CM21-230509Tue

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Announcements

1. Quiz #2 is taken today.
2. Handouts, annotated handouts are available at http://cisl.postech.ac.kr/class/eece574/schedule.htm . Passwords are ...
3. Lecture videos are available at cisl.postech YouTube channel. Clickable time stamps, please.
4. HWs will be assigned. Check the PLMS.

Corrections; Clarifications; Common mistakes,misunderstandings, misconceptions, myth; Frequently Asked Questions; Conventional wisdom

Overview w/ Chapter numbers from the main textbook by Papoulis, (2)-(7) from Peebles

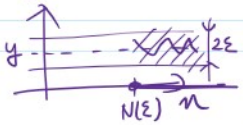
Pa 9.1 Definitions
Pe 6 Random Process--Temporal Characteristics
Pa 7.4 Stochastic Convergence and Limit Theorems
Pa 9.2 Systems with Stochastic Inputs
Pa 9.3 The Power Spectrum
Pe 7 Random Process--Spectral Characteristics
Pe 8 Linear Systems with Random Inputs
Pe 9 Optimal Linear Systems

Review

1. correlation function-related results and definitions
   a. positive semi-definiteness of a correlation function
   b. alpha-dependent random process
   c. correlation alpha-dependent random process
   d. white noise in terms of correlation function
   e. mean-square periodic random process
   f. cyclostationary random process
2. Equality of two real or complex numbers
3. Approximate Equality of two real or complex numbers
   - Bounding
   - Construct a sequence and show it converges.
     i. definition of the limit of a convergent sequence
     ii. definition of a Cauchy sequence

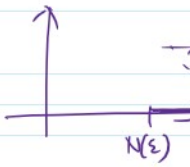
$x=y \Leftrightarrow x-y=0 \Leftrightarrow \forall \varepsilon>0,\ |x-y|\le\varepsilon$

$x\simeq y \quad (x_n)_n \quad x_n\to y$



Def. $\forall \varepsilon>0,\ \exists N(\varepsilon)\in\mathbb{N},\ n\ge N(\varepsilon)\Rightarrow |x_n-y|\le\varepsilon$

Def. A sequence of real numbers $(x_n)_{n\in\mathbb{N}}$ is Cauchy if



$\forall \varepsilon>0,\ \exists N(\varepsilon)\in\mathbb{N},\ m,n\ge N(\varepsilon)\Rightarrow |x_m-x_n|\le\varepsilon$

$x_n=\text{floor}(\sqrt{2}\,10^n)\times 10^{-n}$

$x_1=1.4$
$x_2=1.41$
$\vdots$
$x_n\to\sqrt{2}$

(ii) $x_n=\sqrt{n}$

$x_n-x_{n+1}\to 0$

Preview

1. two real- or complex-valued functions
   a. Equality
   b. Approximate Equality
      - Construct a sequence and show it converges.
      - 5 modes of convergence
2. two random variables
   a. Equality
   b. Approximate Equality
      - Construct a sequence and show it converges.
      - 4 modes of convergence

SEQUENCES OF RANDOM VARIABLES 7 – 46

Equality vs. Approximate Equality of Two Real Functions

The graph of X: $\{(s, X(s)) : s\in D\}$

$\{(s, Y(s)) : s\in D\}$

- Equality of Two Real Functions
  - Given two real functions $X:\mathbb{R}\to\mathbb{R}$ and $Y:\mathbb{R}\to\mathbb{R}$, we have $X=Y$ xor $X\neq Y$ on $\mathcal{D}\subset\mathbb{R}$.
  - Def. (everywhere equality) Given two real functions $X:\mathbb{R}\to\mathbb{R}$ and $Y:\mathbb{R}\to\mathbb{R}$, they are equal everywhere (e) on $\mathcal{D}\subset\mathbb{R}$ if

$$X(s)=Y(s), \forall s\in\mathcal{D}.$$

  * The equality of two functions involves the equality of two real numbers $X(s)$ and $Y(s)$ for each $s\in\mathcal{D}$.
  * Or it involves the equality of a real number $X(s)-Y(s)$ to zero for each $s\in\mathcal{D}$.
  * $\mathcal{D}$ could be $\mathbb{R}$ or an uncountable set. Then, the equality of two functions

Def. $X:\mathcal{D}\to\mathcal{R}_1$ & $Y:\mathcal{D}\to\mathcal{R}_2$

Recall

$f: \mathbb{R}\to\mathbb{R}$

$\mathbb{N}$ $\mathbb{Q}$

name domain range

and $Y(s)$ for each $s \in \mathcal{D}$.

* Or it involves the equality of a real number $X(s) - Y(s)$ to zero for each $s \in \mathcal{D}$.
* $\mathcal{D}$ could be $\mathbb{R}$ or an uncountable set. Then, the equality of two functions involves uncountably infinite number of equalities of two real numbers.
* If there exists any single $s_o \in \mathcal{D}$ such that $X(s_0) \neq Y(s_0)$, then $X \neq Y$ everywhere on $\mathcal{D}$.
* $X(s) = Y(s)$ can be replaced by $X(s) - Y(s) \leq 0, X(s) - Y(s) \geq 0$.
* $X(s) = Y(s)$ can be replaced by $|X(s) - Y(s)| \leq \epsilon, \forall \epsilon > 0$.

Def. $X: \Omega \to \mathcal{R}_1$ & $Y: \Omega \to \mathcal{R}_2$ s.t $\Omega \subset \mathbb{R}$, $\mathcal{R}_1, \mathcal{R}_2 \subset \mathbb{R}$ are equal if $\forall s \in \Omega$, $X(s) = Y(s)$.

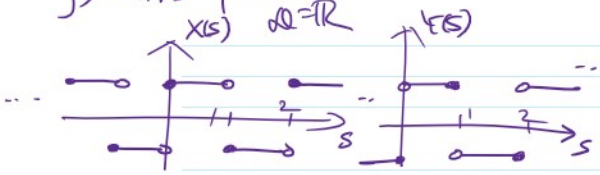
infinite # of equalities b/w two real #s. a single equality b/w two sets.

* $X(s) = Y(s), \forall s \in \mathcal{D}$ can be replaced by $\{s \in \mathcal{D} : X(s) = Y(s)\} = \mathcal{D}$.
* Lemma. Two functions $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$ are equal everywhere iff

$$\{s \in \mathcal{D} : X(s) \neq Y(s)\} = \emptyset.$$

* iff $\{s \in \mathcal{D} : \forall \epsilon > 0, |X(s) - Y(s)| \leq \epsilon\} = \mathcal{D}$.
* This definition of 'equal everywhere' is too strong. Thus, people have introduced a weaker notion of the equality of two functions.

cf) Gibbs' phenomenon



$\{s \in \Omega : X(s) \neq Y(s)\} = \emptyset$

$m(\quad) = 0$

Introduction to real analysis 해석학
Measure Theory
Probability Theory I, II

a decent Ph.D. in ← control, sig. proc, commun.

- Def. (almost everywhere equality) Given two functions $X : \Omega \to \mathbb{R}$ and $Y : \Omega \to \mathbb{R}$, they are equal almost everywhere (a.e.) if

$$m(\{s \in \mathcal{D} : X(s) \neq Y(s)\}) = 0, \iff m(\{s \in \mathcal{D} : \forall \varepsilon > 0, |X(s) - Y(s)| > \varepsilon\}) = 0$$

where $m(\cdot)$ is the Borel measure.

  - The Borel measure $m(\cdot)$ gives the size of a real set.
    * $m([0, 1]) = 1$
    * $m([0, 1] \backslash \mathbb{Q}) = 1$

    Thus, $X = Y$ a.e., iff $X(s) \neq Y(s)$ only on a set of measure zero.
  - When $m(\mathcal{D}) < \infty$ a.e. equal iff $m(\mathcal{D}) = m(\{s \in \mathcal{D} : X(s) = Y(s)\})$.

    Lemma. Two functions $X : \mathbb{R} \to \mathbb{R}$ and $Y : \mathbb{R} \to \mathbb{R}$ are equal almost everywhere (a.e.) iff

$$\forall \varepsilon > 0, m(\{s \in \mathcal{D} : |X(s) - Y(s)| > \varepsilon\}) = 0.$$

    * When $m(\mathcal{D}) < \infty$ a.e. equal iff $\forall \varepsilon > 0, m(\{s \in \mathcal{D} : |X(s) - Y(s)| \leq \varepsilon\}) = m(\mathcal{D})$.
  - Lemma. Two functions $X : \mathbb{R} \to \mathbb{R}$ and $Y : \mathbb{R} \to \mathbb{R}$ are equal almost everywhere (a.e.) iff

$$\int_{-\infty}^{\infty} |X(s) - Y(s)|^p ds = 0$$

    for some $p \geq 1$, where the integral is the Lebesgue integral.

← many sets parameterized by ε.