CM18-230427Thu

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Announcements

1. Handouts, annotated handouts are available at http://cisl.postech.ac.kr/class/eece574/schedule.htm .
   Passwords are ...
2. Lecture videos are available at cisl.postech YouTube channel. Clickable time stamps, please.
3. HWs will be assigned. Check the PLMS.

Corrections; Clarifications; Common mistakes,misunderstandings, misconceptions, myth; Frequently Asked Questions; Conventional wisdom

1. 1/(4T)?
2. Two sufficient conditions!!

Overview w/ Chapter numbers from the main textbook by Papoulis, (2)-(7) from Peebles

9. General Concepts
   1. RP concept
   (2) Stationarity and Independence
   (5) Gaussian RPs
   (6) Poisson RPs
   (7) Complex RPs
   (2) Time average and Ergodicity
   (3) Correlation functions
   (4) Measurement of correlation functions
7. Sequence of Random Variables
   9.1
   - positive semi-definiteness of a correlation function
   - alpha-dependent random process
   - correlation alpha-dependent random process
   - white noise in terms of correlation function
   - mean-square periodic random process
   - cyclostationary random process

10. Random Walks and Other Applications
11. Spectral Representation
12. Spectrum Estimation
13. Mean-Square Estimation

Review

1. mean/power/autocorrelation/distribution ergodic in the MS sense
2. distribution ergodic
   a. n and s condition

Preview

1. correlation functions
2. measurement of correlation functions

6.3 CORRELATION FUNCTION

- the autocorrelation function
- the cross-correlation function
- other correlation-type functions
- their properties

Autocorrelation Function and Its Properties

- Throughout this section, we assume $X(t)$ and $X[n]$ are real-valued random processes.
- Example. Consider $X(t) = a\cos(2\pi f_0 t + \Theta)$, where Θ is a uniform random variable on $[0, 2\pi)$.
  - $X(t)$ is a zero-mean WSS random process because $\mu_X(t) = 0, \forall t$ and ~~$R_{XX}(t+\tau, t) = (a^2/2)\cos(2\pi f_0 \tau)$.~~
  - $R_{XX}(t_1, t_2)$ has a periodic component.
- Def. The autocorrelation function of a real-valued CT random process $X(t)$ is defined by

$$R_{XX}(t_1, t_2) = \mathbf{E}[X(t_1)X(t_2)].$$

  - It is often written by letting $t_1 = t + \tau$ and $t_2 = t$ with τ a real number as

$$R_{XX}(t+\tau, t) = \mathbf{E}[X(t+\tau)X^*(t)]$$

where t is time and τ is offset.

- Def. A real-valued CT random process is called a second-order process if

$$\mathbf{E}[X^2(t)] < \infty, \forall t.$$

- Remark. Suppose that a real-valued CT random process $X(t)$ is at least WSS.
  - There exists a real-valued single-variable function $R_{XX}(\cdot)$ such that

$$R_{XX}(t-\tau, t) = R_{XX}(\tau).$$

  - $R_{XX}(\tau)$ has even symmetry, i.e.

$$R_{XX}(-\tau) = R_{XX}(\tau).$$

  - $R_{XX}(\tau)$ is bounded by its value at the origin as

$$|R_{XX}(\tau)| \le R_{XX}(0),$$

where the upper bound $R_{XX}(0) = \mathbf{E}[X^2(t)]$ is equal to the mean-squared value called the power in the process.

  - If $X(t)$ has a periodic component, then $R_{XX}(\tau)$ will have a periodic component with the same period.
  - If $\mathbf{E}[X(t)] = 0, \forall t$ and $X(t)$ has no periodic component and is ergodic, then

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = 0.$$

Recall the above example having a periodic component.

  - If $X(t) = \boxed{X} + N(t)$ has $\mathbf{E}[X(t)] = X \neq 0$ and has no periodic component, and $N(t)$ is zero-mean with $R_{NN}(\tau) \to 0$ as $|\tau| \to \infty$, then

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = \bar{X}^2.$$

$$\lim_{|\tau| \to \infty} R_{XX}(\tau) = 0$$

Example: Given that the autocorrelation function is

$$R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$$

for a stationary ergodic process $X(t)$ with no periodic components, find its mean and variance.

(solution)

From Property (4), the mean value is $\mathbf{E}[X(t)] = \bar{X} = \sqrt{25} = -5$.

From Property (4), the mean value is $\mathbf{E}[X(t)] = \bar{X} = \sqrt{25} = \pm 5$.

The variance is

$$\begin{aligned}\sigma_X^2 &= \mathbf{E}[X^2(t)] - (\mathbf{E}[X(t)])^2 \\ &= R_{XX}(0) - \bar{X}^2 \\ &= (25+4) - 25 \\ &= 4.\end{aligned}$$

Example: Let $X(t)$ be a WSS random process with autocorrelation function

$$R_{XX}(\tau) = e^{-a|\tau|}, \quad a > 0$$

Determine the autocorrelation function of $Y(t) = X(t)\cos(\omega_0 t + \Theta)$.

(solution)

$$\begin{aligned}R_{YY}(t, t+\tau) &= \mathbf{E}[Y(t)Y(t+\tau)] \\ &= \frac{1}{2} R_{XX}(\tau)\cos(\omega_0 \tau) \\ &= R_{YY}(\tau)\end{aligned}$$

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Cross-Correlation Function and Its Properties

- The cross-correlation function of two real-valued random processes $X(t)$ and $Y(t)$ is defined by

$$R_{XY}(t, t+\tau) = \mathbf{E}[X(t)Y(t+\tau)]$$