Digitally Modulated Signals and Their PSDs: OQPSK, MSK, and DS-CDMA

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- Review. A digitally modulated transmitted signal $X(t)$ is assumed a **real-valued bandpass** random process.
  - PSD of complex envelope $X_l(t)$ and PSD of $X(t)$
  - ASK, OOK, M-ASK
  - PSK, BPSK, M-PSK
  - PSD of ASK and PSK with i.i.d. symbols and uniform constellation points
  - PSDs of PAM and QAM signals with WSS symbol sequences

- Preview.
  - Offset QPSK (OQPSK) and Minimum Shift Keying (MSK)
  - PSDs of OQPSK and MSK
  - DS-CDMA (direct-sequence code-division multiple-access) unlink Tx signal and Its PSD

- Offset QPSK (OQPSK) and Minimum Shift Keying (MSK)
  - Def. OQPSK = Staggered QPSK (SQPSK)

$$X(t) = \left(\sum_{m=-\infty}^{\infty} a_I[m] p_T(t - mT) \right) \cos(2\pi f_c t) - \left(\sum_{n=-\infty}^{\infty} a_Q[n] p_T(t - nT - T/2) \right) \sin(2\pi f_c t)$$

  - Def. MSK is a pulse-shaped OQPSK

$$X(t) = \left(\sum_{m=-\infty}^{\infty} a_I[m] p(t - mT) \right) \cos(2\pi f_c t) - \left(\sum_{n=-\infty}^{\infty} a_Q[n] p(t - nT - T/2) \right) \sin(2\pi f_c t)$$

    where the transmit pulse is the **(half-)sine pulse** given by

$$p(t) = \begin{cases} \sin\left(\frac{\pi t}{T}\right), & \text{for } 0 \le t < T \\ 0, & \text{elsewhere.} \end{cases}$$

- PSDs
  - Q. Find the PSDs of OQPSK and MSK signals under the assumptions
    * that $\phi_{II}[l] = \phi_{QQ}[l] = \delta[l]$ and
    * that the in-phase bit sequence and the quadrature bit sequence are uncorrelated.
  - A.
    * Similar to QPSK with real-valued Tx pulse, $X(t)$ can be viewed as a summation of two PAM signals.
    * Recall that phase rotation and delay do not affect the PSD.

    * So, the PSD of the complex envelope of OQPSK is given by $S_{X_l X_l}(f) \propto \mathrm{sinc}^2(fT)$.
    * Similarly, that of MSK is given by $S_{X_l X_l}(f) \propto \left\{\cos(2\pi fT)/(1-16f^2T^2)\right\}^2$.

    * Fig.

- Why MSK over QPSK, OQPSK?
  - The PSD of MSK has a 1.5 times bigger main lobe,
  - but it has less and twice faster-decaying sidelobes than QPSK and OQPSK.
  - Claim. The MSK signal is **a constant-envelope continuous- phase signal**.
    * Thus, it can be power-efficiently amplified.
    * If a Tx signal is of constant envelope, then it has 0 dB PMEPR.
    * Ideally, QPSK (more generally PSK) and OQPSK are all constant envelope signals. However, realistically, they are not because the phase discontinuity leads to time-varying amplitude.
    * Figs.

    * Phase trajectory. The phase of QPSK jumps at every T by $0, \pi, \pm\pi/2$, while that of OQPSK jumps at every $T/2$ by ...IQIQIQ...

– Proof.

* Consider the interval $0 \le t < T/2$. On this interval, it can be shown that

$$\begin{aligned} X_l(t) &= a_I[0]\sin(\pi t/T) + \mathrm{j}a_Q[-1]\cos(\pi t/T) \\ &= a_I[0]\frac{e^{\mathrm{j}\frac{\pi t}{T}} - e^{-\mathrm{j}\frac{\pi t}{T}}}{2\mathrm{j}} + \mathrm{j}a_Q[-1]\frac{e^{\mathrm{j}\frac{\pi t}{T}} + e^{-\mathrm{j}\frac{\pi t}{T}}}{2} \\ &= \frac{-\mathrm{j}}{2}\left[(a_I[0] - a_Q[-1])e^{\mathrm{j}\frac{\pi t}{T}} - (a_I[0] + a_Q[-1])e^{-\mathrm{j}\frac{\pi t}{T}}\right] \end{aligned}$$

  Thus, $X_l(t)$ is $\pm\mathrm{j}e^{\pm\mathrm{j}\frac{\pi t}{T}}$ on $t \in [nT, nT + T/2), \forall n \in \mathbb{Z}$.

* Similarly, $X_l(t)$ is $\pm e^{\pm\mathrm{j}\frac{\pi t}{T}}$ on $t \in [nT + T/2, (n+1)T), \forall n \in \mathbb{Z}$.

* Thus, $|X_l(t)| = 1, \forall t$. Moreover, $X_l(t)$ is continuous at every $nT/2$.

* Therefore, the conclusion follows.

- Direct-Sequence Spread-Spectrum (DS/SS) Modulation Scheme
  - Def. **Spread-spectrum (SS)** technique is a class of modulation methods that
    * utilizes much larger bandwidth than information transmission rate
    * to attain interference immunity and low probability of interception (LPI).

      · direct-sequence SS (DS/SS, DS-SS, DSSS),
      · frequency-hopping SS (FH/SS, FH-SS, FHSS),
      · time-hopping SS (TH/SS),
      · chirp SS (CSS),
      · hybrids of some of above, etc.

– Def. **DS/SSMA** (direct-sequence spread-spectrum multiple-access) is a multiple-access scheme where multiple users transmit DS/SS signals to a receiver.

  * More often than not, it is called **DS-CDMA** (direct-sequence code-division multiple-access) or in short **CDMA**.
  * IS-95 or cdmaOne of 2.5G, and cdma2000 and WCDMA of 3G, all employ DS/SSMA for their uplinks.
  * Particularly in cdmaOne,
    · Each uplink voice user generates a DS/SS modulated signal of 1.25 MHz bandwidth to transmit around 10 kbps error-control coded vocoder output.
    · Note that $1.25M/10k \approx 125$.
    · The DS/SS signals share the uplink frequency band of 1.25 MHz bandwidth. So, the signals fully overlap in the frequency domain.

• Q. Find the PSD of a DS/SS modulated signal transmitted by a cdmaOne or WCDMA mobile station. What determines the PSD?

– Def. The complex envelope of a **DS/SS signal** is given by

$$X_l(t) = \sum_{n=-\infty}^{\infty} d\left[\lfloor \frac{n}{N} \rfloor\right] s[n] p(t - nT_c)$$

where

* $(d[m])_m$ is a data sequence, and $(s[n])_n$ is called a spreading, signature, or chip sequence/**code**,
  · BPSK data symbol, QPSK data symbol
  · BPSK spreading = BPSK chip symbol
  · QPSK spreading = QPSK chip symbol

* $\lfloor \cdot \rfloor$ denotes flooring operation. Thus, N chip symbols are multiplied by a data symbol.
* N is called the processing gain, spreading gain, or **spreading factor**.
* $p(t)$ is called the chip pulse or the **chip waveform**,
* $1/T_c$ is called the chip or **chipping rate**, and $1/T \triangleq 1/(NT_c)$ is the symbol rate.

- Chip sequence $(s[n])_n$
  - In practice, a deterministic periodic signal is used that is known both to the Tx and the Rx.
    - The period can be equal to ...
  - In particular, a pseudo-random sequence or **pseudo-noise (PN) code** with very large period is used.
    - In cdmaOne, a maximal-length sequence (**m-sequence**) of period $\gg N$ is used that is easy to generate using ...
    - All uplink users employ a same m-sequence but have different **code phase**s.
  - A PN code can be well approximated by a **random sequence** where $s[n]$'s are assumed i.i.d. with
    - $\Pr(s[n] = 1) = \Pr(s[n] = -1) = 1/2$ for BPSK spreading and
    - ...for QPSK spreading.
  - In theory, a random sequence is often used that is known both to the Tx and the Rx.
  - We also adopt the random sequence approximation in order to derive the PSD of a DS/SS signal.

- A.
  - Under the assumptions
    - that $(s[n])_n$ is a random sequence with $\mathbb{E}\{s[m+n]s^*[m]\} = \delta[n], \forall m, \forall n,$
    - and that $(d[m])_m$ is a unit-power WSS random process which is independent of $(s[n])_n$,
  - it can be shown that the sequence $(\hat{d}[m])_m$ defined by

$$\hat{d}[m] \triangleq d\left[\lfloor \frac{m}{N} \rfloor\right] s[m]$$

    satisfies $\mathbb{E}\{\hat{d}[m+n]\hat{d}^*[m]\} = ... = \delta[n]$.
  - Note that $X_l(t)$ **now can be viewed as a QAM signal**

$$X_l(t) = \sum_{m=-\infty}^{\infty} \hat{d}[m] p(t - mT_c)$$

    with a white data sequence $(\hat{d}[m])_m$**.**
  - Thus, the PSD of $X_l(t)$ becomes $S_{X_l X_l}(f) = \frac{1}{T_c}|P(f)|^2$.
  - Therefore, the chip waveform determines the PSD along with the Tx power.

- Q. Why is N called the spreading factor?
- A. (Special Case) If a rectangular chip pulse is used, i.e., $p(t) = p_{T_c}(t)$, then $X_l(t)$ can be factored as

  $$X_l(t) = \left(\sum_{m=-\infty}^{\infty} d[m] p_{NT_c}(t - mNT_c) \right) \cdot \left(\sum_{n=-\infty}^{\infty} s[n] p_{T_c}(t - nT_c) \right)$$

  where

  * the data signal is relatively a narrowband signal, while
  * the spreading signal is relatively a wideband signal.
  * Their product is a wideband signal.
  * The bandwidth of their product is N times that of the data signal under the random sequence assumption on $(s[n])_n$.

- (BPSK data + QPSK chip) vs. (QPSK data + QPSK chip)
- (QPSK data + QPSK chip) $\approx$ 2 (BPSK data + BPSK chip)
- cf. OQPSK spreading = (BPSK data + BPSK chip) + (BPSK data + BPSK chip) with offset $T_c/2$