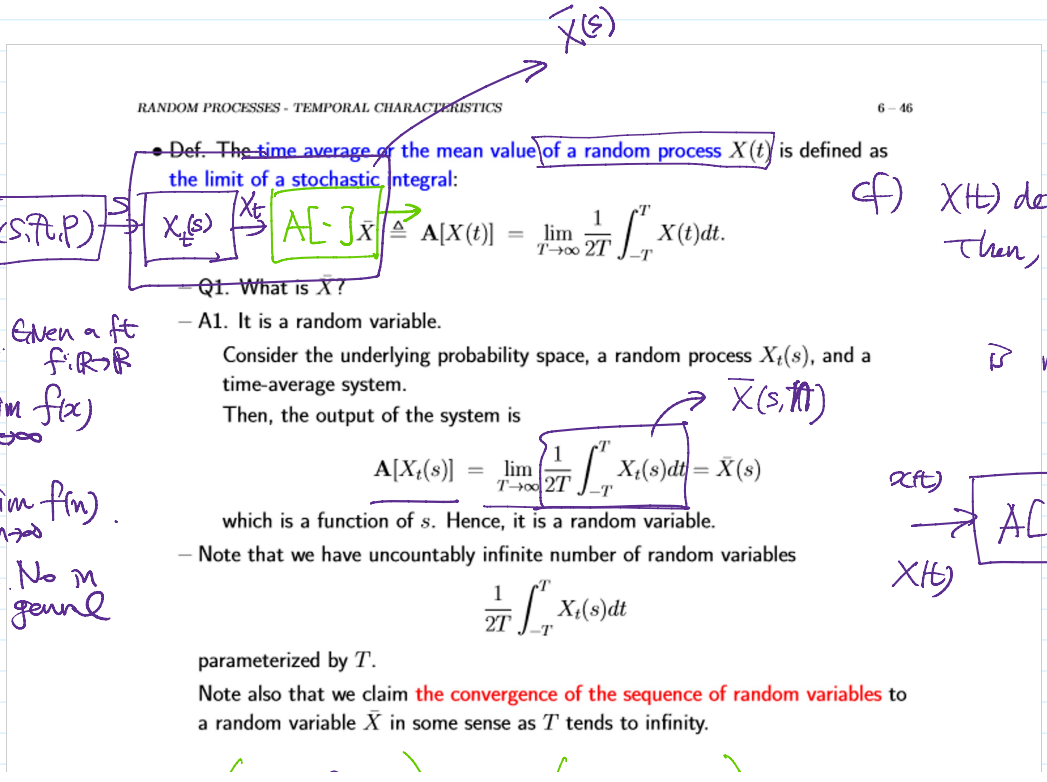
9. General Concepts
   1. RP concept
   (2) Stationarity and Independence
   (5) Gaussian RPs
   (6) Poisson RPs
   (7) Complex RPs
   (2) Time average and Ergodicity
   (3) Correlation functions
   (4) Measurement of correlation functions
7. Sequence of Random Variables
   9.1
   - positive semi-definiteness of a correlation function
   - alpha-dependent random process
   - correlation alpha-dependent random process
   - white noise in terms of correlation function
   - mean-square periodic random process
   - cyclostationary random process

10. Random Walks and Other Applications
11. Spectral Representation
12. Spectrum Estimation
13. Mean-Square Estimation

Review

Preview

1. Time average and ergodicity



RANDOM PROCESSES - TEMPORAL CHARACTERISTICS 6 – 46

• Def. The time average or the mean value of a random process $X(t)$ is defined as the limit of a stochastic integral:

$$\mathbf{A}[X(t)] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X(t)dt.$$

– Q1. What is $\bar{X}$?

– A1. It is a random variable.

Consider the underlying probability space, a random process $X_t(s)$, and a time-average system.

Then, the output of the system is

$$\mathbf{A}[X_t(s)] = \lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}X_t(s)dt = \bar{X}(s)$$

which is a function of s. Hence, it is a random variable.

– Note that we have uncountably infinite number of random variables

$$\frac{1}{2T}\int_{-T}^{T}X_t(s)dt$$

parameterized by T.

Note also that we claim the convergence of the sequence of random variables to a random variable $\bar{X}$ in some sense as T tends to infinity.

Note also that we claim the convergence of the sequence of random variables to a random variable $\bar{X}$ in some sense as T tends to infinity.

Q $0 = \lim_{m\to\infty}\left(\lim_{n\to\infty} f(m,n)\right) = \lim_{n\to\infty}\left(\lim_{m\to\infty} f(m,n)\right)$? = ?

A. $f(m,n) = \frac{m}{m+n}$

– For the convergence of a sequence of random variables, among 4 types/modes of convergence, we here only consider the convergence in the mean-square (MS) sense or the mean-square (MS) convergence defined as

Def. "$X_n \to X$ in the MS sense" $\Leftrightarrow \mathbf{E}[|X_n - X|^2] \to 0$ as $n \to \infty$.

∗ The MS convergence is often denoted by

$$\text{l.i.m.}_{n\to\infty} X_n = X,$$

which is read the limit in the mean of X_n is X.

– Q2. What is $\mathbf{E}[\bar{X}]$?

– A2. It is a number.

Def. $\{x_n\}_{n\in\mathbb{N}}$ converges to x if $\forall \varepsilon > 0$, $\exists N(\varepsilon) \in \mathbb{N}$ s.t. $|x_n - x| \le \varepsilon$, $\forall n \ge N(\varepsilon)$

the limit of this sequence.

Def. (Cauchy).

the set of all sequences of real #s

the set of all convergent seq.

Cauchy sequences

Def. Uniform convergence.

$f_n(x) \to f(x)$

$\int_I \lim_{n\to\infty} f_n(x)\,dx = \int_I f(x)\,dx$?

$\lim_{n\to\infty} \int_I f_n(x)\,dx = \int_I f(x)\,dx$?

a.s. ⇒ in pth mean ($p \ge 1$, $p = 2$) ⇒ in prob ⇒ in distribution

$R_{xx}(\tau) = A\{x(t)\,x(t-\tau)\}$

• Def. The time autocorrelation function of a random process $X(t)$ is defined as

$$\mathcal{R}_{XX}(\tau) \triangleq \mathbf{A}[X(t+\tau)X^*(t)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X(t+\tau)X^*(t)dt.$$

$\lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X_{T,\tau}(s)\,X_T^*(s)\,ds$

T, τ, s

τ, s

$\mathcal{R}_{XX}(\tau)$

– Q1. What is $\mathcal{R}_{XX}(\tau)$?

– A1. It is a random variable parameterized by τ.

Consider the underlying probability space, a random process $X_t(s)$, and a time-average system.

Then, the output of the system is

$$\mathbf{A}[X_{t+\tau}(s)X_t^*(s)] = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} X_{t+\tau}(s)X_t^*(s)dt = \mathcal{R}_{XX,\tau}(s)$$

which is a function of s parameterized by τ.

Hence, it is a random variable parameterized by τ.

– Again note that we have uncountably infinite number of random variables

$$\frac{1}{2T}\int_{-T}^{T} X(t+\tau)X^*(t)dt$$

parameterized by T.

Again note also that we claim the convergence of the sequence of random variables to a random variable $\mathcal{R}_{XX}(\tau)$ in some sense as T tends to infinity.

- Q2. What is $\mathbf{E}[\mathcal{R}_{XX}(\tau)]$?
- A2. It is a function of τ.

- Q. If $X(t)$ is stationary (and the expectation can be brought inside the integrals), then what can we say about
  1. $\mathbf{E}[\bar{X}]$ and
  2. $\mathbf{E}[\mathcal{R}_{XX}(\tau)]$?
  - A1. The statistical/ensemble average of the time average of a random process equals the statistical mean path that is a constant function, i.e.,
  $$\mathbf{E}[\bar{X}] \triangleq \mathbf{E}\{\mathbf{A}[X(t)]\} = \mathbf{A}\{\mathbf{E}[X(t)]\} = \mathbf{E}[X(t)].$$
  - A2. The statistical/ensemble average of the time autocorrelation function equals the autocorrelation function, i.e.,
  $$\mathbf{E}[\mathcal{R}_{XX}(\tau)] \triangleq \mathbf{E}\{\mathbf{A}[X(t+\tau)X^*(t)]\} = \mathbf{A}\{\mathbf{E}[X(t+\tau)X^*(t)]\} = R_{XX}(\tau).$$
  - The stationarity itself does not directly link the time average and statistical average.
  - The equalities above still require the ensemble averages of the time averages to link the time average and statistical average.
  - Example. $P(\{H\}) = 1/2, P(\{T\}) = 1/2$. $X_t(H) = 1, \forall t$, $X_t(T) = -1, \forall t$